

$\beta(\phi_1)$ from B Decays with Charm:

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OUTLINE

- We already have measurement of $\sin 2\beta$ from $B_d \rightarrow J/\psi K_s$.
But 4 fold ambiguity:

$$\left(\beta, \frac{\pi}{2} - \beta\right) \quad \left(\beta + \pi, \frac{\pi}{2} - \beta + \pi\right)$$

- We want to measure the angle β in many processes to test the SM.
Also measure both $\sin 2\beta$ and $\cos 2\beta$.

- This talk is about measuring the CKM angle $\beta(\phi_1)$ from various B decays to charm final states.

- I will concentrate on $B \rightarrow J/\psi K_s^{(*)}$, $B \rightarrow D^{(*)} \bar{D}^{(*)} K_s$ - These are $b \rightarrow c\bar{c}s$ transitions.

- $B \rightarrow D^{(*)} \bar{D}^{(*)} (b \rightarrow c\bar{c}d)$ and $B \rightarrow D^{(*)} h^0 (b \rightarrow c\bar{u}d)$

$$b \rightarrow c\bar{c}s:$$

The general amplitude for $B^0 \rightarrow M_1 M_2$ involving $b \rightarrow c\bar{c}s$ transitions can be written as

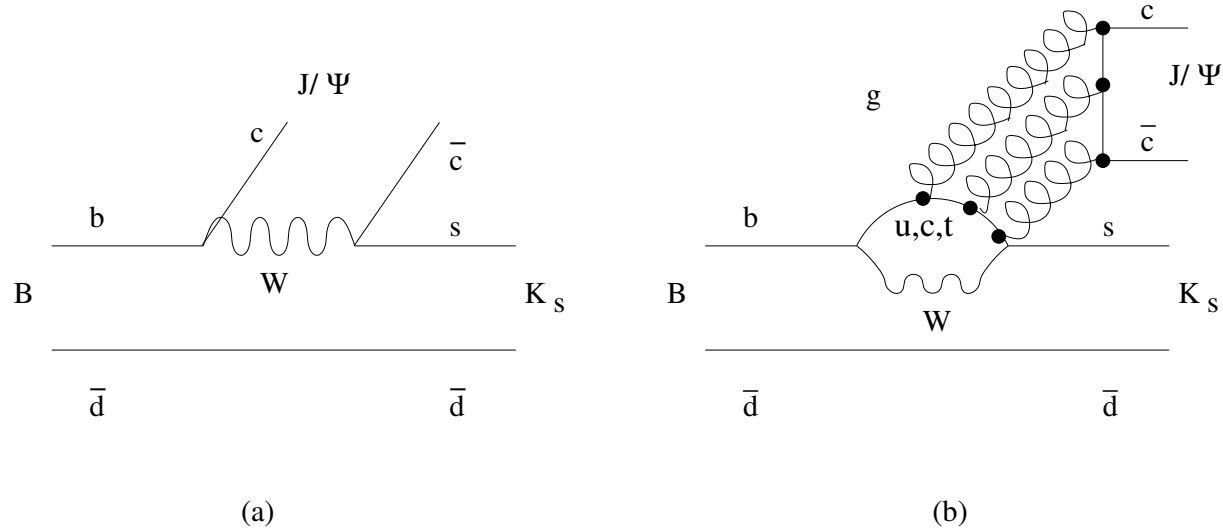
$$\begin{aligned} A(B^0 \rightarrow M_1 M_2) &= A_u V_{ub}^* V_{us} + A_c V_{cb}^* V_{cs} \\ &+ A_t V_{tb}^* V_{ts} \end{aligned}$$

- Use $V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} = 0$ to eliminate the $V_{tb}^* V_{ts}$ term

$$\begin{aligned} A &= (A_u - A_t) V_{ub}^* V_{us} + (A_c - A_t) V_{cb}^* V_{cs} \\ &\approx (A_c - A_t) V_{cb}^* V_{cs} \end{aligned}$$

$$\frac{V_{ub} V_{us}^*}{V_{cb} V_{cs}^*} \sim 2\%$$

$B \rightarrow J/\psi K_s(K^*)$:



$$\begin{aligned}
 A(B \rightarrow J/\psi K_s) &= V_{cb}V_{cs}^* C + V_{tb}V_{ts}^* P_t + V_{cb}V_{cs}^* P_c + V_{ub}V_{us}^* P_u \\
 &= V_{cb}V_{cs}^* (C + P_c - P_t) + V_{ub}V_{us}^* (P_u - P_t) \\
 &\approx V_{cb}V_{cs}^* (C + P_c - P_t)
 \end{aligned}$$

: Single amplitude decay- allows us to obtain $\sin 2\beta$.

$$B(t) \rightarrow J/\psi K^*$$

This is a VV decay and there are three amplitudes.

A_0 , A_{\parallel} - CP even and A_{\perp} which is CP odd.

The time dependent angular distribution allows us to measure both $\sin 2\beta$ and $\cos 2\beta$.

But time independent angular distributions are needed to fix the coefficients of $\sin 2\beta$ and $\cos 2\beta$ in the time dependent distributions.

$$B(t) \longrightarrow J/\psi K^*$$

The time dependent differential decay rate is:

$$Dist = \frac{\Gamma_0}{2} e^{-\Gamma_0 t} \mathcal{A}(\omega; \mathbf{A}) \times \left\{ 1 + \eta \left[\cos \Delta m t \frac{\mathcal{P}(\omega; \mathbf{A})}{\mathcal{A}(\omega; \mathbf{A})} + \sin \Delta m t \left(\frac{\mathcal{S}(\omega; \mathbf{A})}{\mathcal{A}(\omega; \mathbf{A})} \sin 2\beta + \frac{\mathcal{C}(\omega; \mathbf{A})}{\mathcal{A}(\omega; \mathbf{A})} \cos 2\beta \right) \right] \right\}$$

with $\eta = +1$ ($\eta = -1$) for an initial B^0 (\bar{B}^0) meson.

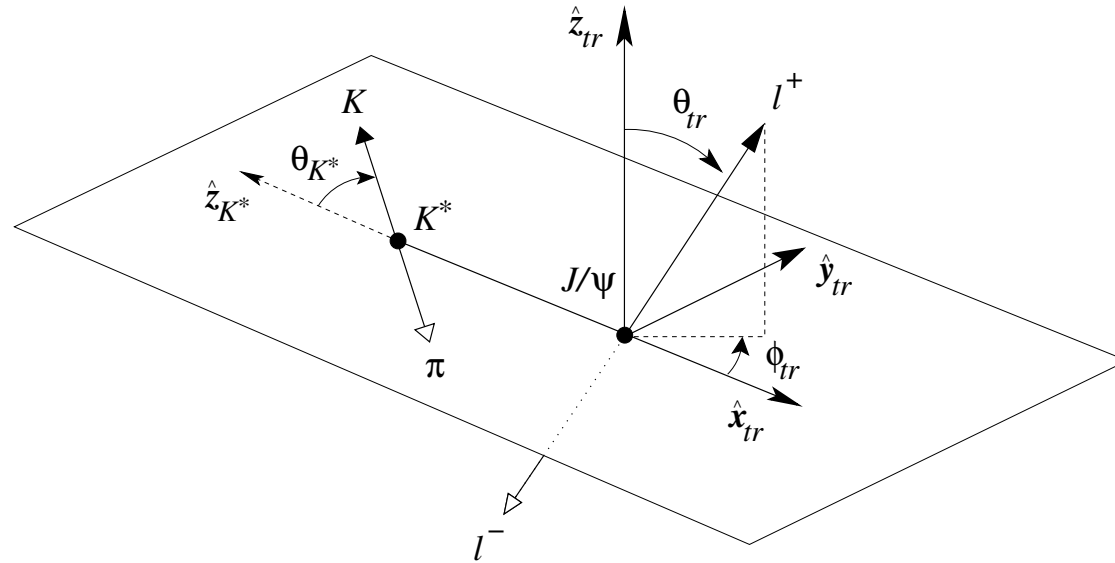
$$\begin{aligned} \mathcal{A}(\omega; \mathbf{A}) &\equiv f_1(\omega)|A_0|^2 + f_2(\omega)|A_{\parallel}|^2 + f_3(\omega)|A_{\perp}|^2 \\ &+ f_5(\omega)|A_0||A_{\parallel}| \cos(\delta_{\parallel} - \delta_0), \end{aligned}$$

$$\mathcal{P}(\omega; \mathbf{A}) \equiv f_4(\omega)|A_{\parallel}||A_{\perp}| \sin(\delta_{\perp} - \delta_{\parallel}) + f_6(\omega)|A_0||A_{\perp}| \sin(\delta_{\perp} - \delta_0),$$

$$\begin{aligned} \mathcal{S}(\omega; \mathbf{A}) &\equiv f_1(\omega)|A_0|^2 + f_2(\omega)|A_{\parallel}|^2 - f_3(\omega)|A_{\perp}|^2 \\ &+ f_5(\omega)|A_0||A_{\parallel}| \cos(\delta_{\parallel} - \delta_0), \end{aligned}$$

$$\mathcal{C}(\omega; \mathbf{A}) \equiv -f_4(\omega)|A_{\parallel}||A_{\perp}| \cos(\delta_{\perp} - \delta_{\parallel}) - f_6(\omega)|A_0||A_{\perp}| \cos(\delta_{\perp} - \delta_0).$$

Angular distribution- $B \rightarrow J/\psi K^*$



$$\frac{1}{\Gamma} \frac{d^3\Gamma}{d \cos \theta_{K^*} d \cos \theta_{tr} d \phi_{tr}} = f_1(\omega) |A_0|^2 + f_2(\omega) |A_{\parallel}|^2 + f_3(\omega) |A_{\perp}|^2 + f_4(\omega) \text{Im}(A_{\parallel}^* A_{\perp}) + f_5(\omega) \text{Re}(A_{\parallel} A_0^*) + f_6(\omega) \text{Im}(A_{\perp} A_0^*),$$

$f_i(\omega)$ are known angular functions of $\omega \equiv (\theta_{K^*}, \theta_{tr}, \phi_{tr})$ $\beta(\phi_1)$ from B Decays with Charm: - p.

$$B \rightarrow J/\psi K^*$$

$A_i = |A_i|e^{i\delta_i}$. The angular analysis can fix $|A_i|$ and the relative strong phases through

$$\text{Im}(A_{\parallel}^* A_{\perp}) = |A_{\parallel}| |A_{\perp}| \sin(\delta_{\perp} - \delta_{\parallel}),$$

$$\text{Re}(A_{\parallel} A_0^*) = |A_{\parallel}| |A_0| \cos(\delta_{\parallel} - \delta_0),$$

$$\text{Im}(A_{\perp} A_0^*) = |A_{\perp}| |A_0| \sin(\delta_{\perp} - \delta_0),$$

$$\begin{aligned} \mathcal{A}(\omega; \mathbf{A}) &\equiv f_1(\omega) |A_0|^2 + f_2(\omega) |A_{\parallel}|^2 + f_3(\omega) |A_{\perp}|^2 \\ &+ f_5(\omega) |A_0| |A_{\parallel}| \cos(\delta_{\parallel} - \delta_0), \end{aligned}$$

$$\mathcal{P}(\omega; \mathbf{A}) \equiv f_4(\omega) |A_{\parallel}| |A_{\perp}| \sin(\delta_{\perp} - \delta_{\parallel}) + f_6(\omega) |A_0| |A_{\perp}| \sin(\delta_{\perp} - \delta_0),$$

$$\begin{aligned} \mathcal{S}(\omega; \mathbf{A}) &\equiv f_1(\omega) |A_0|^2 + f_2(\omega) |A_{\parallel}|^2 - f_3(\omega) |A_{\perp}|^2 \\ &+ f_5(\omega) |A_0| |A_{\parallel}| \cos(\delta_{\parallel} - \delta_0), \end{aligned}$$

$$\mathcal{C}(\omega; \mathbf{A}) \equiv -f_4(\omega) |A_{\parallel}| |A_{\perp}| \cos(\delta_{\perp} - \delta_{\parallel}) - f_6(\omega) |A_0| |A_{\perp}| \cos(\delta_{\perp} - \delta_0).$$

$$B \rightarrow J/\psi K^*$$

The differential decay rate is invariant under the transformation

$$(\delta_{\parallel} - \delta_0, \delta_{\perp} - \delta_0) \longleftrightarrow (\delta_0 - \delta_{\parallel}, \pi + \delta_0 - \delta_{\perp}).$$

This ambiguity translates into a sign ambiguity in the extraction of $\cos 2\beta$.

BABAR(hep-ex/0411016)

$$|A_0|^2 = 0.566 \pm 0.012 \pm 0.005,$$

$$|A_{\parallel}|^2 = 0.204 \pm 0.015 \pm 0.005,$$

$$|A_{\perp}|^2 = 0.230 \pm 0.015 \pm 0.004,$$

$$\text{Solution I : } (\delta_{\parallel} - \delta_0, \delta_{\perp} - \delta_0) \simeq (2.7, 0.2),$$

$$\text{Solution II : } (\delta_{\parallel} - \delta_0, \delta_{\perp} - \delta_0) \simeq (-2.7, \pi - 0.2).$$

$$B(t) \longrightarrow J/\psi K\pi$$

The $K\pi$ system originating from $B \rightarrow J/\psi(K\pi)$ can, in principle, have any integer spin.

Below 1.3 GeV, the S and P waves dominate (Aston 1987).

$$\begin{aligned} Dist(S + P) &\equiv \cos^2 \lambda Dist(P) + \sin^2 \lambda f_7(\omega) \\ &+ \frac{1}{2} \sin 2\lambda \left[\right. \\ &\quad f_8(\omega) \cos(\delta_{\parallel} - \delta_S) |A_{\parallel}| \\ &\quad + f_9(\omega) \sin(\delta_{\perp} - \delta_S) |A_{\perp}| \\ &\quad \left. + f_{10}(\omega) \cos(\delta_S - \delta_0) |A_0| \right]. \end{aligned}$$

$$\cos \lambda \equiv \frac{A_P}{\sqrt{A_P^2 + |A_S|^2}},$$

$$\sin \lambda \equiv \frac{|A_S|}{\sqrt{A_P^2 + |A_S|^2}}, \quad (1)$$

$$B(t) \longrightarrow J/\psi K \pi$$

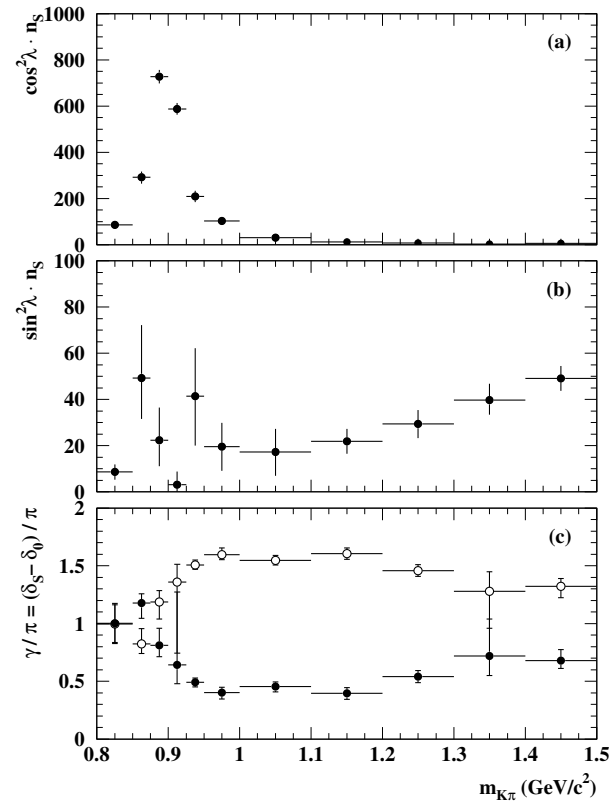
$$\begin{aligned}\delta_{\parallel} - \delta_S &= (\delta_{\parallel} - \delta_0) + (\delta_0 - \delta_S) \\ \delta_{\perp} - \delta_S &= (\delta_{\perp} - \delta_0) + (\delta_0 - \delta_S)\end{aligned}$$

Hence for the two possible solutions for $(\delta_i - \delta_0)$ with $i = \parallel, \perp$ we can fit for $\delta_S - \delta_0$. If we know how $\delta_S - \delta_0$ with $m_{K\pi}$ then we can choose the correct solutions for $(\delta_i - \delta_0)$.

As one goes through the K^* resonance δ_P will change rapidly (going through $\pi/2$ and increasing beyond K^* mass) while δ_S changes slowly. Hence S-P relative $\delta_S - \delta_0$ phase decreases in the region of the K^* (892).

Assume that the strong interactions of the $K\pi$ with the J/ψ do not alter this behavior- there is theory uncertainty here !

Resolving Ambiguity



$$\delta_{\parallel} - \delta_0 = (-2.73 \pm 0.10 \pm 0.05),$$

$$\delta_{\perp} - \delta_0 = (+2.96 \pm 0.07 \pm 0.05),$$

$$\cos 2\beta = +2.72_{-0.79}^{+0.50} \pm 0.27 \quad (2)$$

using $\sin 2\beta$ as an input.

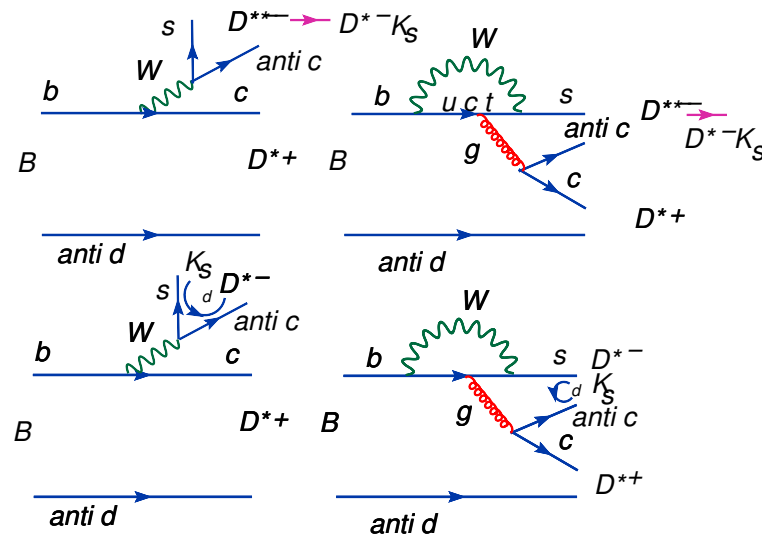
$B(t) \rightarrow D^{*+} D^{*-} K_s$ (Browder et.al.)

- We define the following amplitudes

$$a^{\lambda_1, \lambda_2} \equiv A(B^0(p) \rightarrow D_{\lambda_1}^{+*}(p_+) D_{\lambda_2}^{-*}(p_-) K_s(p_k))$$

$$\bar{a}^{\lambda_1, \lambda_2} \equiv A(\bar{B}^0(p) \rightarrow D_{\lambda_1}^{+*}(p_+) D_{\lambda_2}^{-*}(p_-) K_s(p_k))$$

where B^0 and \bar{B}^0 represent unmixed neutral B and λ_1 and λ_2 are the polarization indices of the D^{*+} and D^{*-} respectively.



$$B(t) \rightarrow D^{*+} D^{*-} K_s$$

The time-dependent amplitudes for an oscillating state $B^0(t)$ which has been tagged as a B^0 meson at time $t = 0$ is given by

$$A^{\lambda_1, \lambda_2}(t) = a^{\lambda_1, \lambda_2} \cos\left(\frac{\Delta m t}{2}\right) + i e^{-2i\beta} \bar{a}^{\lambda_1, \lambda_2} \sin\left(\frac{\Delta m t}{2}\right),$$

and the time-dependent amplitude squared summed over polarizations and integrated over the phase space angles is:

$$|A(s^+, s^-; t)|^2 = \frac{1}{2} [G_0(s^+, s^-) + G_c(s^+, s^-) \cos \Delta m t - G_s(s^+, s^-) \sin \Delta m t]$$

$$G_0(s^+, s^-) = |a(s^+, s^-)|^2 + |\bar{a}(s^+, s^-)|^2,$$

$$G_c(s^+, s^-) = |a(s^+, s^-)|^2 - |\bar{a}(s^+, s^-)|^2,$$

$$\begin{aligned} G_s(s^+, s^-) &= 2\Im(e^{-2i\beta} \bar{a}(s^+, s^-) a^*(s^+, s^-)) \\ &= -2 \sin(2\beta) \Re(\bar{a} a^*) + 2 \cos(2\beta) \Im(\bar{a} a^*) \end{aligned}$$

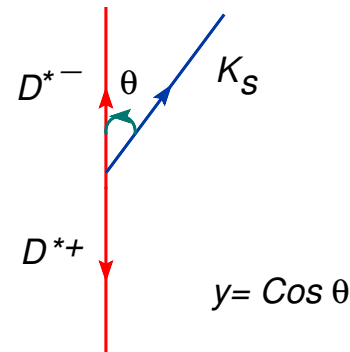
$$s^+ = (p_+ + p_k)^2, \quad s^- = (p_- + p_k)^2.$$

$$B(t) \rightarrow D^{*+} D^{*-} K_s$$

The amplitude for $\bar{B}^0(t) \rightarrow D^{*-} D^{*+} K_s$ can be obtained with $s^+ \leftrightarrow s^-$, $a \leftrightarrow \bar{a}$ and $\beta \rightarrow -\beta$. Then:

$$|\bar{A}(s^-, s^+; t)|^2 = \frac{1}{2} [G_0(s^-, s^+) - G_c(s^-, s^+) \cos \Delta m t + G_s(s^-, s^+) \sin \Delta m t] .$$

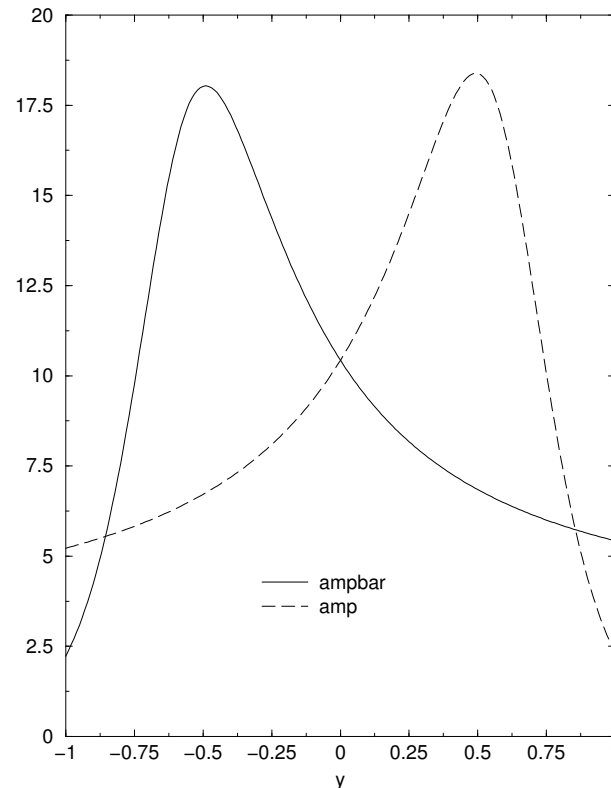
• Replace $(s^+, s^-) \rightarrow (y, E_k)$ where E_k is the K_s energy in the rest frame of the B .



• $y = \cos \theta$ with θ being the angle between the momentum of K_s and D^{*+} in a frame where the two D^* are moving back to back along the z-axis. $s^+ \leftrightarrow s^-$ corresponds to $y \leftrightarrow -y$.

$$B(t) \rightarrow D^{*+} D^{*-} K_S$$

To be sensitive to $\cos 2\beta$ the term $\Im(\bar{a}a^*)$ must be nonzero. This implies \bar{a} and a have to be different with different strong phases.



The presence of an intermediate resonance makes \bar{a} and a different.

$$B(t) \rightarrow D^{*+} D^{*-} K_s$$

In the approximation of neglecting the amplitude proportional to $V_{ub}V_{us}^*$ relative to $V_{cb}V_{cs}^*$ there is no direct CP violation. This leads to the relation

$$a^{\lambda_1, \lambda_2}(\vec{p}_{k1}, E_k) = \bar{a}^{-\lambda_1, -\lambda_2}(-\vec{p}_{k1}, E_k)$$

where \vec{p}_{k1} is the momentum of the of the K_s in the boosted frame. The above relations then leads to

$$G_0(-y, E_k) = G_0(y, E_k)$$

$$G_c(-y, E_k) = -G_c(y, E_k)$$

$$G_{s1}(-y, E_k) = G_{s1}(y, E_k)$$

$$G_{s2}(-y, E_k) = -G_{s2}(y, E_k)$$

where we have defined

$$G_{s1}(y, E_k) = \Re(\bar{a}a^*)$$

$$G_{s2}(-y, E_k) = \Im(\bar{a}a^*)$$

$$B(t) \rightarrow D^{*+} D^{*-} K_s$$

Carrying out the integration over the phase space variables y and E_k one gets the following expressions for the time-dependent total rates for $B^0(t) \rightarrow D^{*+} D^{*-} K_s$ and the CP conjugate process

$$\begin{aligned}\Gamma(t) &= \frac{1}{2} [I_0 + 2 \sin(2\beta) \sin(\Delta mt) I_{s1}] \\ \bar{\Gamma}(t) &= \frac{1}{2} [I_0 - 2 \sin(2\beta) \sin(\Delta mt) I_{s1}]\end{aligned}$$

where I_0 and I_{s1} are the integrated $G_0(y, E_k)$ and $G_{s1}(y, E_k)$ functions. One can then extract $\sin(2\beta)$ from the rate asymmetry

$$\frac{\Gamma(t) - \bar{\Gamma}(t)}{\Gamma(t) + \bar{\Gamma}(t)} = D \sin(2\beta) \sin(\Delta mt); \quad D = \frac{2I_{s1}}{I_0}$$

D is the dilution factor.

$$I_0 = 2\Gamma(0) = 2\bar{\Gamma}(0)$$

$$B(t) \rightarrow D^{*+} D^{*-} K_s$$

The $\cos(2\beta)$ term can be probed by integrating over half the range of the variable y which can be taken for instance to be $y \geq 0$ for B decay and $y \leq 0$ for \bar{B} decay.

$$\frac{\Gamma(t, y \geq 0)}{I_0} = \frac{1}{2} \left[\frac{J_0}{I_0} + \frac{J_c}{I_0} \cos(\Delta mt) + 2 \sin(2\beta) \sin(\Delta mt) \frac{J_{s1}}{I_0} - 2 \cos(2\beta) \sin(\Delta mt) \frac{J_{s2}}{I_0} \right]$$

$$\frac{\bar{\Gamma}(t, y \leq 0)}{I_0} = \frac{1}{2} \left[\frac{J_0}{I_0} + \frac{J_c}{I_0} \cos(\Delta mt) - 2 \sin(2\beta) \sin(\Delta mt) \frac{J_{s1}}{I_0} - 2 \cos(2\beta) \sin(\Delta mt) \frac{J_{s2}}{I_0} \right]$$

where J_0 , J_c , J_{s1} and J_{s2} , are the integrated $G_0(y, E_k)$, $G_c(y, E_k)$, $G_{s1}(y, E_k)$ and $G_{s2}(y, E_k)$ functions integrated over the range $y \geq 0$.

One can measure $\cos(2\beta)$ by fitting to the time distribution of $\frac{\Gamma(t) + \bar{\Gamma}(t)}{I_0}$.

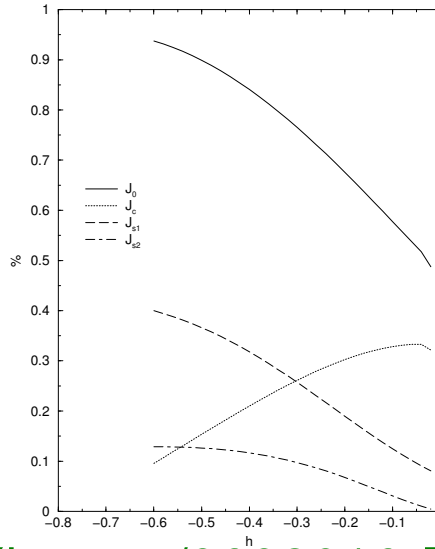
$$B(t) \rightarrow D^{*+} D^{*-} K_s$$

$a^{\lambda_1, \lambda_2}(y, E_k)$ and $\bar{a}^{\lambda_1, \lambda_2}(y, E_k)$ are calculated using factorization and chiral perturbation theory:

Experimentally factorization works well for $B \rightarrow D^{(*)} D^{(*)}$ and the K_s energy is small enough for chiral perturbation to be applicable.

Expect corrections to theory input to change the numerical value of $\cos 2\beta$ somewhat but not the **sign**. Comparing the $\sin 2\beta$ value from this method with $B(t) \rightarrow J/\psi K_s$ will tell us about the reliability of the theory input.

Measurements



The fits to the data yield (hep-ex/0608016-BaBar)

$$\begin{aligned}
 \frac{J_c}{J_0} &= 0.76 \pm 0.18(stat) \pm 0.07(syst) \\
 \frac{2J_{s1}}{J_0} \sin 2\beta &= 0.10 \pm 0.24(stat) \pm 0.06(syst) \\
 \frac{2J_{s2}}{J_0} \cos 2\beta &= 0.38 \pm 0.24(stat) \pm 0.05(syst)
 \end{aligned} \tag{3}$$

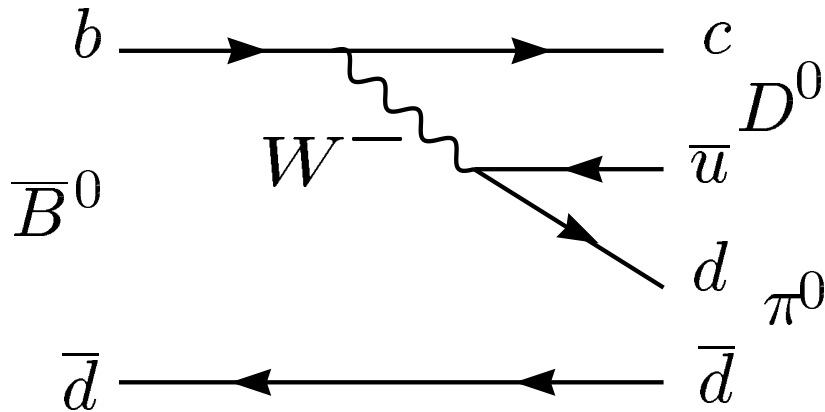
New Resonance ?

$\cos 2\beta$ is preferred to be positive at the 94 % confidence level as the theoretical parameter J_{s2}/J_0 is positive.

J_c/J_0 is significantly different from zero, this implies that there is a sizable resonant contribution to the decay from a unknown D_{s1}^+ state with large width.

$B(t) \rightarrow Dh^0$ (Bondar et al)

Consider the decays of the B meson to $Dh^0(\pi^0, \eta..)$. The dominant decay is through $b \rightarrow c\bar{u}d$



Next consider multibody D meson decay. As an example use $D \rightarrow K_S \pi^+ \pi^-$.

$$\text{Amp}(\bar{D}^0 \rightarrow K_S \pi^+ \pi^-) = f(m_+^2, m_-^2)$$

$$\text{Amp}(D^0 \rightarrow K_S \pi^+ \pi^-) = f(m_-^2, m_+^2)$$

$$m_{+,-}^2 = (p_{K_S} + p_{\pi^\pm})^2$$

$B(t) \rightarrow Dh^0$ (Bondar et al)

The amplitude for the B decay at time t_{sig} is then given by

$$M_{\bar{B}^0}(\Delta t) = [\bar{B}^0 \rightarrow D^0(K_s\pi^+\pi^-)h^0] \cos(\Delta m\Delta t/2) - i\frac{p}{q}\eta_{h^0}(-1)^l [B^0 \rightarrow \bar{D}^0(K_s\pi^+\pi^-)h^0] \sin(\Delta m\Delta t/2).$$

$$M_{\bar{B}^0}(\Delta t) = f(m_-^2, m_+^2) \cos(\Delta m\Delta t/2) - i\frac{p}{q}\eta_{h^0}(-1)^l f(m_+^2, m_-^2) \sin(\Delta m\Delta t/2).$$

Then

$$M_{\bar{B}^0}(\Delta t) = f(m_-^2, m_+^2) \cos(\Delta m\Delta t/2) - ie^{-i2\phi_1}\eta_{h^0}(-1)^l f(m_+^2, m_-^2) \sin(\Delta m\Delta t/2),$$

$$M_{B^0}(\Delta t) = f(m_+^2, m_-^2) \cos(\Delta m\Delta t/2) - ie^{+i2\phi_1}\eta_{h^0}(-1)^l f(m_-^2, m_+^2) \sin(\Delta m\Delta t/2),$$

Need to model $f(m_+^2, m_-^2)$ to extract the phase $2\phi_1(2\beta)$. $\beta(\phi_1)$ from B Decays with Charm: – p.2

Modeling D Decay

Three body D decays is modeled as the coherent sum of two-body decay amplitudes plus non-resonant part:

$$f(m_{K_S\pi^+}^2, m_{K_S\pi^+}^2) = \sum_{J=1}^N a_j e^{i\alpha_j} A_j(m_{K_S\pi^+}^2, m_{K_S\pi^+}^2) + b e^{i\beta},$$

where N is the number of resonances, $A_j(m_{K_S\pi^+}^2, m_{K_S\pi^+}^2)$, a_j and α_j are the matrix element, amplitude and phase, respectively, for the j -th resonance, and b and β are the amplitude and phase for the non-resonant component.

Measurements

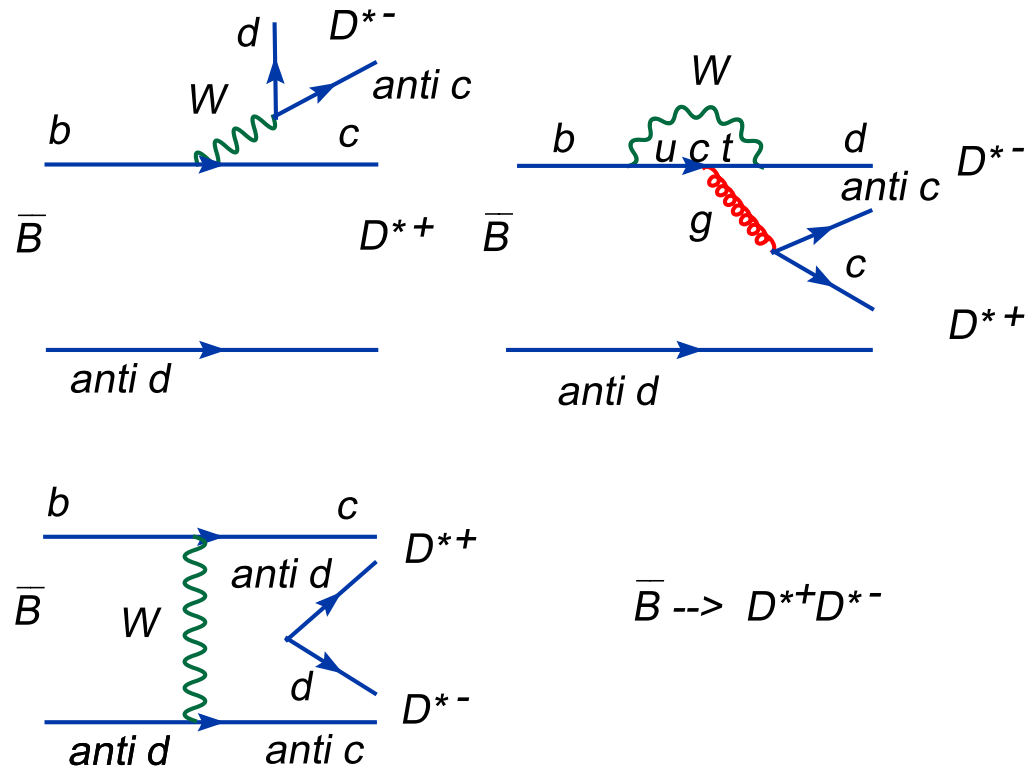
Resonance	Amplitude	Phase ($^{\circ}$)
$K^*(892)^+\pi^-$	1.418	170
$K_0^*(1430)^+\pi^-$	1.818	23
$K_2^*(1430)^+\pi^-$	0.909	194
$K^*(892)^-\pi^+$ (DCS)	0.100	341
$K_S\rho^0$	0.909	20
$K_S\omega$	0.034	134
$K_S f_0(980)$	0.309	208
$K_S f_0(1370)$	1.636	105
$K_S f_2(1270)$	0.636	328
$K_S\pi^+\pi^-$ non-resonant	1.0	0

$$\cos 2\beta = 0.54 \pm 0.54 \pm 0.08 \pm 0.18,$$

$$\sin 2\beta = 0.45 \pm 0.36 \pm 0.05 \pm 0.07.$$

$$\bar{B}(t) \rightarrow D^{(*)} \bar{D}^{(*)}$$

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$$\begin{aligned}
 A^D &= (T + E + P_c) V_{cb}^* V_{cd} + P_u V_{ub}^* V_{ud} + (P_t + P_{EW}^C) V_{tb}^* V_{td} \\
 &= (T + E + P_c - P_t - P_{EW}^C) V_{cb}^* V_{cd} + (P_u - P_t - P_{EW}^C) V_{ub}^* V_{ud} \\
 &\equiv \mathcal{A}_{ct} e^{i\delta^{ct}} + \mathcal{A}_{ut} e^{i\gamma} e^{i\delta^{ut}},
 \end{aligned}$$

$$\bar{B}(t) \rightarrow D^{(*)} \bar{D}^{(*)}$$

- The time-dependent measurement of $B^0(t) \rightarrow M_1 M_2$ allows one to obtain the three observables

$$\begin{aligned}
 B &\equiv \frac{1}{2} (|A|^2 + |\bar{A}|^2) = \mathcal{A}_{ct}^2 + \mathcal{A}_{ut}^2 \\
 &+ 2\mathcal{A}_{ct} \mathcal{A}_{ut} \cos \delta \cos \gamma \\
 a_{dir} &\equiv \frac{1}{2} (|A|^2 - |\bar{A}|^2) = -2\mathcal{A}_{ct} \mathcal{A}_{ut} \sin \delta \sin \gamma \\
 a_{indir} &\equiv \text{Im} (e^{-2i\beta} A^* \bar{A}) \\
 &- \mathcal{A}_{ct}^2 \sin 2\beta - 2\mathcal{A}_{ct} \mathcal{A}_{ut} \cos \delta \sin(2\beta + \gamma) \\
 &- \mathcal{A}_{ut}^2 \sin(2\beta + 2\gamma)
 \end{aligned}$$

- The three independent observables depend on five theoretical parameters: \mathcal{A}_{ut} , \mathcal{A}_{ct} , δ , β , γ .

$$\bar{B}(t) \rightarrow D^{(*)} \bar{D}^{(*)}$$

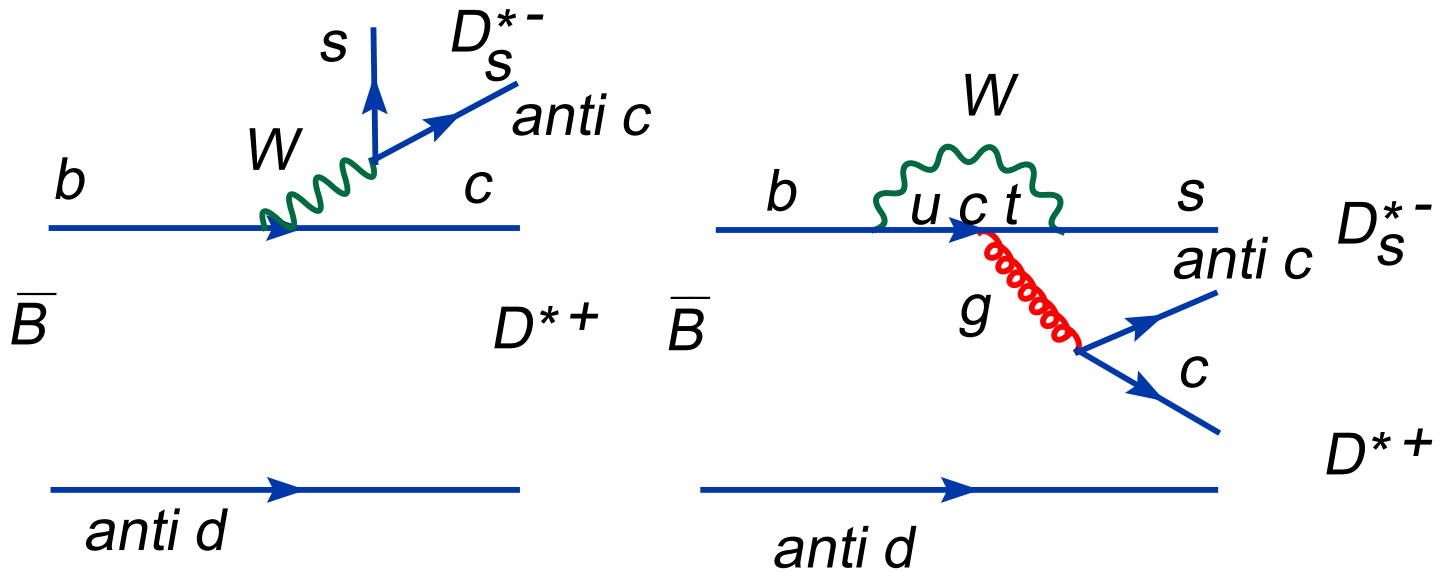
Therefore one cannot obtain CP phase information from these measurements- Penguin Pollution. Hence theoretical input is necessary to get the CKM phase information

$$\mathcal{A}_{ct}^2 = \frac{a_R \cos(2\beta + 2\gamma) - a_{indir} \sin(2\beta + 2\gamma) - B}{\cos 2\gamma - 1}$$
$$a_R^2 = B^2 - a_{dir}^2 - a_{indir}^2 .$$

Obtain \mathcal{A}_{ct} from a partner process and use it as a theory input. We can then obtain γ if given β and vice versa.

$$\bar{B} \rightarrow D^{(*)} \bar{D}_s^{(*)}$$

Partner process is $\bar{B} \rightarrow D^{(*)} \bar{D}_s^{(*)}$



$$\bar{B} \rightarrow D_S^{*-} D^{*+}$$

$$\begin{aligned} A^{D_s} &= (T' + P'_c) V_{cb}^* V_{cs} + P'_u V_{ub}^* V_{us} + (P'_t + P'_{EW}) V_{tb}^* V_{ts} \\ &= (T' + P'_c - P'_t - P'_{EW}) V_{cb}^* V_{cs} + (P'_u - P'_t - P'_{EW}) V_{ub}^* V_{us} \\ &\approx (T' + P'_c - P'_t - P'_{EW}) V_{cb}^* V_{cs} \equiv \mathcal{A}'_{ct} e^{i\delta'_{ct}}. \end{aligned}$$

Theory Input

Note the measurement of the total rate for $\bar{B} \rightarrow D^{(*)} \bar{D}_s^{(*)}$ yields \mathcal{A}'_{ct}

Theory input(SU(3))

$$\Delta \equiv \frac{\sin \theta_c \mathcal{A}'_{ct}}{\mathcal{A}_{ct}} = \frac{\sin \theta_c |(T' + P'_c - P'_t - P'_{EW}) V_{cb}^* V_{cs}|}{|(T + E + P_c - P_t - P_{EW}^C) V_{cb}^* V_{cd}|} = 1,$$

$$\mathcal{A}_{ct}^2 = (\sin \theta_c \mathcal{A}'_{ct})^2 = \frac{a_R \cos(2\beta + 2\gamma) - a_{indir} \sin(2\beta + 2\gamma) - B}{\cos 2\gamma - 1}$$

Neglecting exchange diagrams we can estimate SU(3) breaking as

$$\Delta \equiv \frac{\sin \theta_c \mathcal{A}'_{ct}}{\mathcal{A}_{ct}} = \frac{f_{D_s^*}}{f_{D^*}}$$

Conclusions

- I discussed the extraction of $\sin 2\beta$ and $\cos 2\beta$ from $B \rightarrow J/\psi K_s^{(*)}$, $B \rightarrow D^{(*)} \bar{D}^{(*)} K_s$ - these are $b \rightarrow c\bar{c}s$ transitions and from $B \rightarrow D^{(*)} h^0$ ($b \rightarrow c\bar{u}d$)
- All methods require some modeling of hadronic decays. But measurement for all three decays give the same positive sign for $\cos 2\beta$.
- The decay $\bar{B} \rightarrow D^{(*)} \bar{D}^{(*)}$ ($b \rightarrow c\bar{c}d$) cannot be used to obtain CP phase information. However with a partner process $\bar{B} \rightarrow D^{(*)} \bar{D}_s^{(*)}$ and theory input (SU(3)) one can obtain γ with β as input and vice versa.