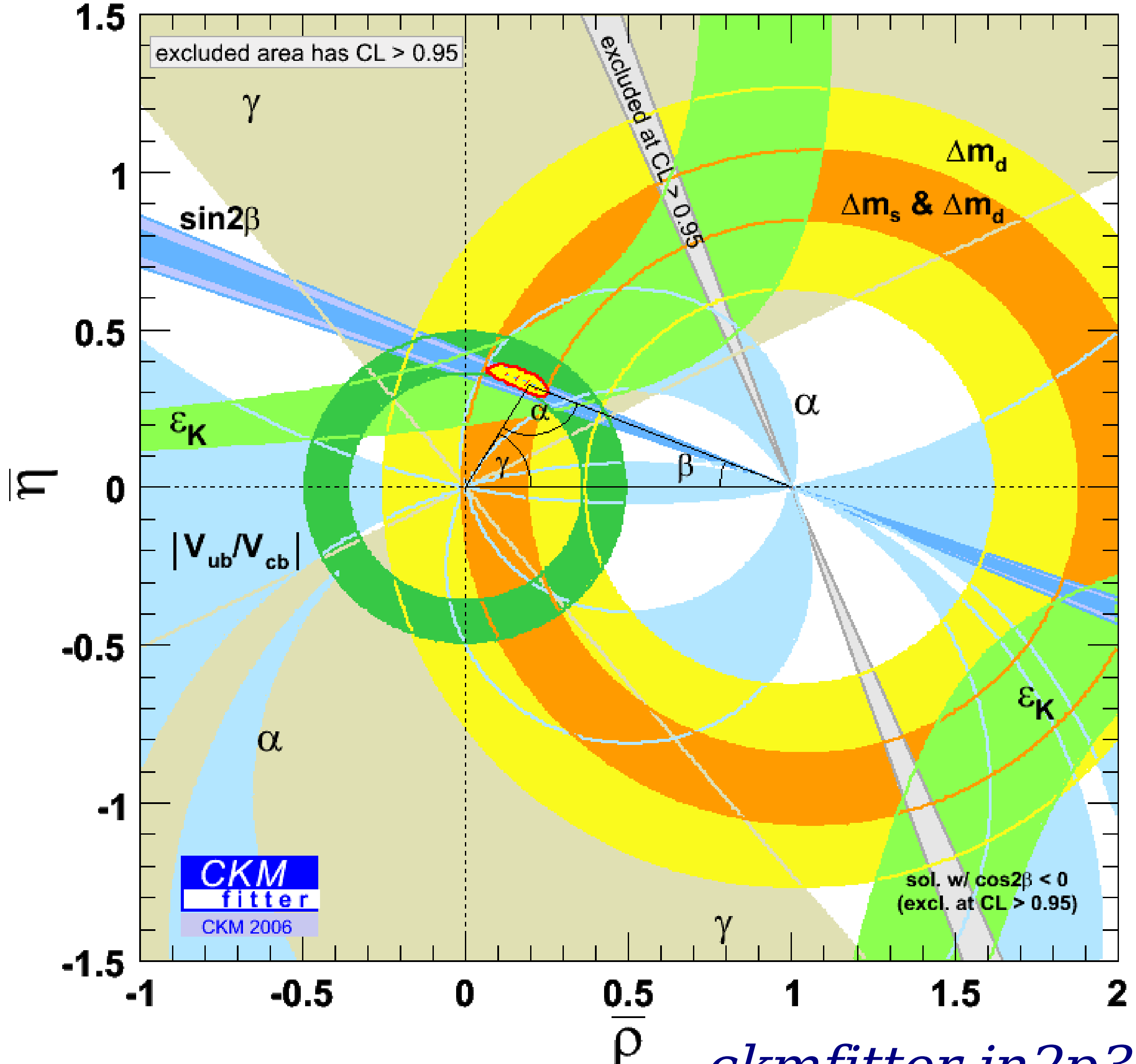
CKMfitter: the SM fit

K.Trabelsi
(KEK)



ckmfitter.in2p3.fr

Global CKM fit

statistical method : frequentist approach Rfit

J.Charles et al. [CKMfitter group], Eur. Phys. J. C41 (2005)

use all constraints on which we have a good theoretical control

charged current couplings

$|V_{ud}|, |V_{us}|, |V_{cb}|$ — *fix the length scale of the UT and constraint on A and* λ

ϵ_K

$|V_{ub}|$ — *sides of the UT*

Δm_d

Δm_s

β/ϕ_1

α/ϕ_2 — $\pi\pi$, $\rho\rho$, $\rho\pi$ *and* $SU(2)$

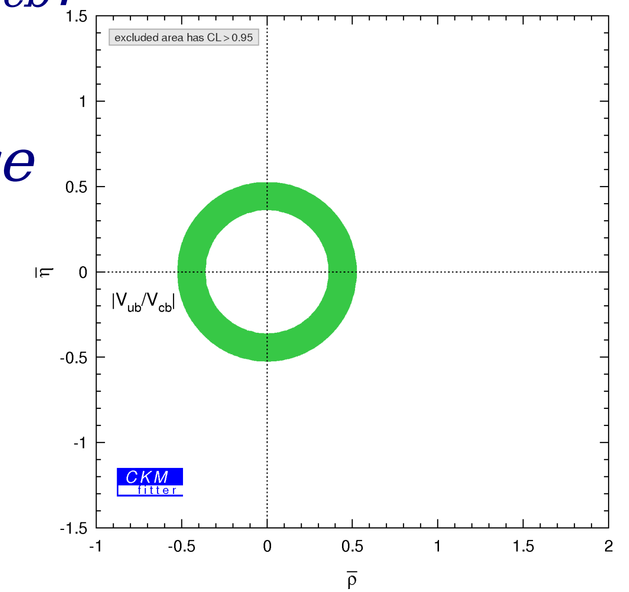
γ/ϕ_3 — *GLW/ADS/GGSZ*

$B \to \tau\nu$

quantities sensitive to $(\bar{\rho}, \bar{\eta})$ $\to$ *precision respectively* $20\,\%$ *and* $7\,\%$

$|V_{ub}|$ measurement

$+|V_{cb}|$



extraction difficult

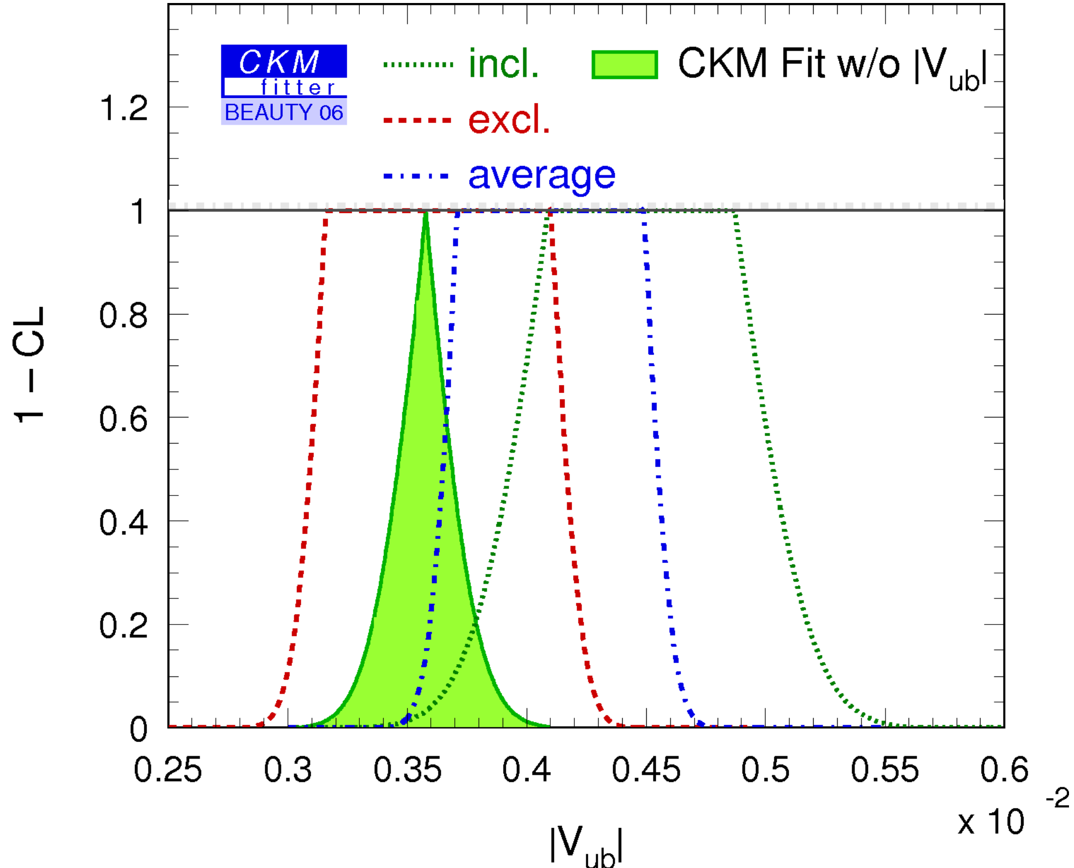
partial BR available only in restricted phase space

average done between incl. and excl.

$$|V_{ub}| = (4.10 \pm 0.09(\exp) \pm 0.39(\mathit{theo})) \times 10^{-3}$$

$$|V_{ub}| = (3.59^{+0.17}_{-0.18}) \times 10^{-3}$$

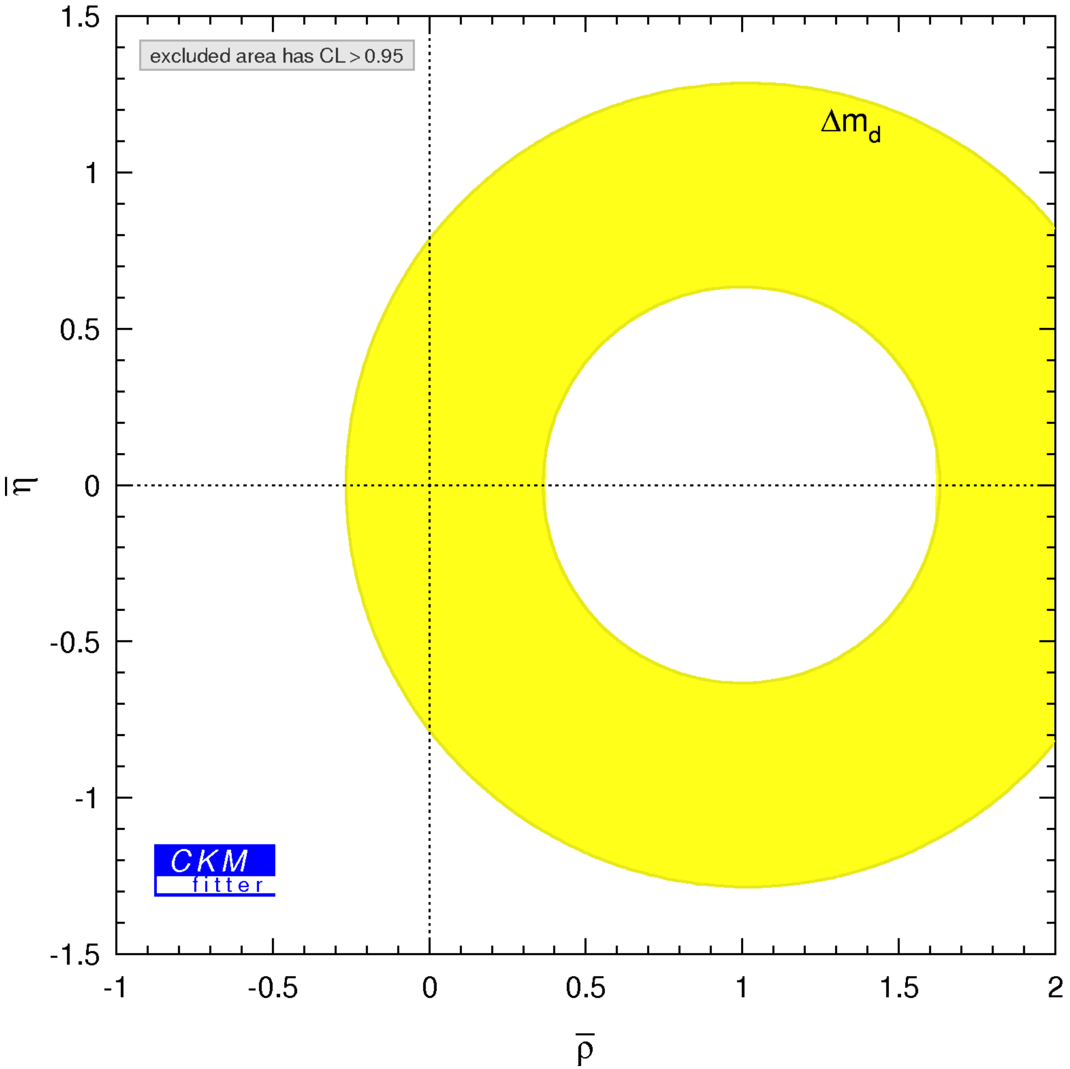
$(\textit{dominated by the } \sin 2\beta \textit{ measurement})$



Impact of Δm_d

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_{B_d} m_W^2 \eta_B S_0(x_t) f_{B_d}^2 B_d |V_{td} V_{tb}^*|^2$$

$$\propto A^2 \lambda^6 [(1-\bar{\rho})^2 + \bar{\eta}^2]$$



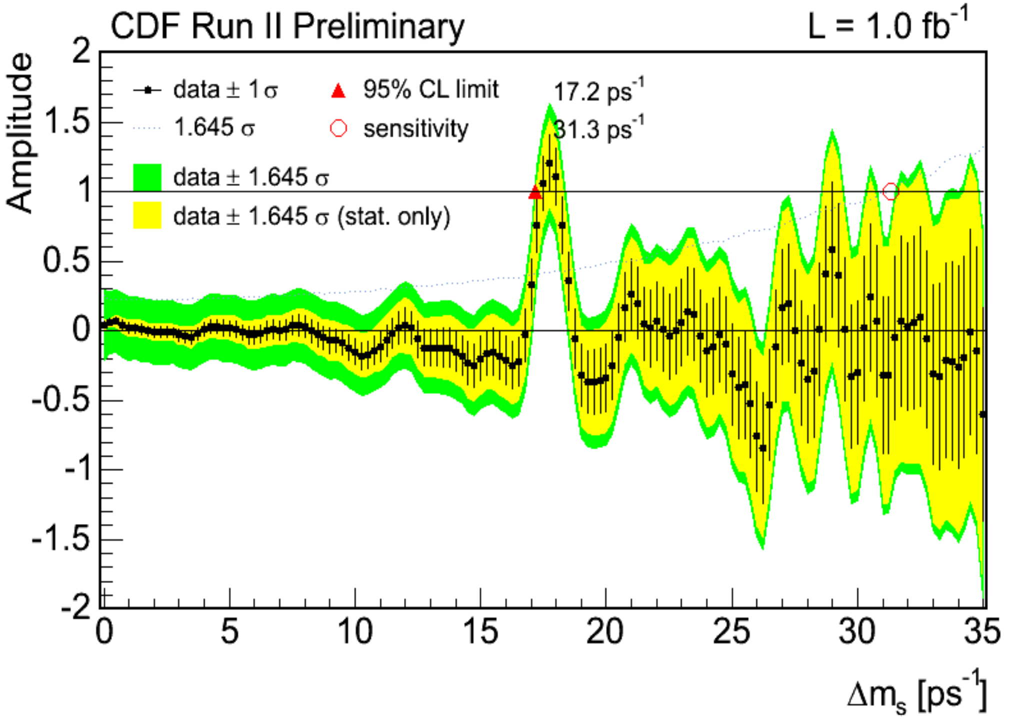
Δm_s results (CDF/D0)

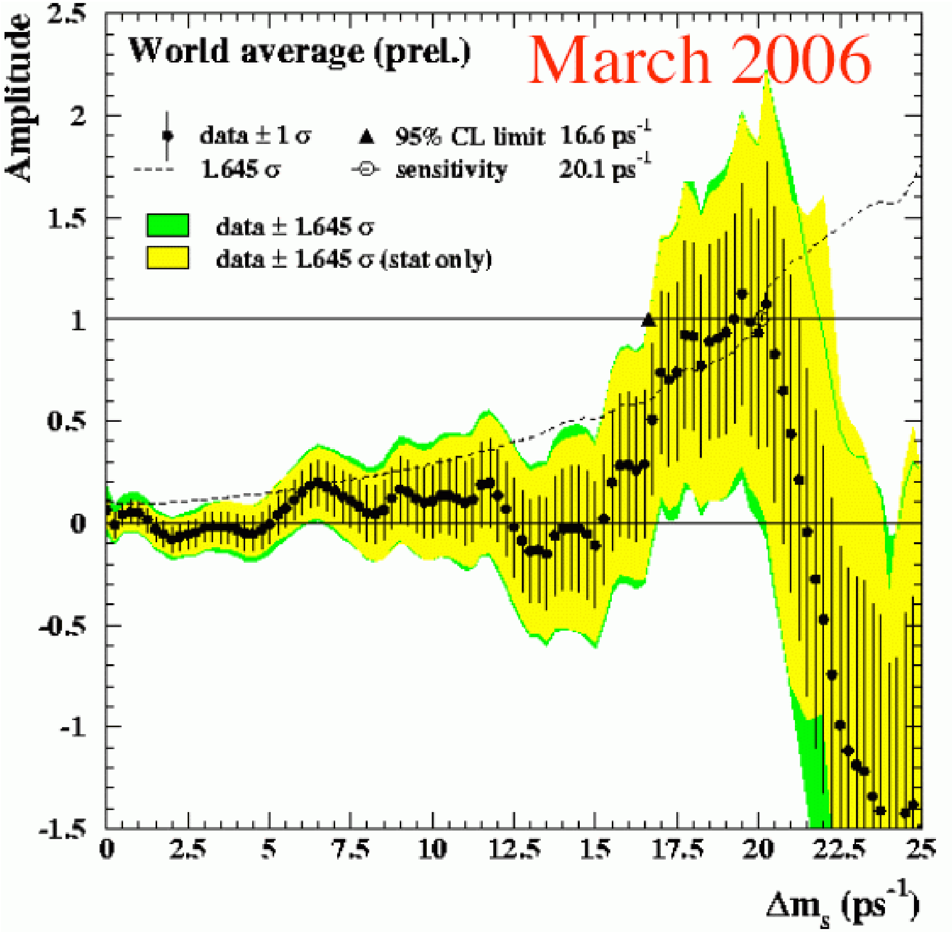
D0: [hep–ex/0603029]

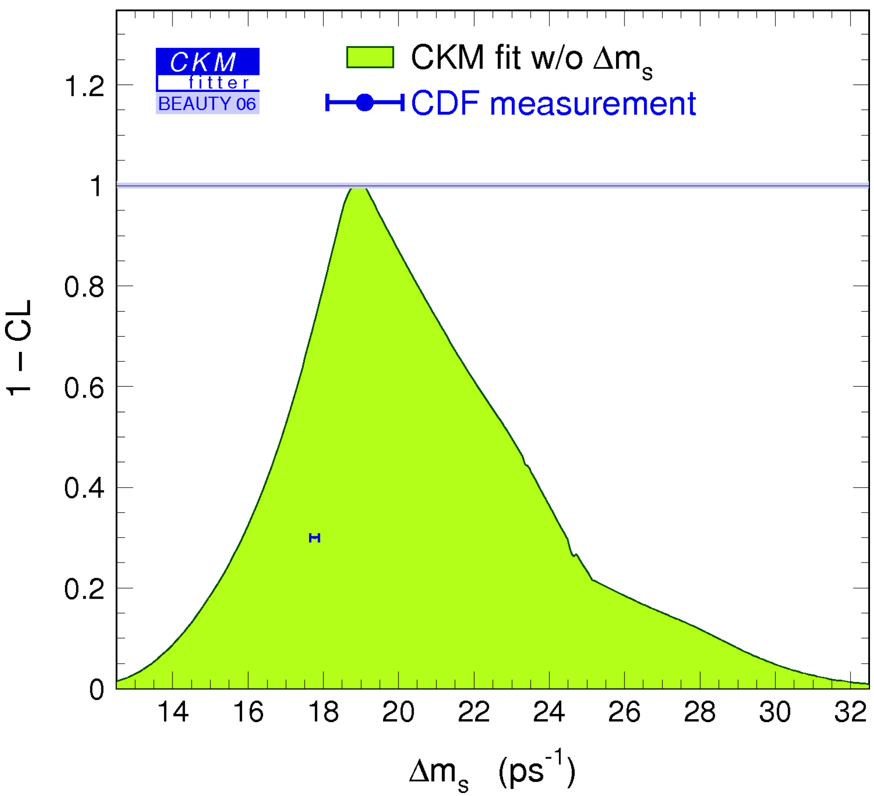
$17 < \Delta m_s < 21\ ps^{-1}$ @ 90 % C.L.

CDF: [hep-ex/0609040]

$\Delta m_s = 17.77 \pm 0.10 \pm 0.07\ ps^{-1}$







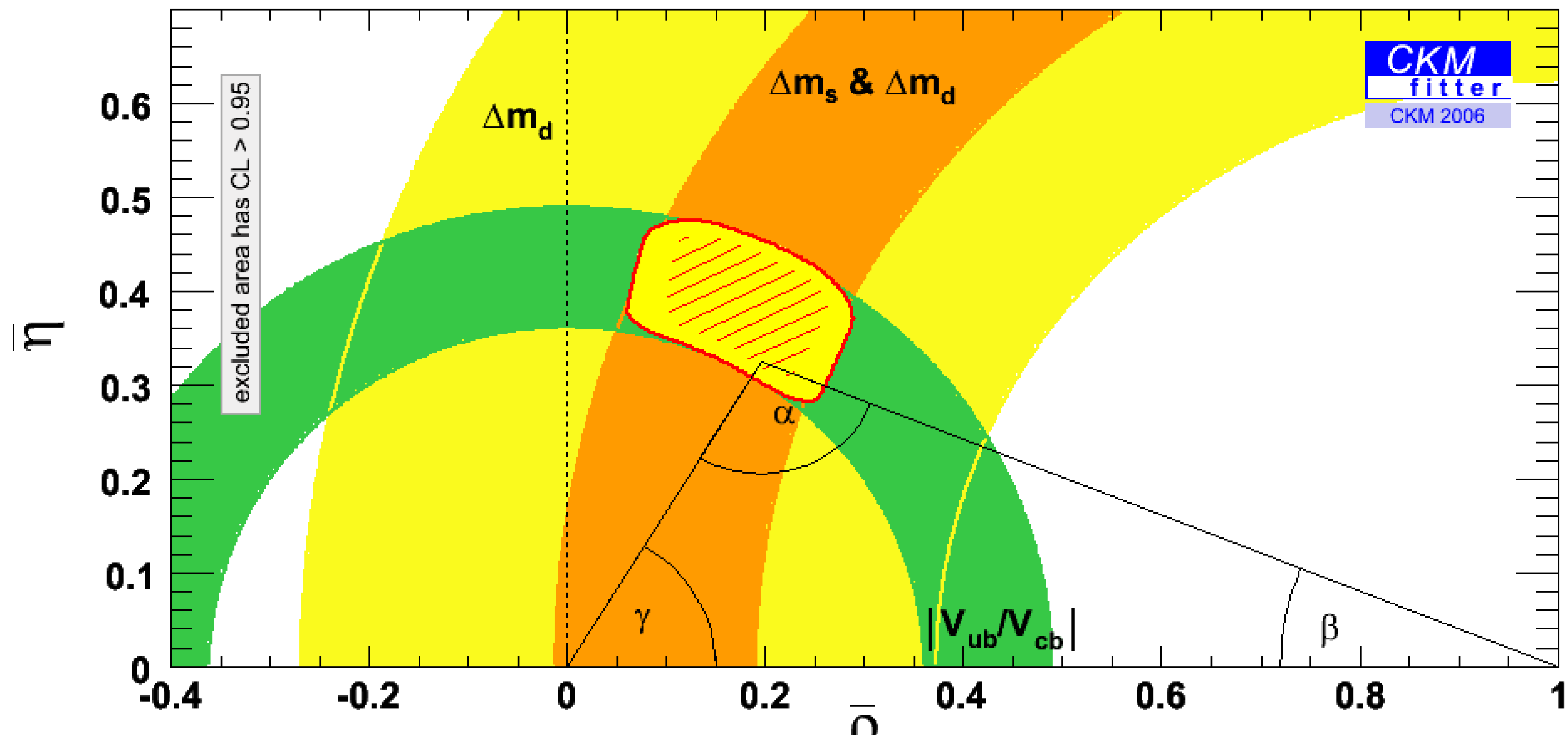
Impact of Δm_s results

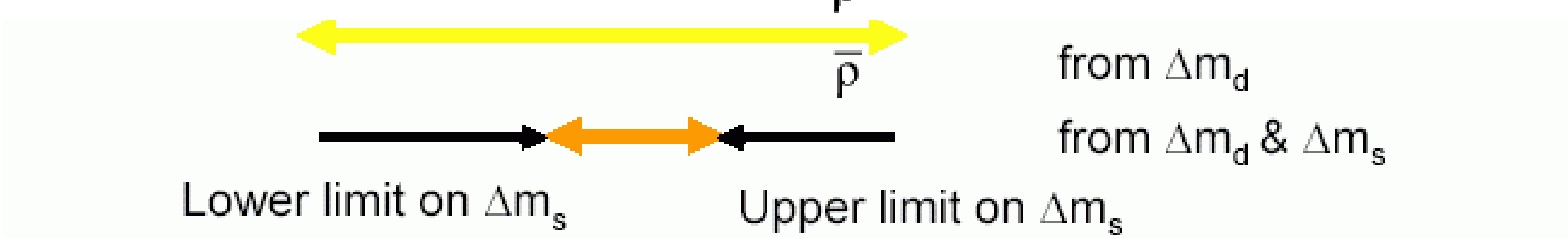
$$\Delta m_s = \frac{G_F^2}{6\pi^2} m_{B_s} m_W^2 \eta_B S_0(x_t) f_{B_s}^2 B_s |V_{ts} V_{tb}^*|^2$$

$$f_{B_s}^2 B_s = \frac{f_{B_s}^2 B_s}{f_{B_d}^2 B_d} f_{B_d}^2 B_d = \xi^2 f_{B_d}^2 B_d$$

$\xi : SU(3) - \textit{breaking corrections}$

$$\xi = 1.24 \pm 0.04 \pm 0.05$$





constraint now completly dominated by theoretical uncertainty on ξ

$B^+ \to \tau^+ \nu$

$$BF(B\to\tau\nu) = \frac{G_F^2 m_B}{8\pi} m_\tau^2 \left(1-\frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

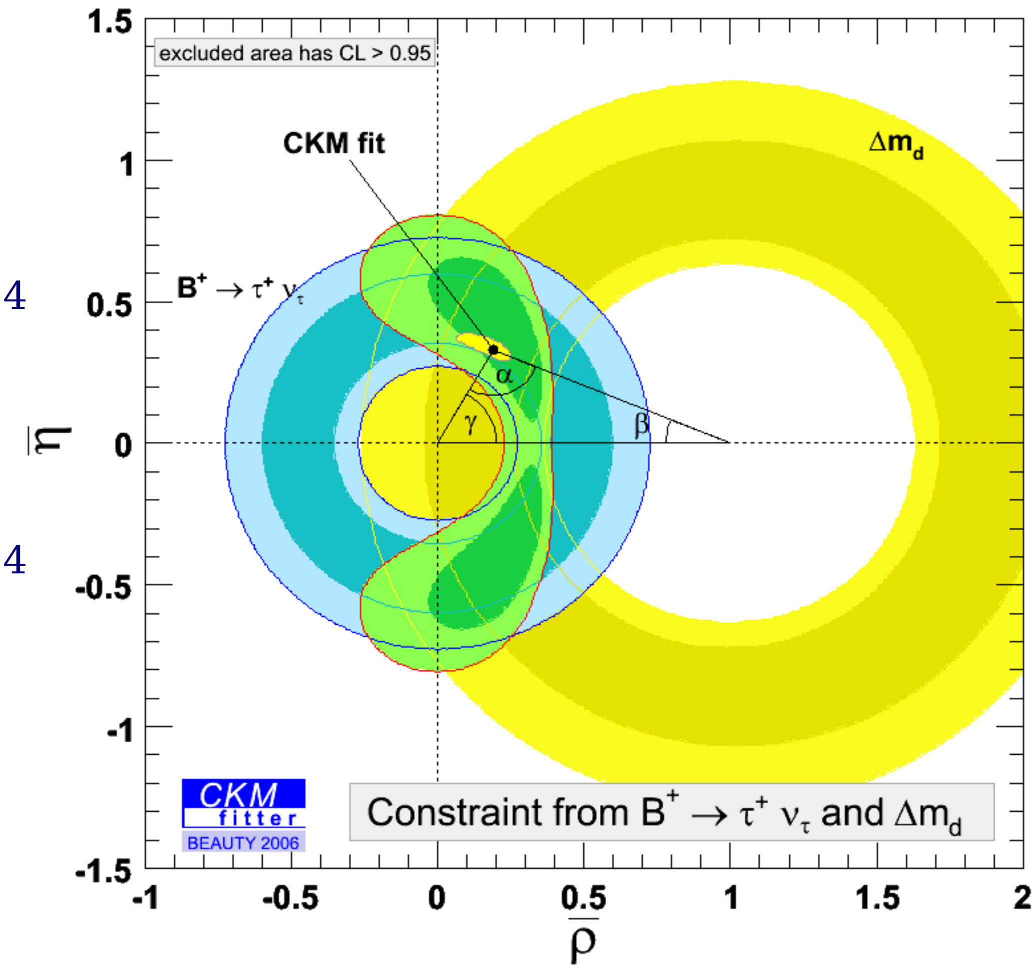
$$f_B = 191 \pm 27\ MeV$$

(320m)

$$BF(B\to\tau\nu) = (1.06^{+0.34+0.18}_{-0.28-0.16})\times 10^{-4}$$

(447m)

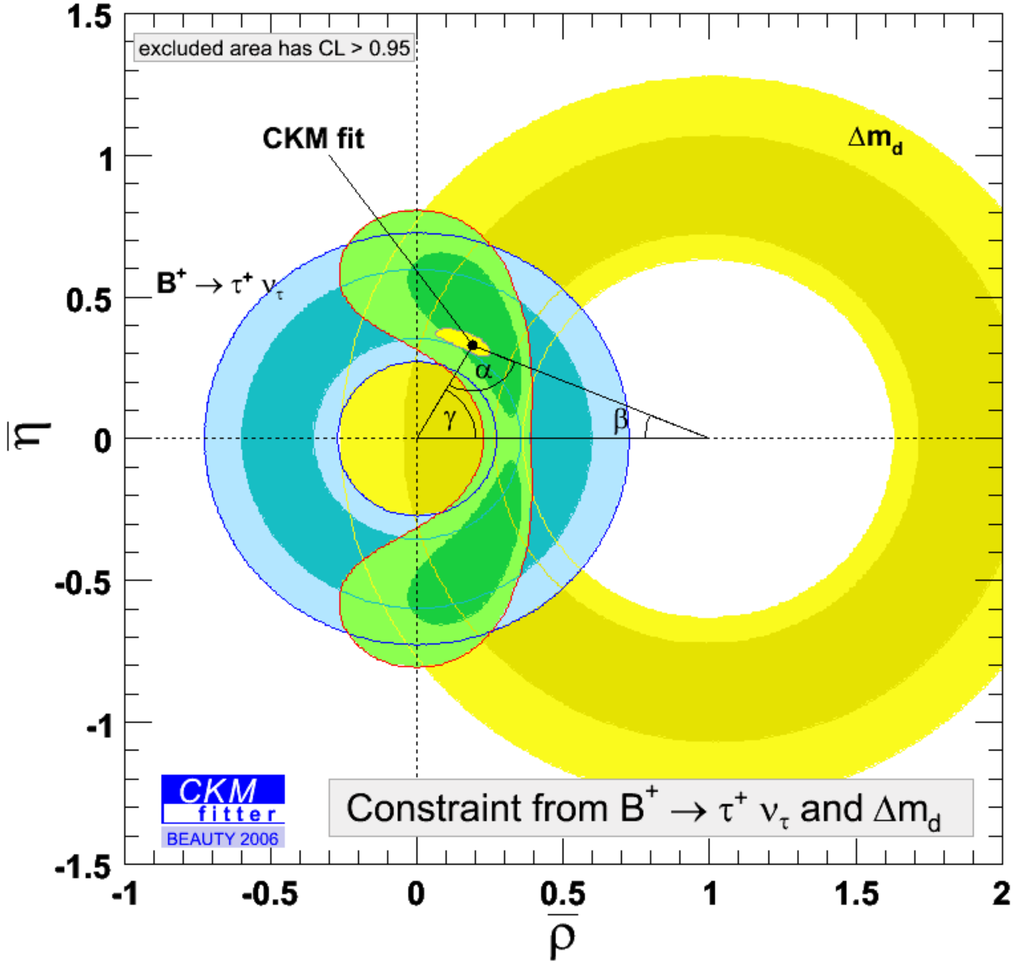
$$BF(B\to\tau\nu) = (1.79^{+0.56+0.39}_{-0.49-0.46})\times 10^{-4}$$



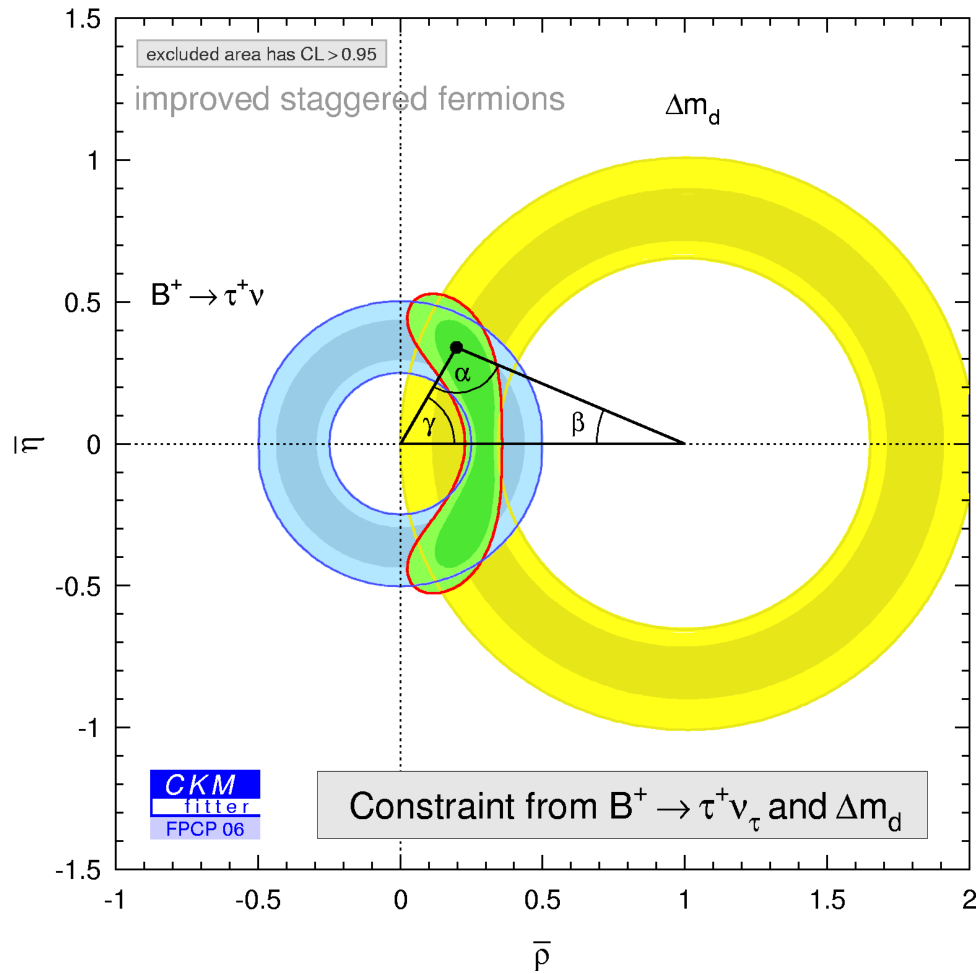
together with Δm_d: removes f_B (lattice QCD) dependence

Constraints on $(\bar{\rho}, \bar{\eta})$ from Δm_d and $B \rightarrow \tau \nu$

[*hep−lat*/0510113]

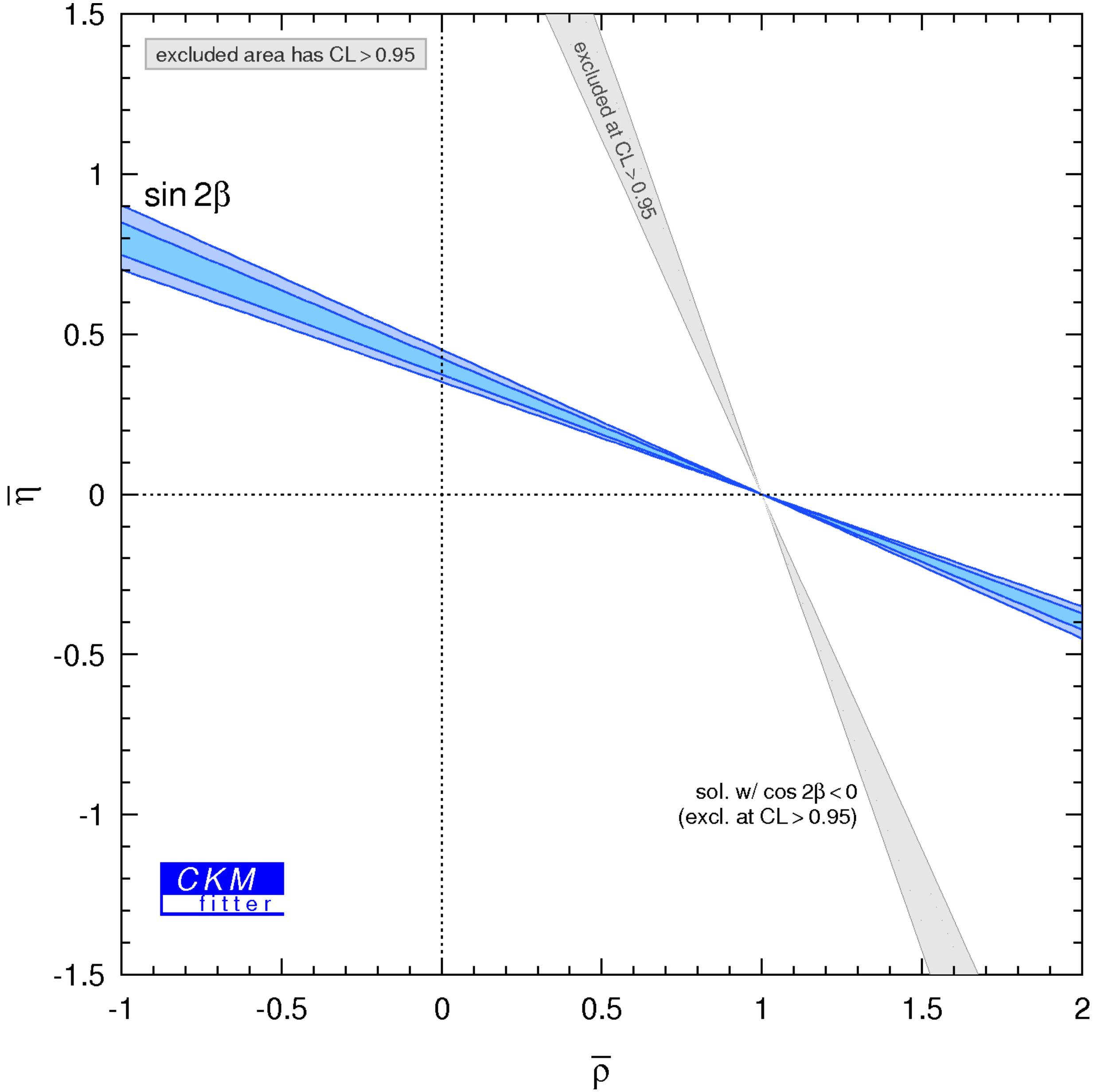


using traditional approaches

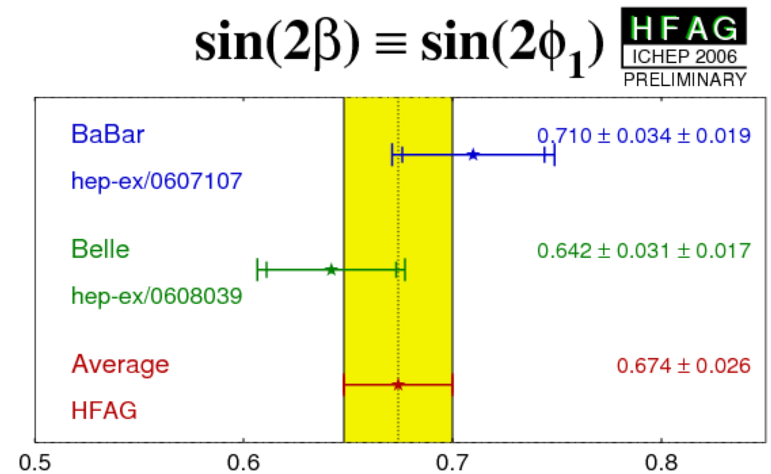


using improved staggered fermions

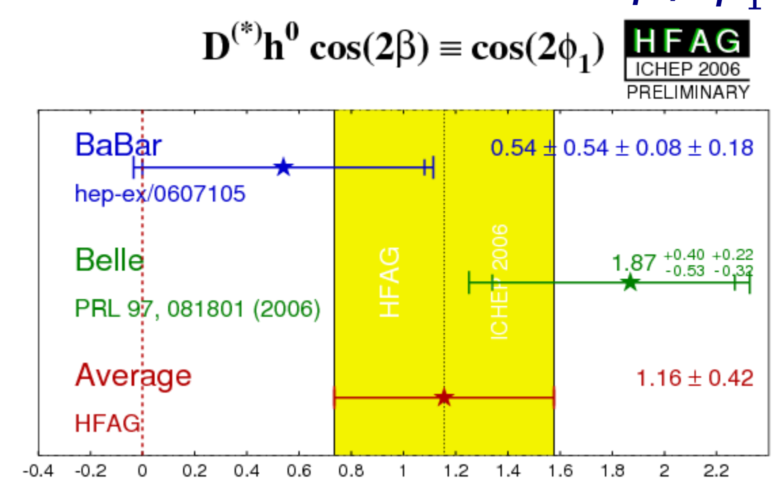
Angle β / ϕ_1



charmonium modes



Dalitz $D^{(*)} h^0 \to \cos 2\beta / \phi_1$



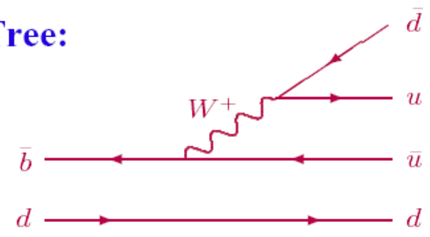
$\cos 2\beta / \phi_1 > 0$

Belle: 98.3 %

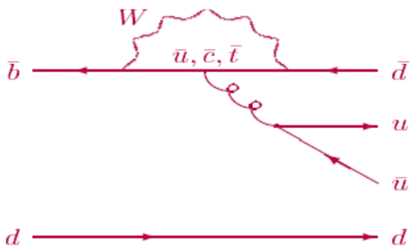
BaBar: 87 %

$Angle\ \alpha/\phi_2$

Tree:



Penguin:



$$A_{\pi^+\pi^-}(t) = S_{\pi^+\pi^-}\sin(\Delta m t) - C_{\pi^+\pi^-}\cos(\Delta m t)$$
$$= \sqrt{1-C^2_{\pi^+\pi^-}}\,\sin 2\alpha_{eff}\sin(\Delta m t) - C_{\pi^+\pi^-}\cos(\Delta m t)$$

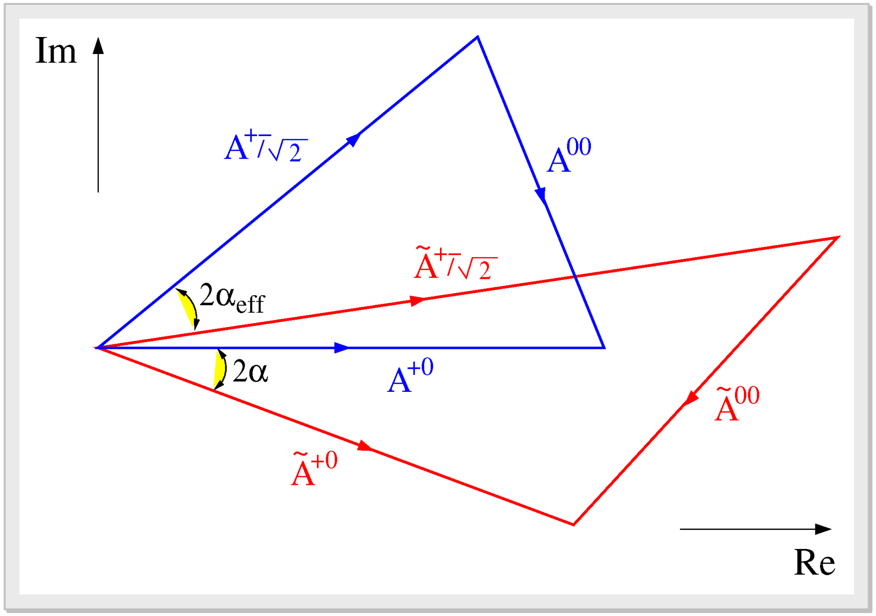
from time dependent CP, we can measure α_{eff},
but we want α !

$$\alpha_{eff} = \alpha + \theta$$

Isospin analysis: [*Gronau–London, PRL65,* 3381 (1990)]

$$A_{+-} + \sqrt{2}A_{00} = \sqrt{2}A_{+0}$$
$$\overline{A}_{+-} + \sqrt{2}\,\overline{A}_{00} = \sqrt{2}\,\overline{A}_{+0}$$

α can be resolved up to an 8-fold ambiguity

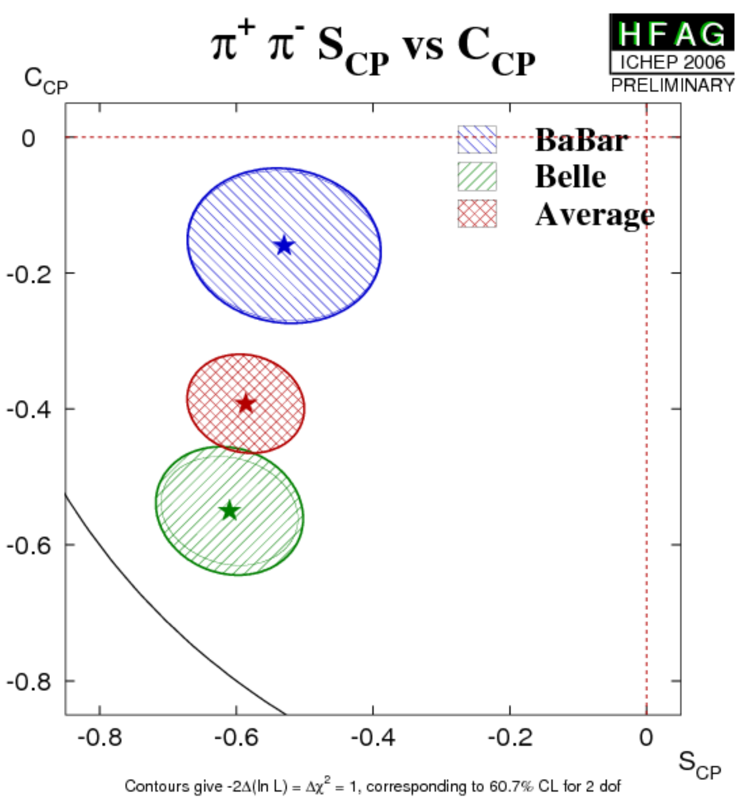


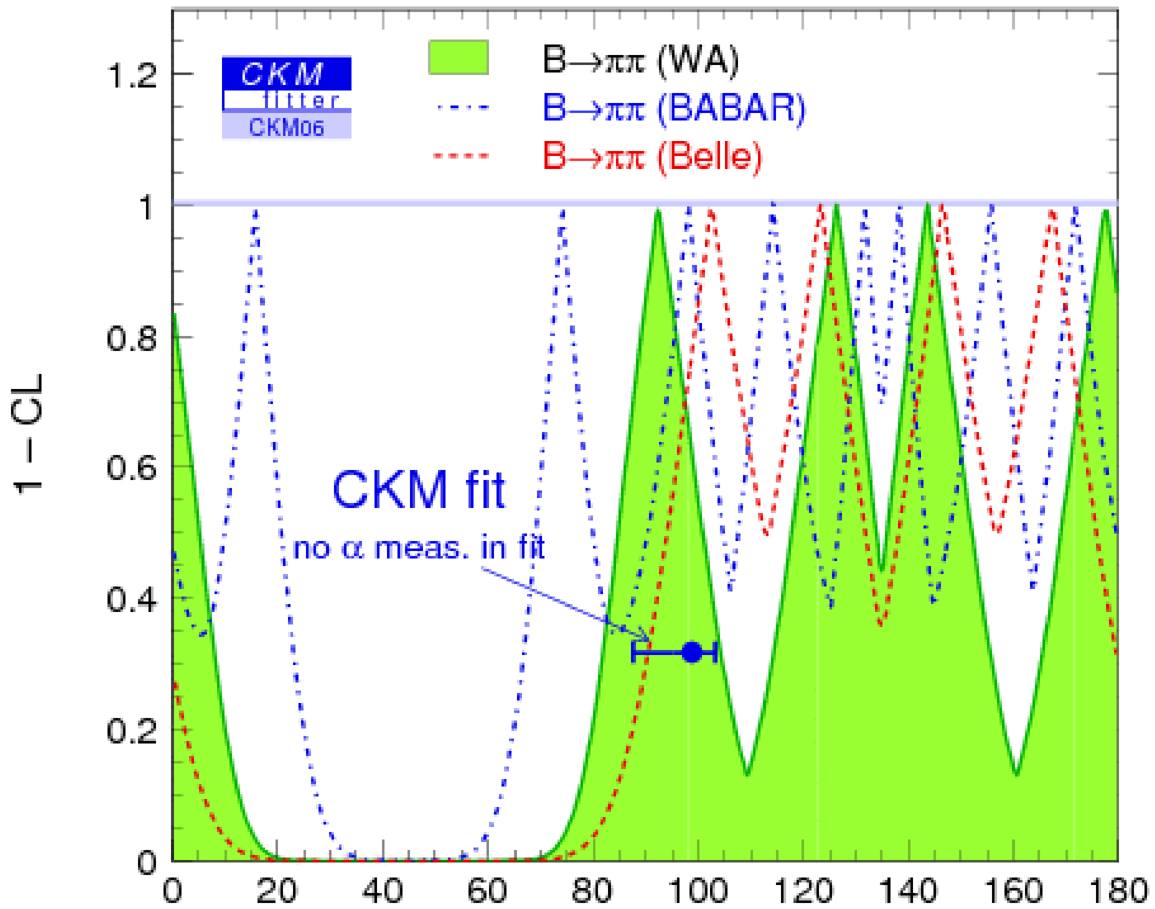
$\rho^+\rho^-$: ~ 100 % *longitudinally polarized* (*similar isospin analysis*)

$\pi\pi$ system (6 observables for 6 parameters)

$$(Br(B\to\pi^+\pi^-), S_{\pi^+\pi^-}, C_{\pi^+\pi^-}, Br(B\to\pi^+\pi^0), Br(B\to\pi^0\pi^0), C_{\pi^0\pi^0})$$

	BaBar(347m)	Belle(532m)	Average
$S_{\pi\pi}$	$-0.53\pm0.14\pm0.02$	$-0.61\pm0.10\pm0.04$	-0.59 ± 0.09
$C_{\pi\pi}$	$-0.16\pm0.11\pm0.03$	$-0.55\pm0.08\pm0.05$	-0.39 ± 0.07





agreement: 2.3σ

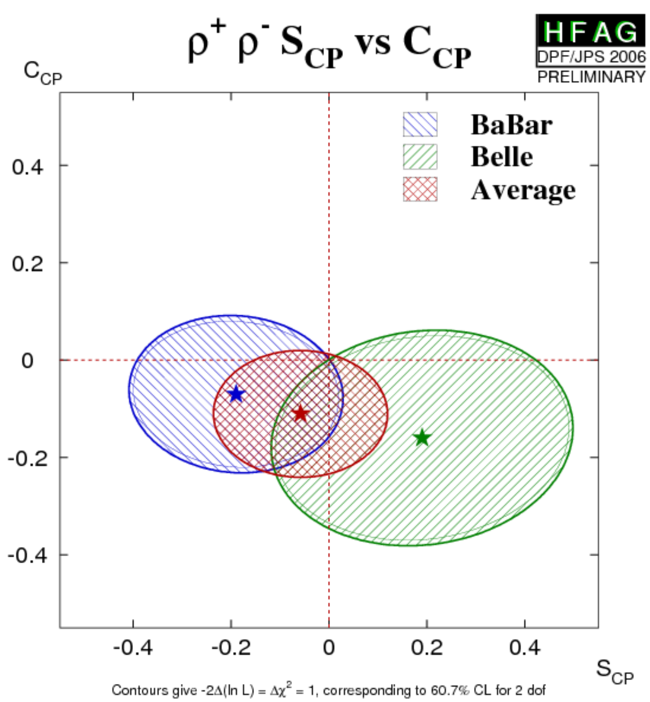
$$\alpha/\phi_2 = (92^{+12}_{-10})^o$$

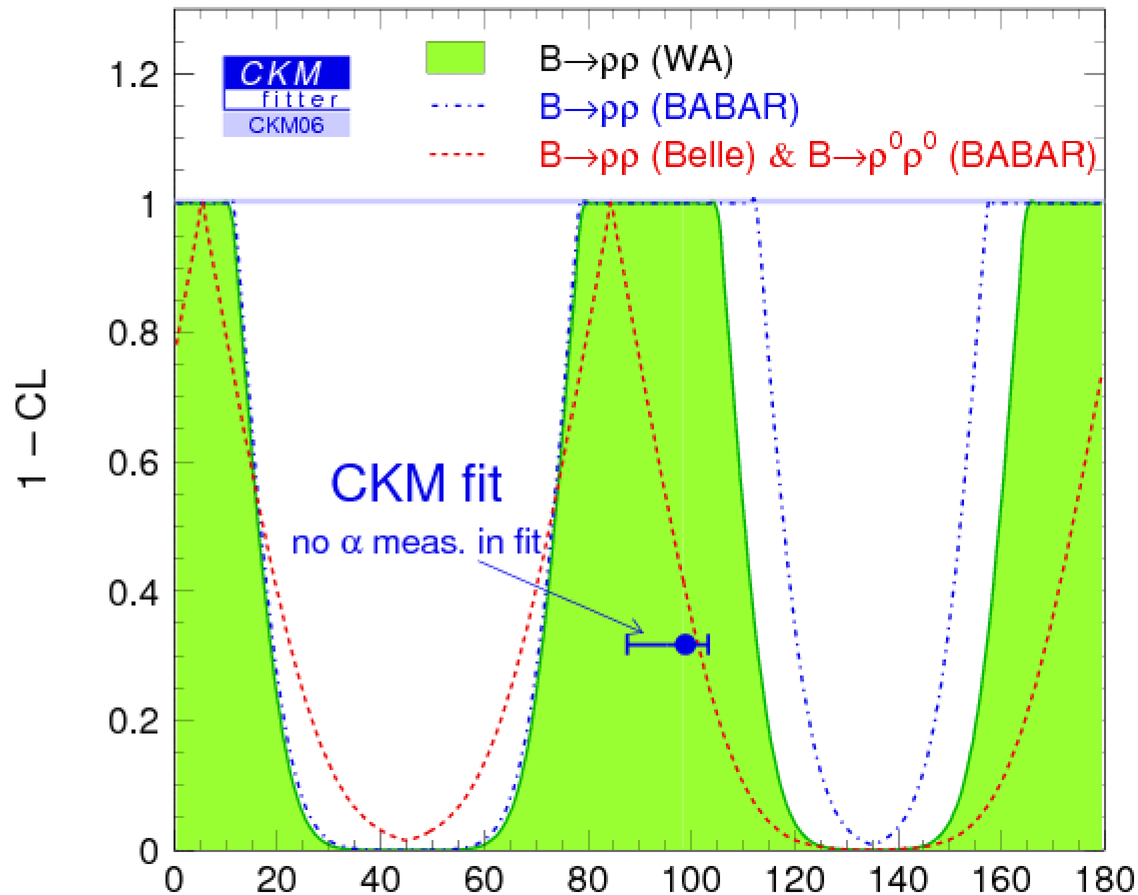
$\rho\rho$ system (5 observables for 6 parameters)

$$(Br(B\to\rho^+\rho^-), S_{\rho^+\rho^-}, C_{\rho^+\rho^-}, Br(B\to\rho^+\rho^0), Br(B\to\rho^0\rho^0)) + f_L$$

	BaBar(350m)	Belle(535m)	Average
$S_{\rho\rho}$	$-0.19\pm0.21\pm0.07$	$0.19\pm0.30\pm0.07$	-0.06 ± 0.18
$C_{\rho\rho}$	$-0.07\pm0.15\pm0.06$	$-0.16\pm0.21\pm0.07$	-0.11 ± 0.13

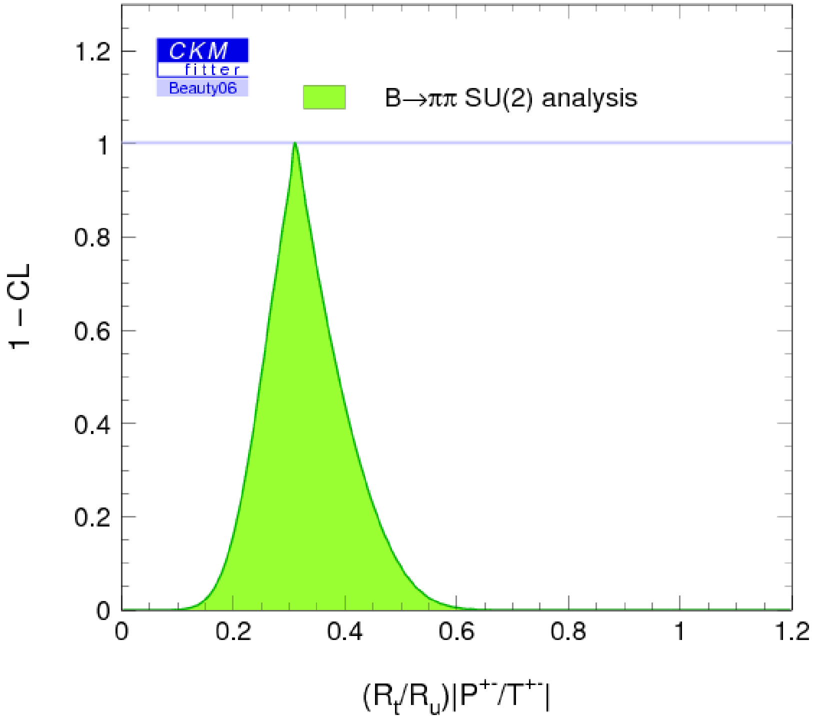
$B\to\rho^0\rho^0$	BaBar(350m)
f_L	$0.86^{+0.11}_{-0.13}\pm0.06$
BR	$(1.2\pm0.4\pm0.3)\times10^{-6}$

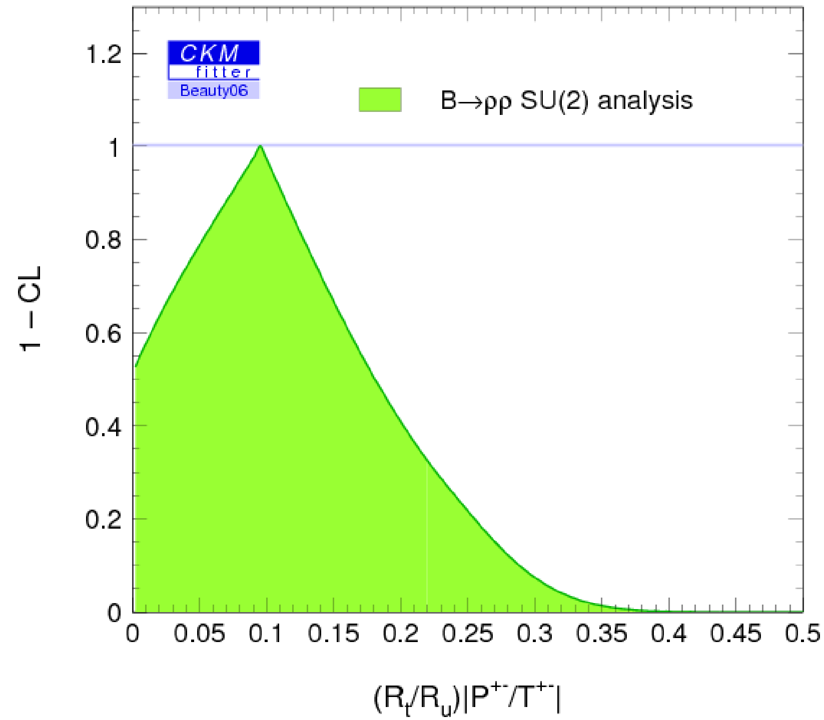


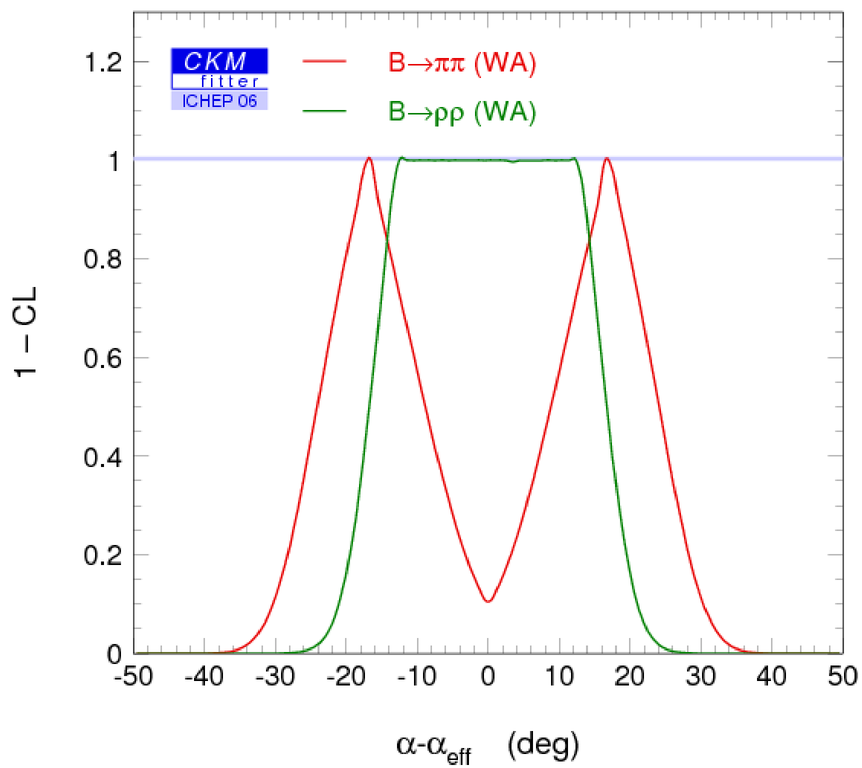


$$\alpha/\phi_2 = (92\pm21)^o$$

P/T







$\pi\pi$

$|\alpha-\alpha_{eff}| < 32.2^o\ (95\ \%\ CL)$

$\rho\rho$

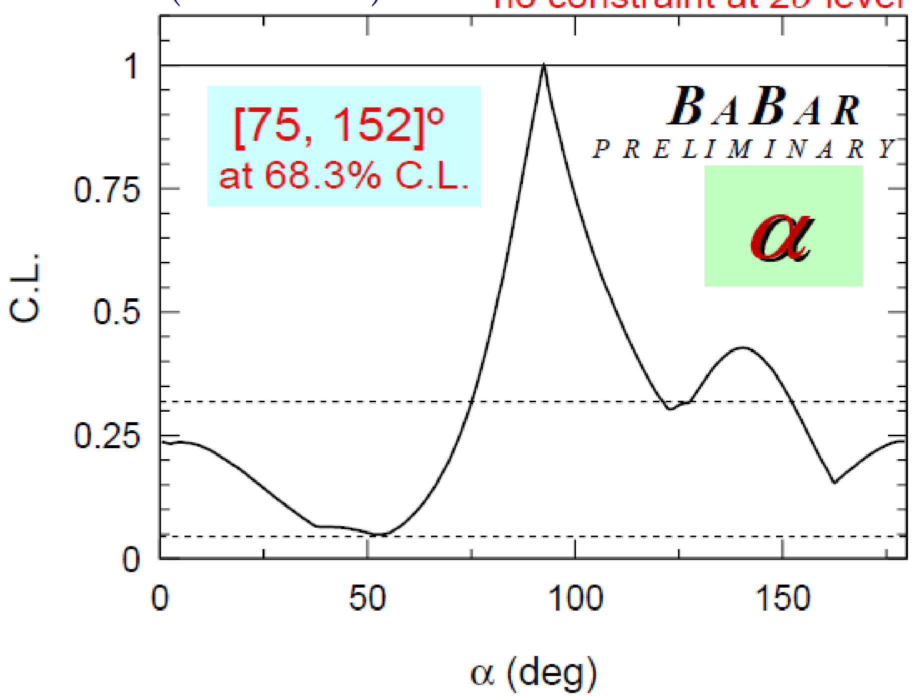
$|\alpha-\alpha_{eff}| < 22.5^o\ (95\ \%\ CL)$

$\rho\pi$ system

- non CP−eigenstate $B \to \rho\pi$ $(\pi\pi\pi^0)$
- Amplitude interference in Dalitz plot [Snyder, Quinn, PRD48, 2139 (1993)]
- Experiments determine 27 coefficients of the form factor bilinears from the fit to data

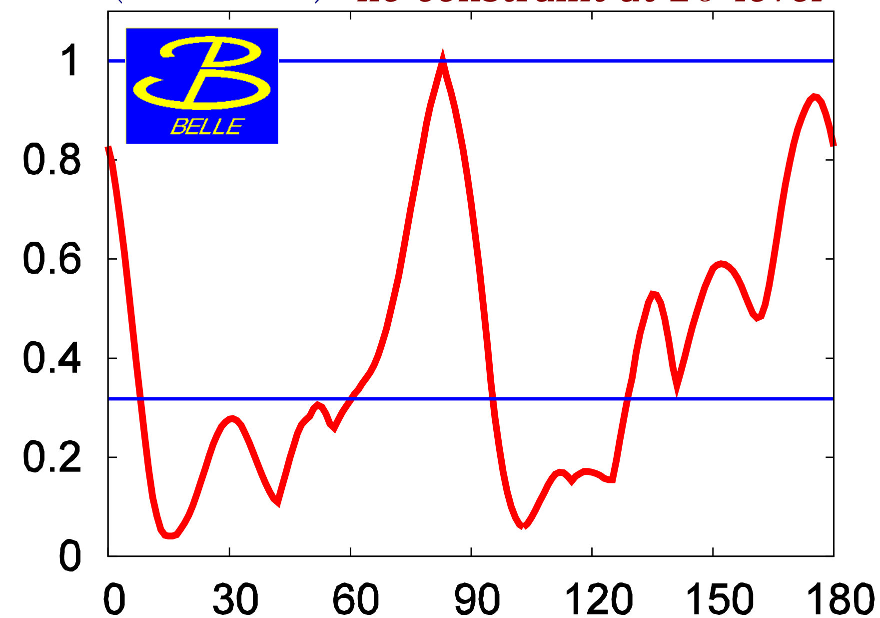
→ quasi-two-body parameters, α/ϕ_2... obtained from subsequents fits

(347m) no constraint at 2σ level



$75^\circ < \alpha < 152^\circ$ @ 68 % C.L.

(449m) no constraint at 2σ level

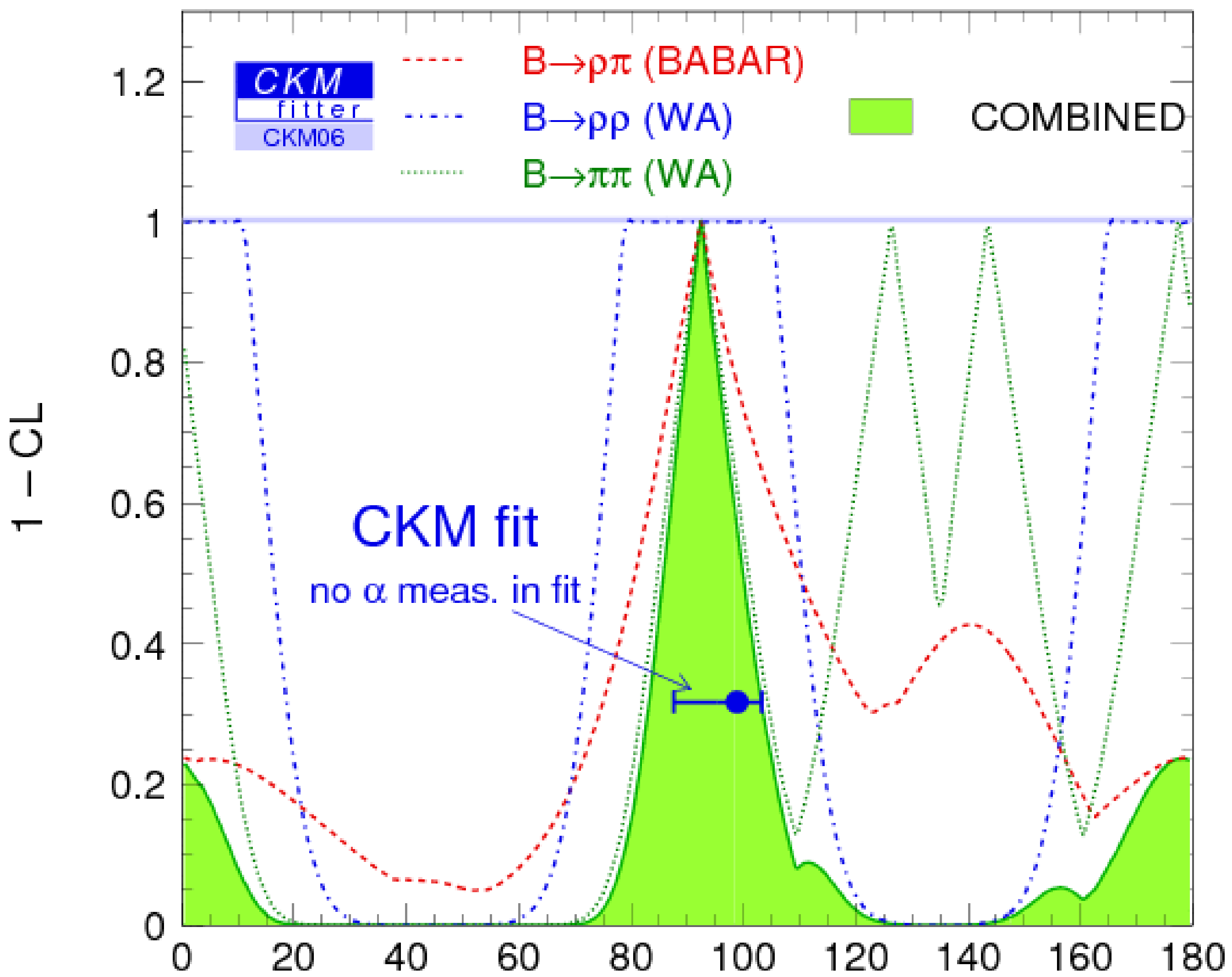


Dalitz analysis + isospin (pentagon) analysis

$\phi_2 = (83^{+12}_{-23})^\circ$

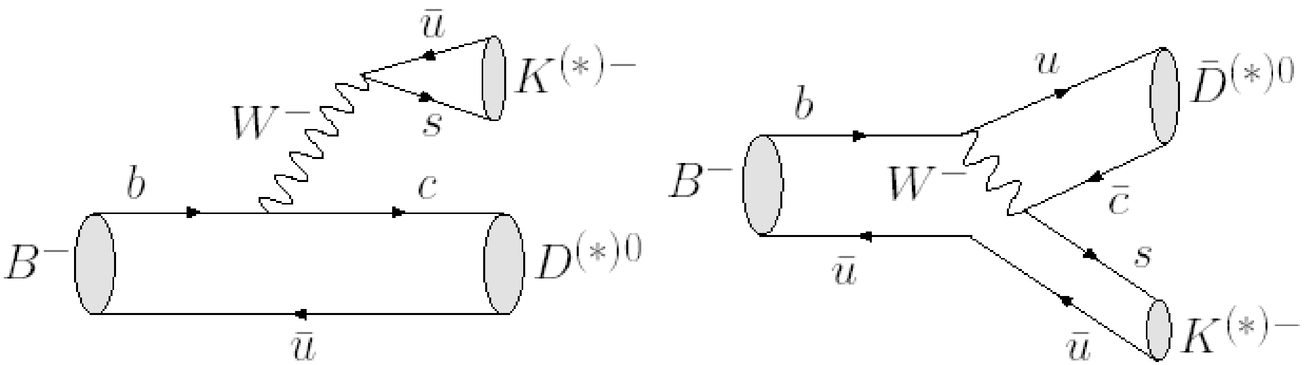
for $\rho\pi$ input, still use only BaBar result...

α/ϕ_2 measurement



$$\alpha/\phi_2 = (93^{+11}_{-9})^o$$

γ / ϕ_3 determination from different measurements



$$r_B = \frac{|A(b \to u)|}{|A(b \to c)|}$$

GLW method

$D \to$ CP eigenstates

$(K^+ K^-, \pi^+ \pi^-, K_S \pi^0 \ldots)$

$$A_{CP\pm} = \frac{\pm 2 r_B \sin\delta \sin\phi_3}{1 + r_B^2 \pm 2 r_B \cos\delta \cos\phi_3}$$

$$R_{CP\pm} = 1 + r_B^2 \pm 2 r_B \cos\delta \cos\phi_3$$

ADS method

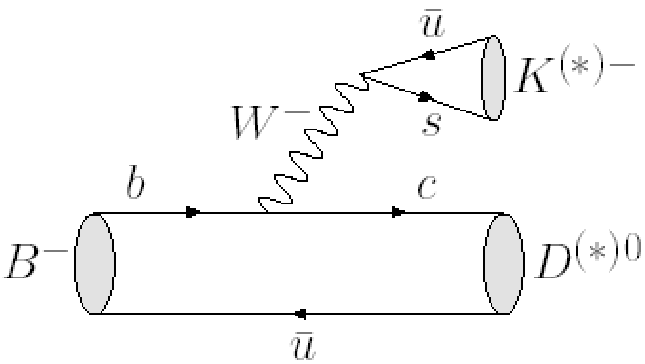
$D \to$ DCSD

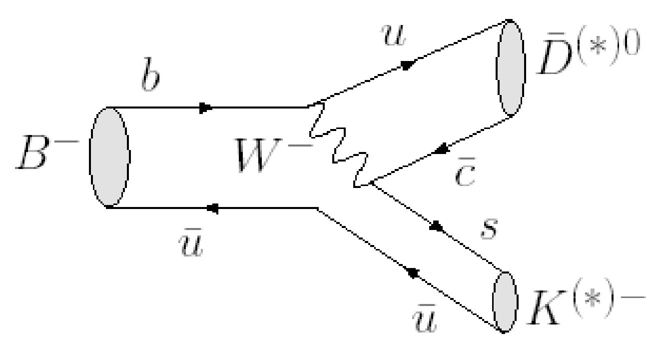
$(K^+ \pi^-, K^+ \pi^- \pi^0 \ldots)$

$$A_{ADS} = \frac{2 r_B r_D \sin(\delta_B + \delta_D) \sin\phi_3}{r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos\phi_3}$$

$$R_{ADS} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos\phi_3$$

γ/ϕ_3 determination from different measurements



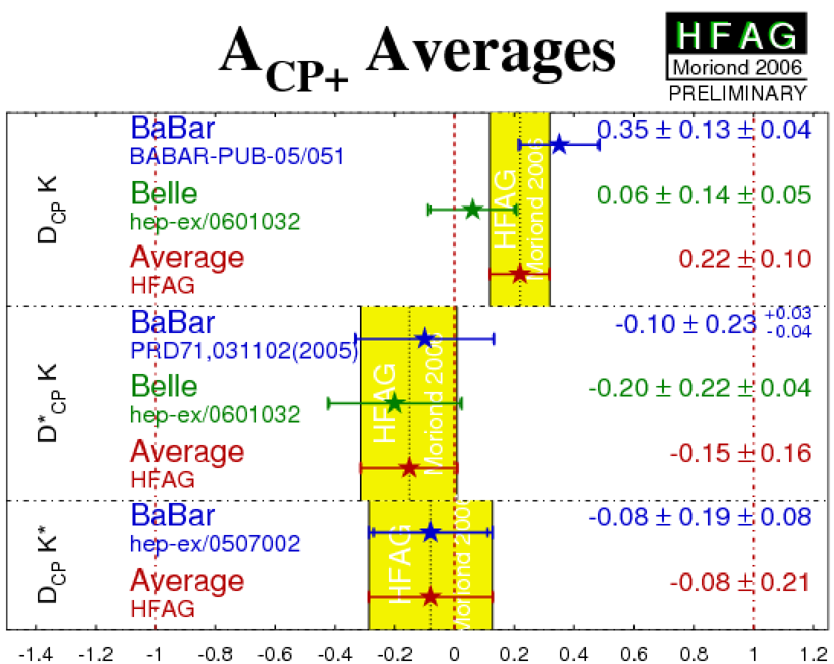


$$r_B = \frac{|A(b \to u)|}{|A(b \to c)|}$$

GLW method

$D \to CP$ eigenstates

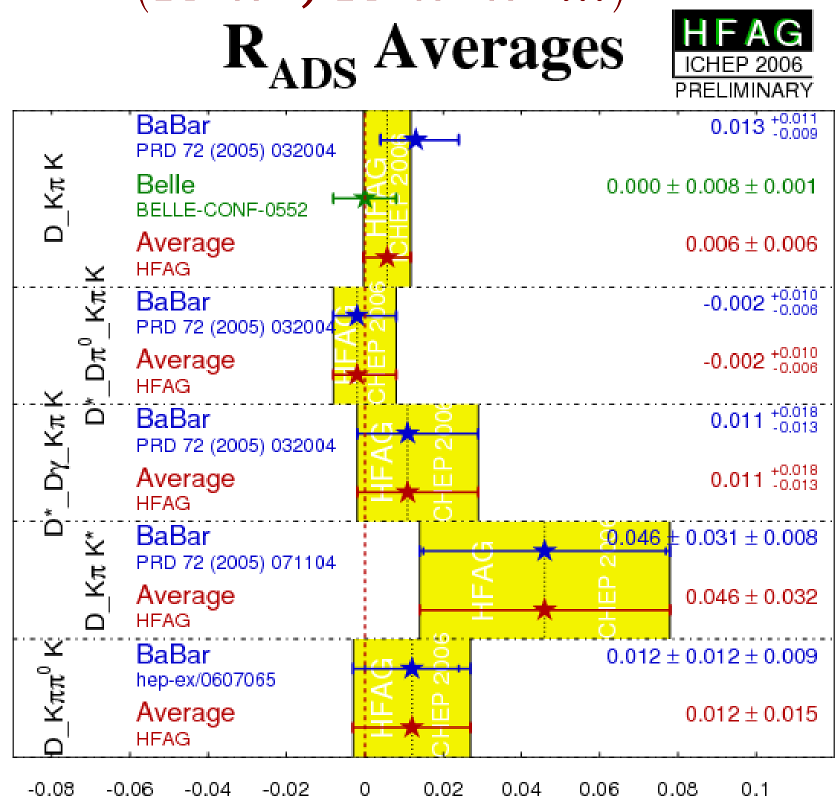
$(K^+K^-, \pi^+\pi^-, K_S\pi^0 \ldots)$



ADS method

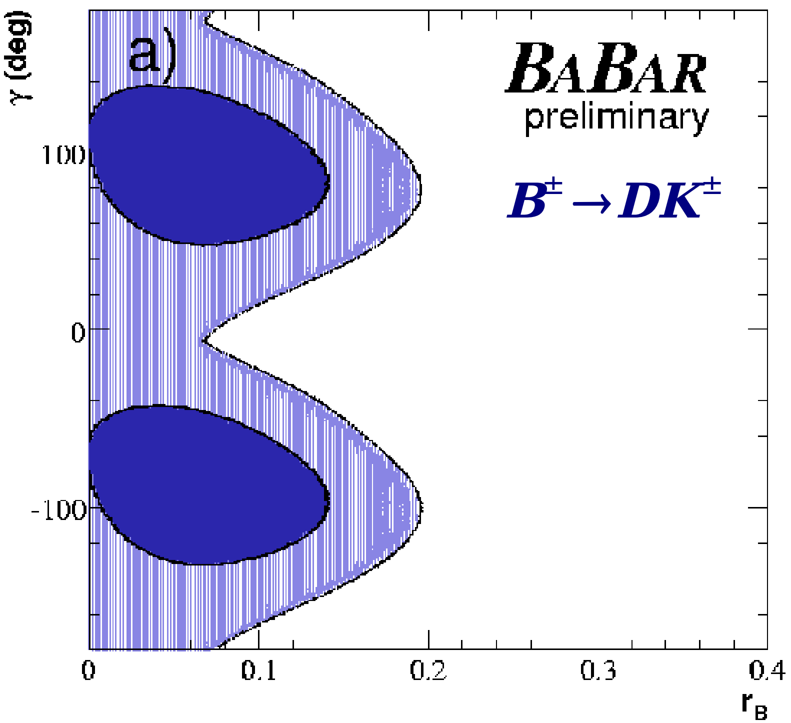
$D \to DCSD$

$(K^+\pi^-, K^+\pi^-\pi^0 \ldots)$

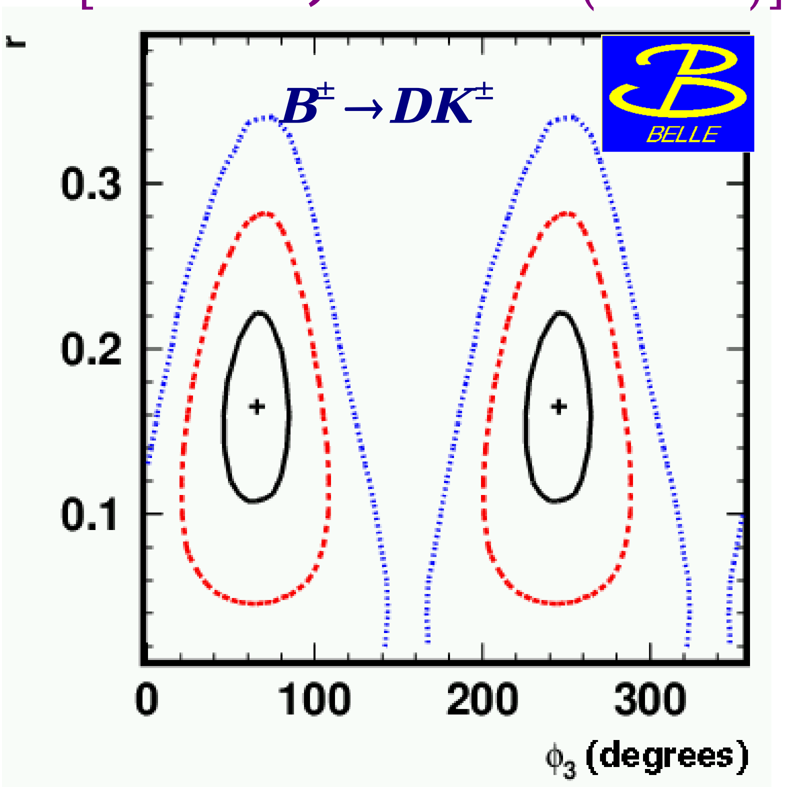


γ/ϕ_3 results GGSZ method: DK, D^*K, DK^*, $D\to K_S\pi\pi$

[*hep–ex*/0607104]



[*PRD73*, 112009 (2006)]



combined:	$\gamma=(92\pm41\pm11\pm12)^\circ$	$\phi_3=(53^{+15}_{-18}\pm3\pm9)^\circ$
DK:	$r_B<0.140$ @68 % CL	$r_B=0.16\pm0.05\pm0.01\pm0.05$
D^*K:	$0.017<r_B<0.203$	$r_B=0.18\pm0.11\pm0.01\pm0.05$
DK^*:		$r_B=0.56\pm0.22\pm0.04\pm0.08$

σ_γ depends on the value of r_B !

γ/ϕ_3 measurement: $(DK,\ D^*K,\ DK^*$ modes$)$

GLW: $(A_{CP+},\ A_{CP-},\ R_{CP+},\ R_{CP-}) \Rightarrow (\phi_3,\ r_B,\ \delta_B)$

ADS: $(R_{ADS}) \Rightarrow (\phi_3,\ r_B,\ \delta_B,\ r_D,\ \delta_D)$

GGSZ: $(x_\pm,\ y_\pm) \Rightarrow (\phi_3,\ r_B,\ \delta_B)$

preliminary...

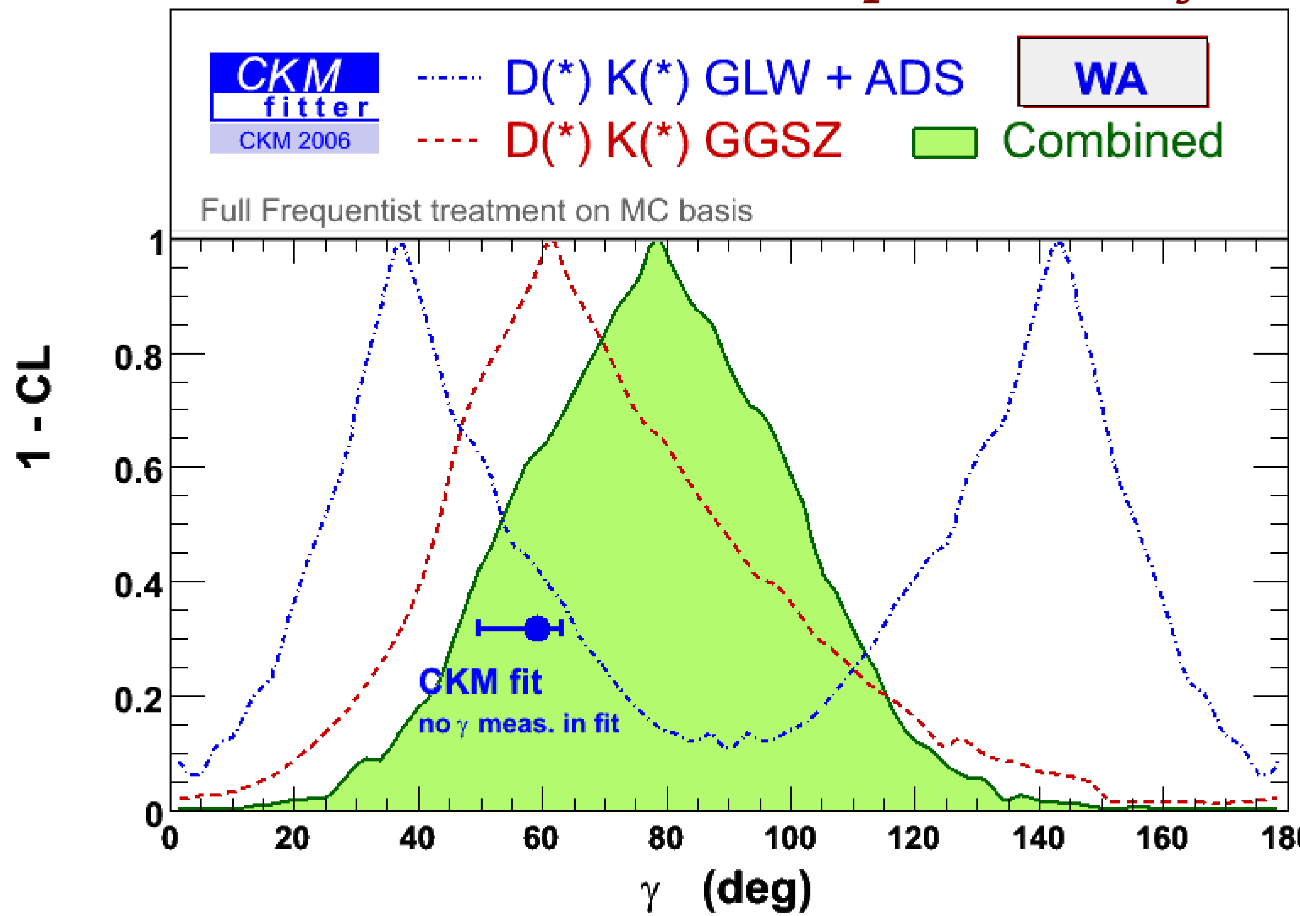
low value of r_B ?

90 % CL

$DK:\ r_B < 0.13$

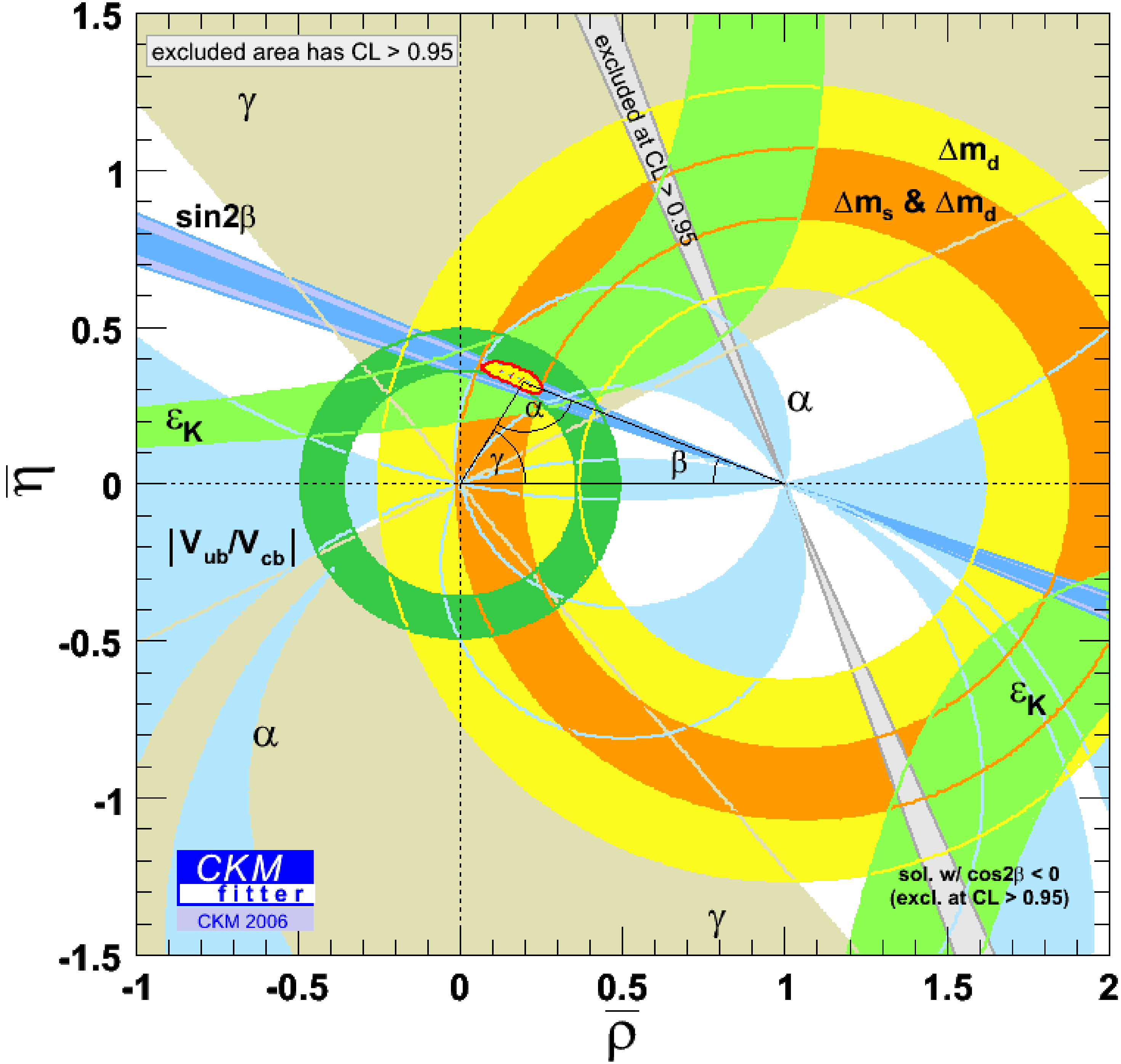
$D^*K:\ r_B < 0.13$

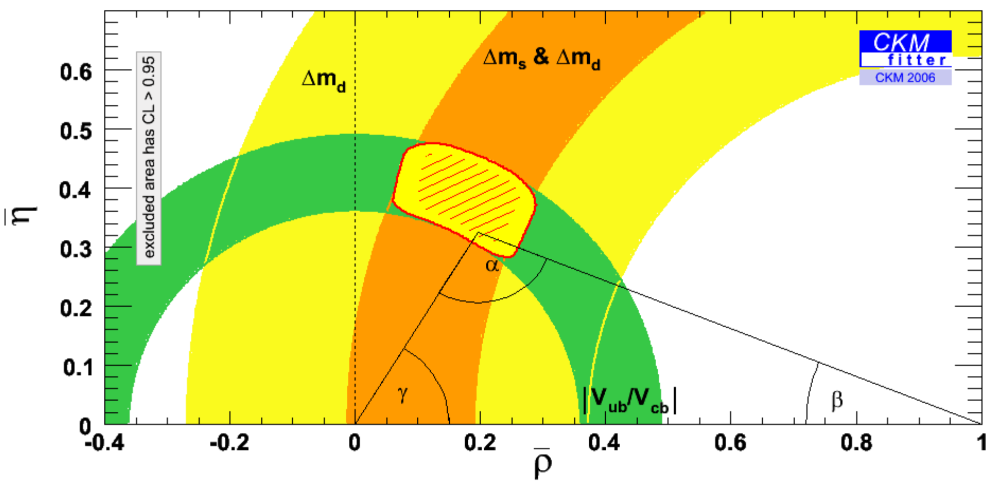
$DK^*:\ r_B < 0.27$

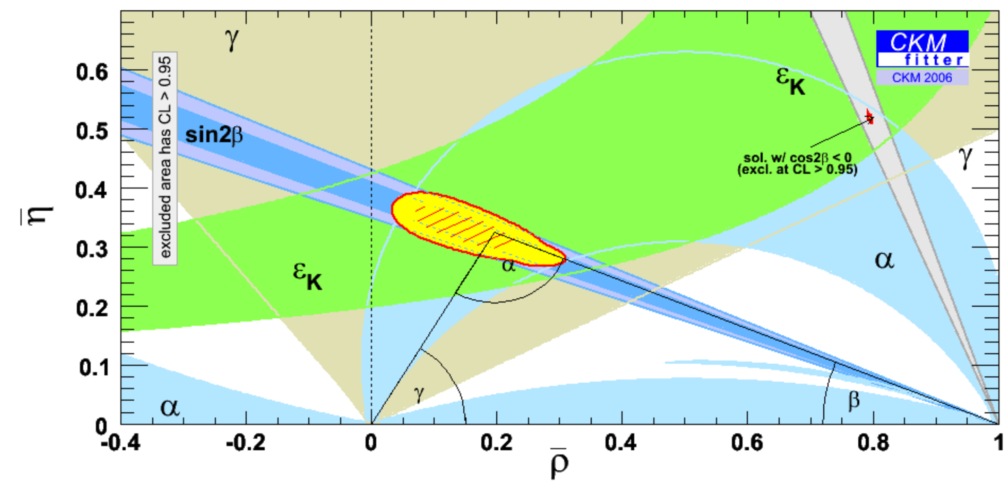


$$\gamma/\phi_3 = (77 \pm 31)^o$$

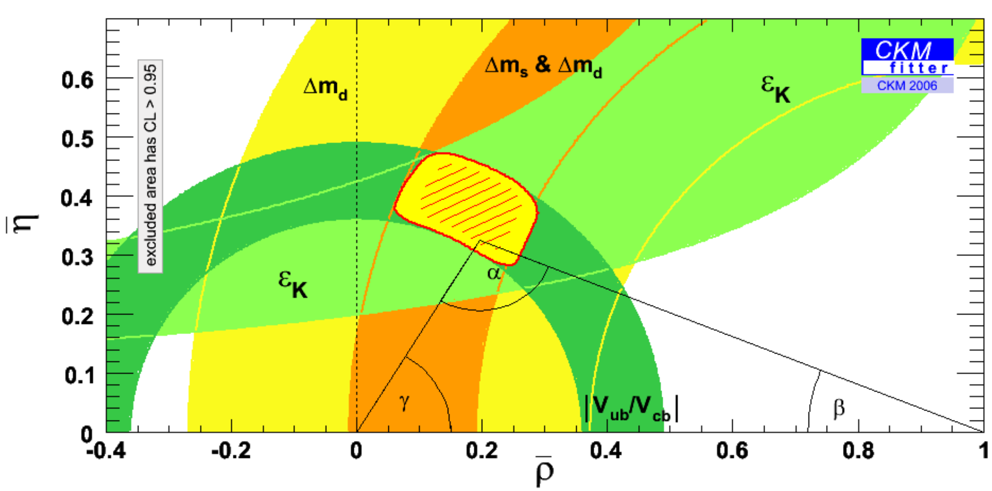
All together...

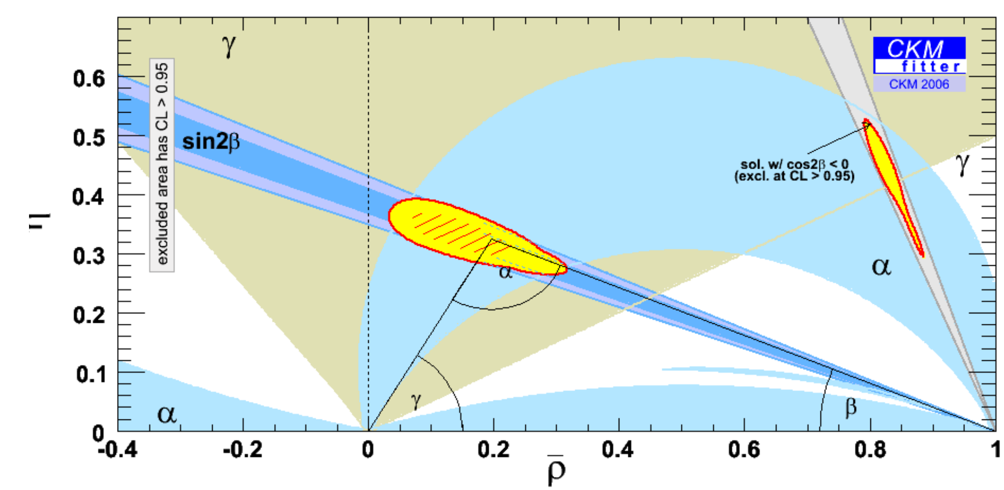




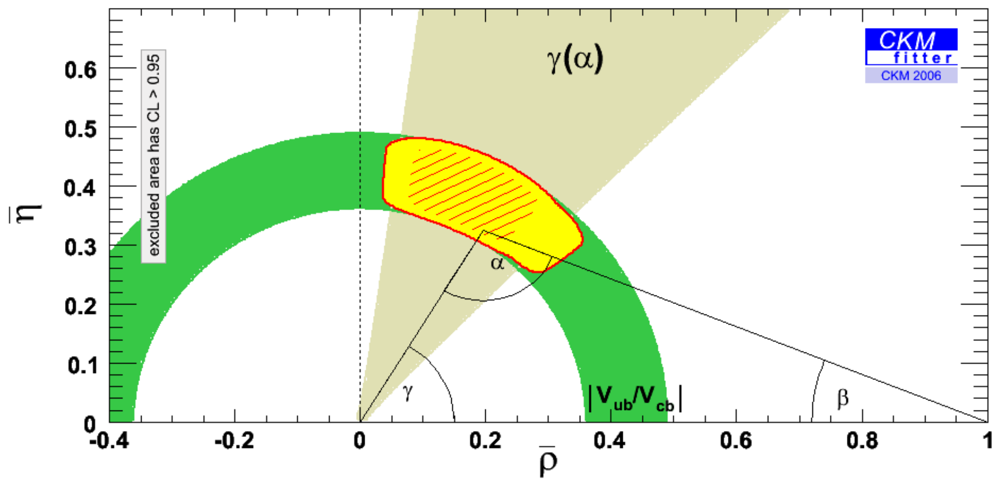


CP conserving *vs* *CP Violating*

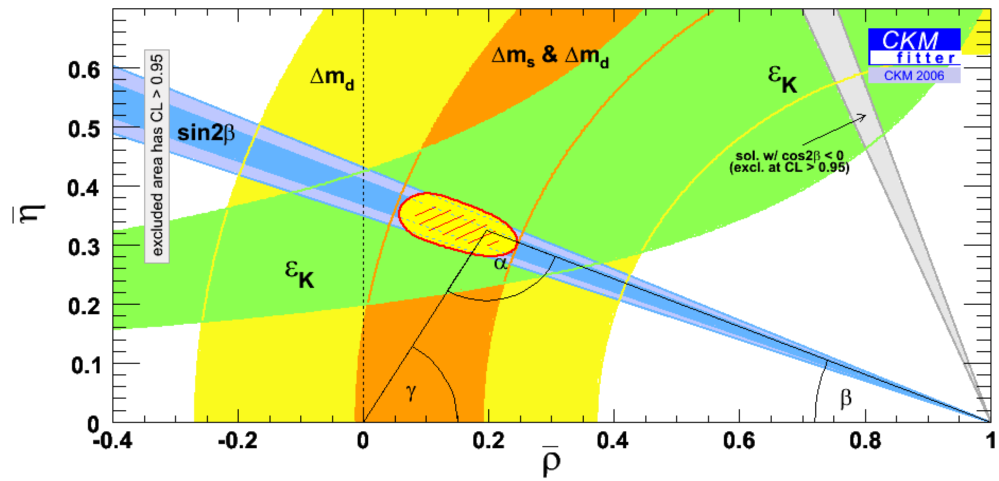




No Angles (with theory) *vs* *Angles (without theory)*


γ(α)
CKM fitter
CKM 2006
excluded area has CL > 0.95
$|V_{ub}/V_{cb}|$
α
γ
β
$\bar{\eta}$
$\bar{\rho}$

tree


Δm_d
Δm_s & Δm_d
ε_K
CKM fitter
CKM 2006
excluded area has CL > 0.95
sin2β
sol. w/ cos2β < 0
(excl. at CL > 0.95)
ε_K
α
γ
β
$\bar{\eta}$
$\bar{\rho}$

loop

Global CKM fit predictions:

Wolfenstein parameters:

$A = 0.806 \pm 0.014 \quad \lambda = 0.2272 \pm 0.0010 \quad \bar{\rho} = 0.195^{+0.024}_{-0.067} \quad \bar{\eta} = 0.326^{+0.032}_{-0.015}$

Jarlskog invariant: $J = (2.90^{+0.29}_{-0.15}) \times 10^{-5}$

UT angles: $\alpha = (98.9^{+4.4}_{-11.3})^\circ, \ \beta = (22.00^{+0.69}_{-0.63})^\circ, \ \gamma = (59.1^{+11.1}_{-4.1})^\circ$

$\sum_{meas} = (191^{+33}_{-32})^\circ$

UT sides: $R_u = (0.380^{+0.011}_{-0.009}), \quad R_t = (0.868^{+0.074}_{-0.026})$

$B - \bar{B}$ mixing: $\Delta m_s = (19.2^{+5.1}_{-3.4})\, ps^{-1}$ [*CKM fit*]

$\Delta m_s = (17.77 \pm 0.10 \pm 0.07)\, ps^{-1}$ [*CDF*]

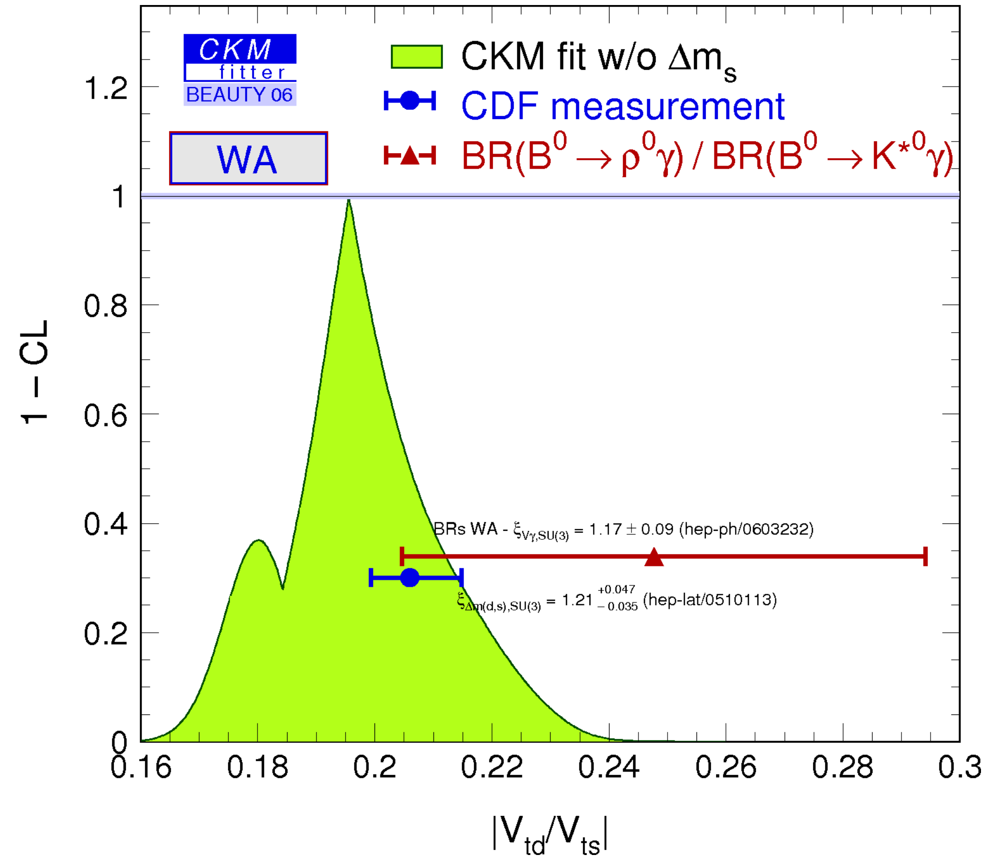
$B^+ \to \tau^+ \nu$: $BF(B^+ \to \tau^+ \nu) = (0.92^{+0.20}_{-0.19}) \times 10^{-4}$ [*CKM fit*]

$BF(B^+ \to \tau^+ \nu) = (1.45^{+0.46}_{-0.43}) \times 10^{-4}$ [*WA*]

Backup slides...

$|V_{td}/V_{ts}|$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d}}{m_{B_s}} \xi_{\Delta m}^{-2} \frac{|V_{td}|^2}{|V_{ts}|^2}$$



$$\frac{B(\bar{B}\to(\rho,\omega)\gamma)}{B(\bar{B}\to\bar{K}^*\gamma)} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1-m^2_{(\rho,\omega)}/m^2_B)^3}{(1-m^2_{K^*}/m^2_B)^3} \xi^2 [1+\Delta R] \quad (A.Ali\,et\,al.)$$

(a) loop diagram

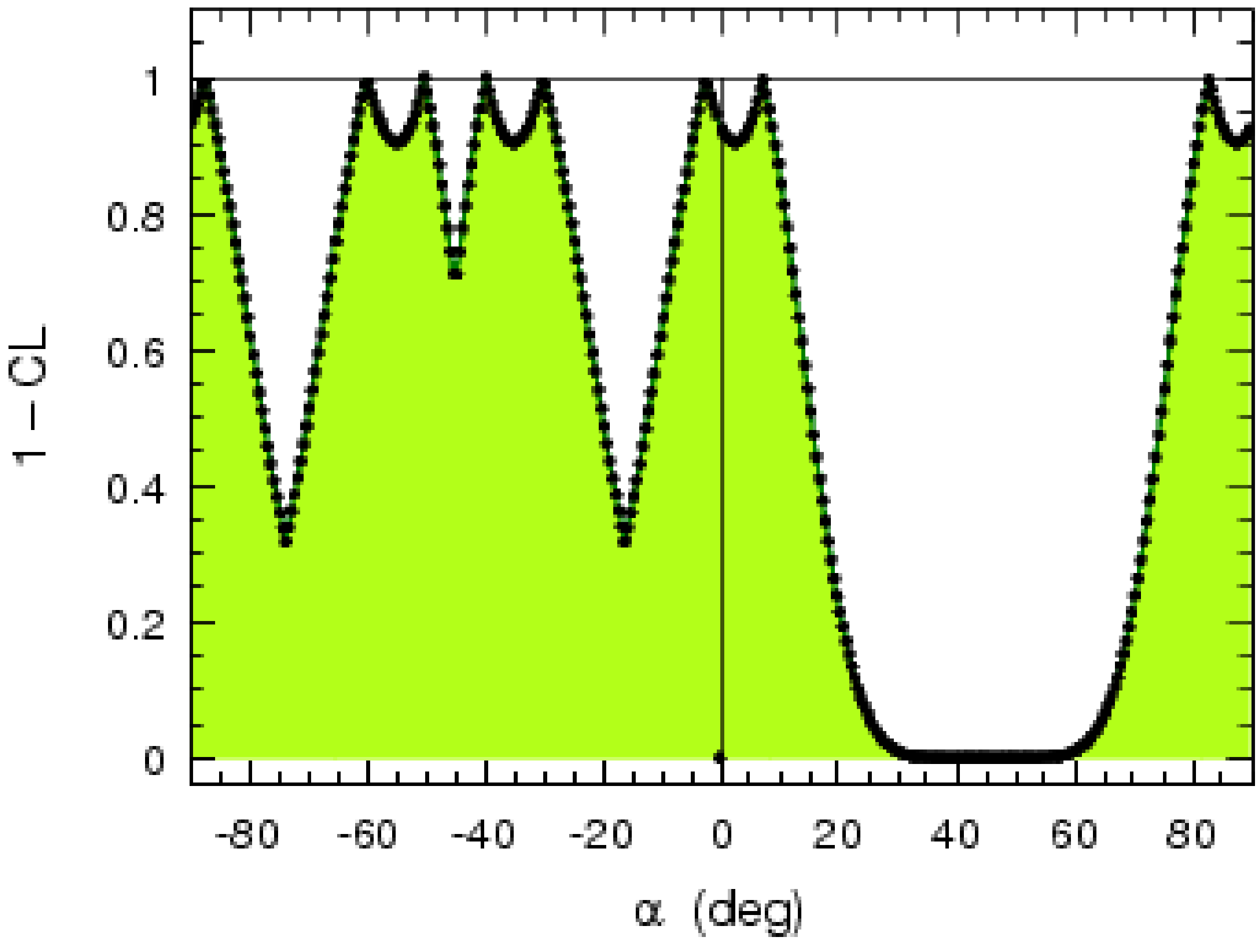
(b) annihilation diagram

- *Form factors for $B\to\rho$ and $B\to\omega$ are different* $(\xi\neq)$

  [ξ**.... *which we assumed to be the same for the ρ^0 and ω mesons...***]
- *annihilation diag for charged mode (probably small contribution)*

The $\alpha \to 0$ limit

($see\ appendix\ A\ of\ hep-ph/0607246$)



In the SM parametrization:

$$C^{+-} = \frac{2\sin\alpha\,\Im(P/T^{+-})}{1+2\cos\alpha\,\Re(P/T^{+-})+|P/T^{+-}|^2}$$

ϕ_3 determination from different measurements

GLW method: *D decays to CP eigenstates*

Relation between $(A_{CP+}, A_{CP-}, R_{CP+}, R_{CP-})$ and (ϕ_3, r_B, δ)

$$A_{CP+} = \frac{2 r_B \sin\delta \sin\phi_3}{1 + r_B^2 + 2 r_B \cos\delta \cos\phi_3} \qquad A_{CP-} = \frac{-2 r_B \sin\delta \sin\phi_3}{1 + r_B^2 - 2 r_B \cos\delta \cos\phi_3}$$

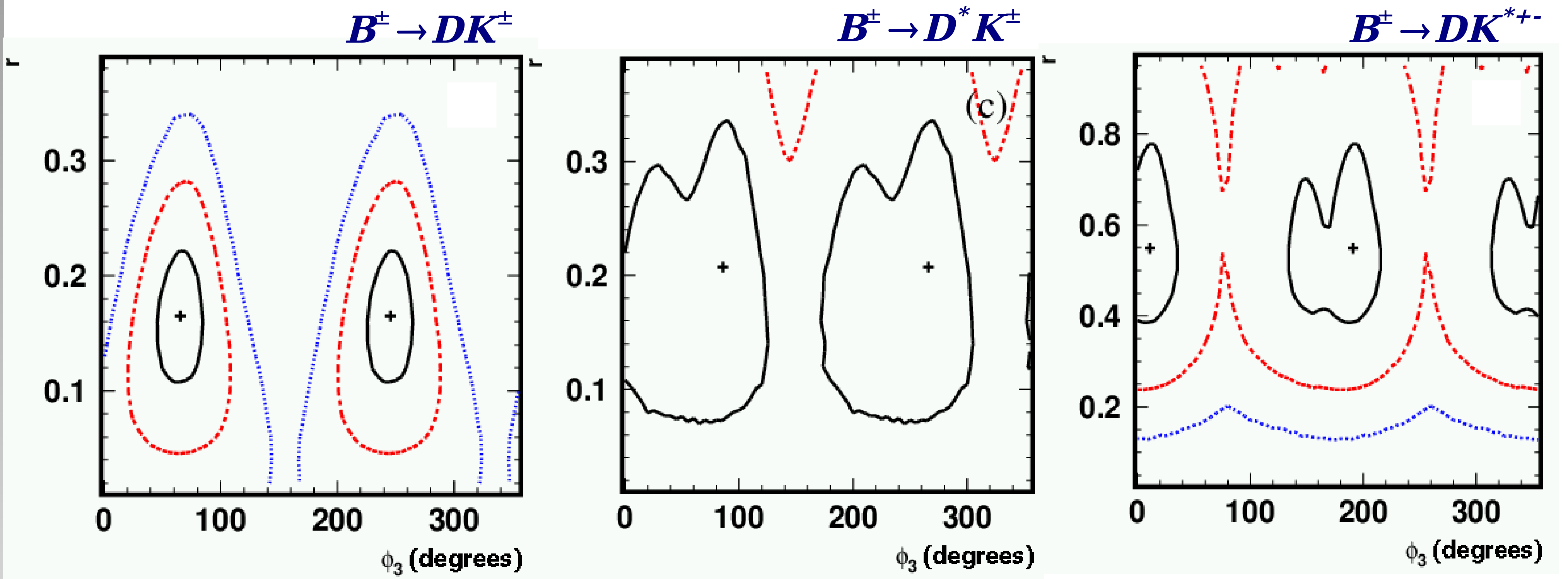
$$R_{CP+} = 1 + r_B^2 + 2 r_B \cos\delta \cos\phi_3 \qquad R_{CP-} = 1 + r_B^2 - 2 r_B \cos\delta \cos\phi_3$$

ADS method: *DCSD*

$$A_{ADS} = \frac{2 r_B r_D \sin(\delta_B + \delta_D) \sin\phi_3}{r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos\phi_3}$$

$$R_{ADS} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos\phi_3$$

$r_B,\ \delta,\ \gamma$ determination (Belle) [PRD73, 112009 (2006)]



$\gamma = 66^{\circ}{}^{+19^\circ}_{-20^\circ}(stat)$

$\gamma = 86^{\circ}{}^{+37^\circ}_{-93^\circ}(stat)$

$\gamma = 11^{\circ}{}^{+23^\circ}_{-57^\circ}(stat)$

Combined the 3 modes:

$$\gamma = 53^{\circ}{}^{+15^\circ}_{-18^\circ}(stat) \pm 3^\circ(syst) \pm 9^\circ(model)$$

$$r_B(DK) = 0.159^{+0.054}_{-0.050} \pm 0.012 \pm 0.049$$

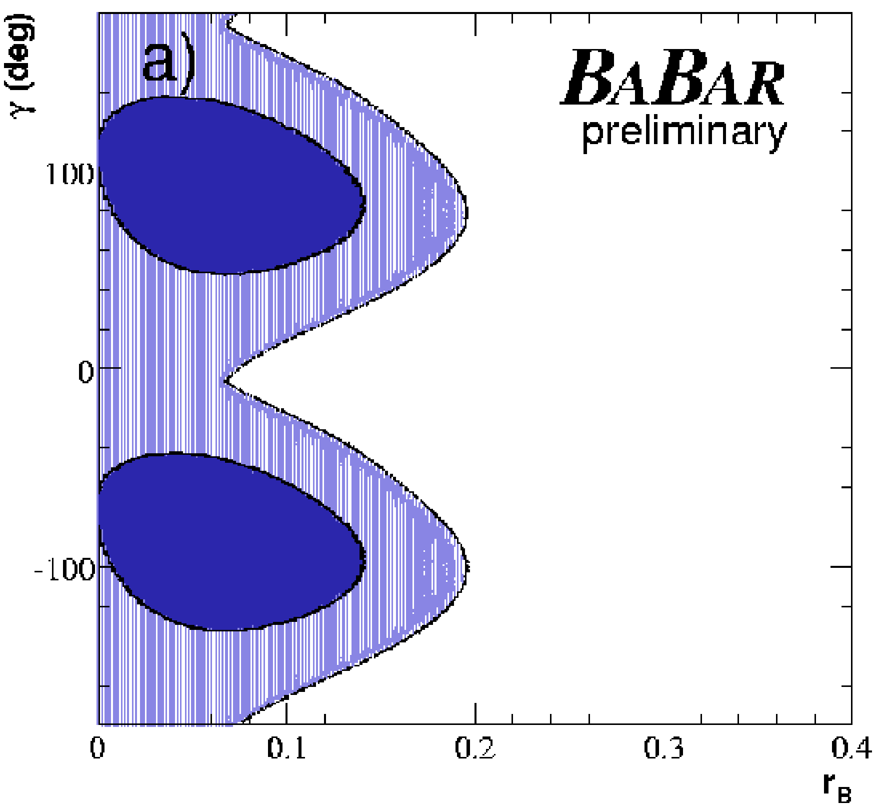
$$r_B(D^*K) = 0.175^{+0.108}_{-0.099} \pm 0.013 \pm 0.049$$

$$r_B(DK^*) = 0.564^{+0.216}_{-0.155} \pm 0.041 \pm 0.084$$

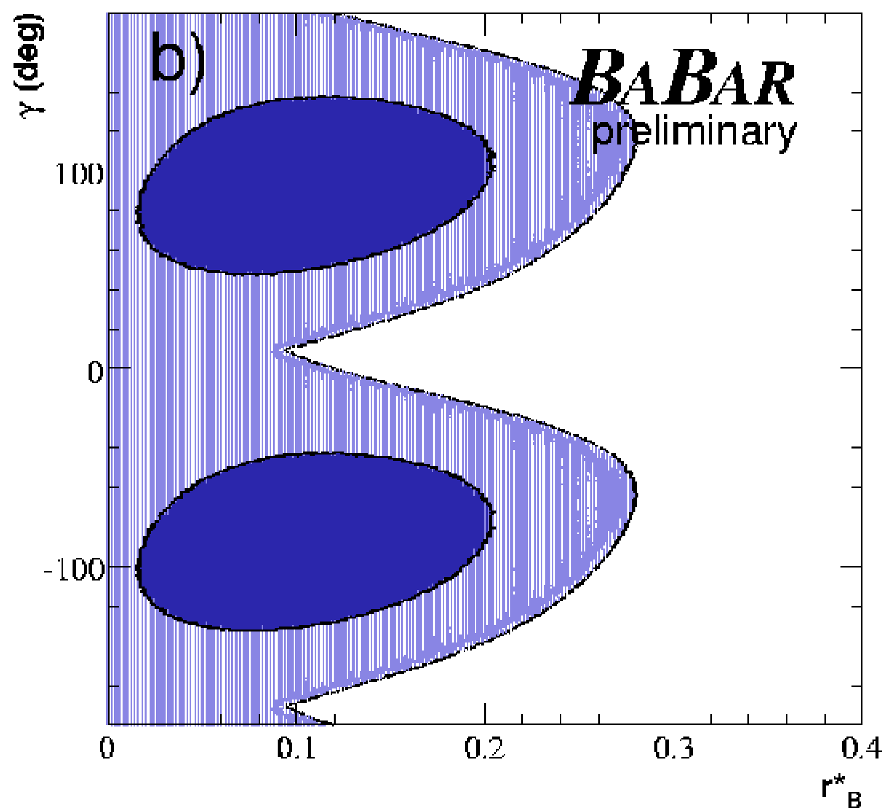
r_B, δ, γ determination (BaBar) [hep–ex/0607104]

→ r, δ, γ are obtained from frequentist (Neyman) statistical treatment

$B^{\pm} \to DK^{\pm}$



$B^{\pm} \to D^{*}K^{\pm}$



$r_B \in [0,\ 0.14]$

$\delta_B = (118 \pm 63 \pm 19 \pm 36)^{\circ}$

$r_B^{*} \in [0.02,\ 0.20]$

$\delta_B^{*} = (-62 \pm 59 \pm 18 \pm 10)^{\circ}$

$$\boldsymbol{\gamma = 92^{\circ} \pm 41^{\circ}(stat) \pm 11^{\circ}(syst) \pm 12^{\circ}(model)}$$

What happened ? (BaBar)

previous analysis: $227\times10^6\, B\bar{B}$ [**PRL95, 121802 (2005)**]

$r_B = 0.12 \pm 0.08 \pm 0.03 \pm 0.04$ $r_B^* = 0.17 \pm 0.10 \pm 0.03 \pm 0.03$

$$\gamma = 70^\circ \pm 31^\circ (stat)^{+12^\circ}_{-10^\circ} (syst)^{+14^\circ}_{-11^\circ} (model)$$

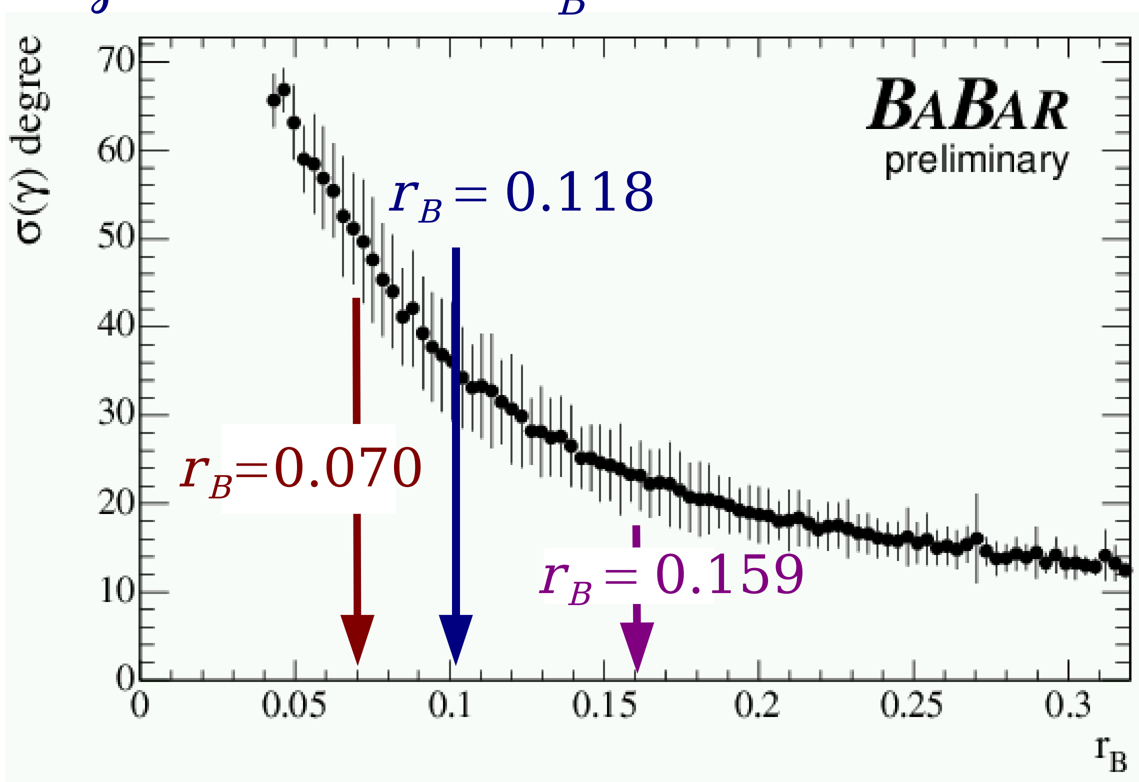
DK, D K* this analysis: $347\times10^6\, B\bar{B}$ [**hep−ex/0607104**]

$0 < r_B < 0.140\ (1\sigma)$ $0.017 < r_B^* < 0.203\ (1\sigma)$

$$\gamma = 92^\circ \pm 41^\circ (stat) \pm 11^\circ (syst) \pm 12^\circ (model)$$

uncertainty on γ scales as $1/r_B$!

e.g. DK



... and for Belle ?

$\underline{\textit{previous}^2 \textit{ analysis}: 152\times10^6 B\overline{B}}$ [***PRD70*, 072003 (2004)**]

$r_B = 0.26^{+0.10}_{-0.14} \pm 0.03 \pm 0.004$ $r_B^* = 0.20^{+0.19}_{-0.17} \pm 0.02 \pm 0.04$

$$\boldsymbol{\gamma = 77^{\circ}{}^{+17^{\circ}}_{-19^{\circ}}(stat) \pm 13^{\circ}(syst) \pm 11^{\circ}(model)}$$

$\underline{\textit{previous analysis}: 275\times10^6 B\overline{B}}$ [***BELLE–CONF–*0476**]

$r_B = 0.21 \pm 0.08 \pm 0.03 \pm 0.004$ $r_B^* = 0.12^{+0.16}_{-0.11} \pm 0.02 \pm 0.04$

$$\boldsymbol{\gamma = 68^{\circ}{}^{+14^{\circ}}_{-15^{\circ}}(stat) \pm 13^{\circ}(syst) \pm 11^{\circ}(model)}$$

$\underline{\textit{this analysis}: 386\times10^6 B\overline{B}}$ [***PRD73*, 112009 (2006)**]

$r_B = 0.159^{+0.054}_{-0.050} \pm 0.012 \pm 0.049$ $r_B^* = 0.175^{+0.108}_{-0.099} \pm 0.013 \pm 0.049$

$$\boldsymbol{\gamma = 53^{\circ}{}^{+15^{\circ}}_{-18^{\circ}}(stat) \pm 3^{\circ}(syst) \pm 9^{\circ}(model)}$$

doesn't always scale as $1/\sqrt{N}$...

$\rightarrow$ *future projection depends of real* r_B *value*

$\sigma_{stat}(\gamma) = 10^{\circ}$ *to* 15° $(r_B \sim 0.1 - 0.15)$ *@ B–factories*

Method reminder...

- measured observables: $x_i \pm \sigma_i \ (i=1,\ldots n)$
- theory prediction: x_i^{th} in fonction of theory parameters, $\mu_j, j=1,\ldots m,\ (r_B, delta\ldots)$ and ϕ_3
- for a fixed ϕ_3^0:

$$\chi^2(\phi_3^0) = \sum_i \frac{(x_i^{th}(\phi_3^0, \mu) - x_i)^2}{\sigma_i^2}$$

substract to global χ^2 where minimize also according ϕ_3

$$\delta\chi^2(\phi_3^0) = \chi^2(\phi_3^0) - \chi^2_{global}$$

- Either compute a confidence level using $CL = 1 - Prob(\delta\chi^2)$
- or generate MC experiments (frequentist)

$$CL(\phi_3^0) = \frac{Number\ of\ experiments\ for\ which\ \delta\chi^2_{MC}(\phi_3^0) < \delta\chi^2_{data}(\phi_3^0)}{Number\ of\ experiments}$$

Finally, minimize CL (nuisance param), plot $1 - CL(\phi_3^0)$ vs ϕ_3^0

Probability of having the data described by the model @ $\phi_3 = \phi_3^0$