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# Extraction of $2\beta+\gamma$ from $B \to D\,\pi$ and related decays

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# Outline

- $2\beta+\gamma$ from time-integrated $B \to D^{(*)}\pi$ decays
- The parameter $R$
- Optimal condition for minimum vertex separation
  - Perfect time resolution and no mistagging
  - Finite time resolution and no mistagging
  - Finite time resolution with mistagging
- A four-fold ambiguity
- Summary

Talk primarily based upon the following work:

Suprun, CWC, and Rosner, PRD **65**, 054025 (2002).

# Notations

- We use the following notations for the decay amplitudes, weak phases, and strong phases:

| Mode | RS decays $(V_{cb}^* V_{ud})$ | WS decays $(V_{cd}^* V_{ub})$ |
|---|---|---|
| $f = D^{(*)-}\pi^+$ | $\langle f \mid B^0 \rangle = A_1 e^{i\phi_1} e^{i\delta_1}$ | $\langle f \mid \overline{B}^0 \rangle = A_2 e^{i\phi_2} e^{i\delta_2}$ |
| $\bar{f} = D^{(*)+}\pi^-$ | $\langle \bar{f} \mid \overline{B}^0 \rangle = A_1 e^{-i\phi_1} e^{i\delta_1}$ | $\langle \bar{f} \mid B^0 \rangle = A_2 e^{-i\phi_2} e^{i\delta_2}$ |

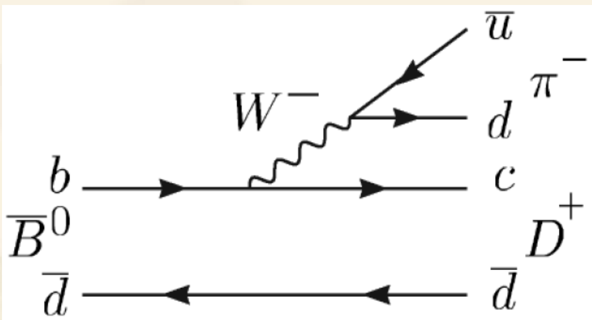
where $\phi_i$ and $\delta_i$ denote weak and strong phases, respectively.

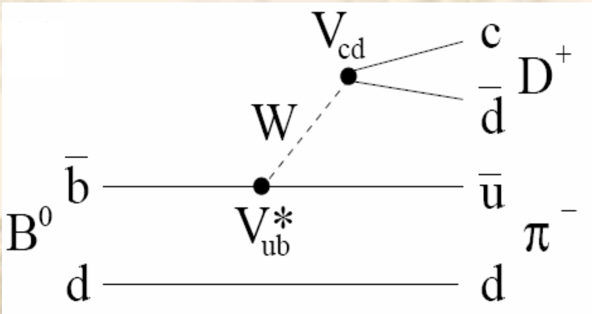
- The amplitudes are in the ratio

$$R \equiv \frac{A_2}{A_1} = \left| \frac{V_{cd}^* V_{ub}}{V_{cb}^* V_{ud}} \right| r \simeq 0.022 r$$

where $r = O(1)$ describes a ratio of hadronic ME's.

- The weak phase $\phi_1 - \phi_2 = \text{Arg}\left( \frac{V_{cb}^* V_{ud}}{V_{cd}^* V_{ub}} \right) = \pi + \gamma$



- Let $t'$ be the difference between the decay time and tagging time, and define quantities involved in time-integrated decays ($\delta \equiv \delta_2 - \delta_1$)

$$A_{\pm}(t') \equiv (1+R^2) \pm (1-R^2)\cos\Delta m t' \ , \qquad \text{(even in } t')$$
$$B_{\pm}(t') \equiv -2R\sin(2\beta+\gamma\pm\delta)\sin\Delta m t' \ . \qquad \text{(odd in } t')$$

- Consider *time-integrated* decays with vertex separations $t' \geq t_0$ (+) or $t' \leq -t_0$ (–). Apart from an overall normalization, different time-integrated numbers of decays are:

$$N_{\pm}^{r}(t_0) \propto \int_{t_0}^{\infty} dt'\, e^{-\Gamma|t'|}[A_{+}(t') \pm B_{-}(t')] \ ,$$
$$N_{\pm}^{w}(t_0) \propto \int_{t_0}^{\infty} dt'\, e^{-\Gamma|t'|}[A_{-}(t') \pm B_{+}(t')] \ ,$$
$$\overline{N_{\pm}^{r}}(t_0) \propto \int_{t_0}^{\infty} dt'\, e^{-\Gamma|t'|}[A_{+}(t') \mp B_{+}(t')] \ ,$$
$$\overline{N_{\pm}^{w}}(t_0) \propto \int_{t_0}^{\infty} dt'\, e^{-\Gamma|t'|}[A_{-}(t') \mp B_{-}(t')] \ .$$

exist 4 linear relations

- Here $\Delta m = m_H - m_L$, $\Delta\Gamma = \Gamma_H - \Gamma_L$, $m = (m_H + m_L)/2$, $\Gamma = (\Gamma_H + \Gamma_L)/2$.

- Define the quantities

$$
\begin{aligned}
N &\equiv N_+^r + N_-^r + \overline{N_+^r} + \overline{N_-^r} + N_+^w + N_-^w + \overline{N_+^w} + \overline{N_-^w} \ , \\
f_1(t_0) &\equiv \left\{ (N_+^r + N_-^r + \overline{N_+^r} + \overline{N_-^r}) - (N_+^w + N_-^w + \overline{N_+^w} + \overline{N_-^w}) \right\} / N \ , \\
f_{2,3}(t_0) &\equiv \left\{ (N_\mp^r + N_-^w + \overline{N_+^r} + \overline{N_\pm^w}) - (N_\pm^r + N_+^w + \overline{N_-^r} + \overline{N_\mp^w}) \right\} / N \ , \\
a(t_0) &\equiv e^{\Gamma t_0} \int_{t_0}^{\infty} dt' \, e^{-\Gamma t'} \cos(\Delta m t') = \frac{1}{\sqrt{1+x_d^2}} \cos(x_d \Gamma t_0 + \Delta) \ , \\
b(t_0) &\equiv e^{\Gamma t_0} \int_{t_0}^{\infty} dt' \, e^{-\Gamma t'} \sin(\Delta m t') = \frac{1}{\sqrt{1+x_d^2}} \sin(x_d \Gamma t_0 + \Delta) \ .
\end{aligned}
$$

total events

RS-WS

$$\left( x_d \equiv \frac{\Delta m_{B_d}}{\Gamma_d} \right)$$

- Then we have

$$
\begin{aligned}
R &= \sqrt{\frac{a(t_0) - f_1(t_0)}{a(t_0) + f_1(t_0)}} \ , \\
SC &\equiv \sin(2\beta + \gamma) \cos\delta = \frac{1+R^2}{2b(t_0)\,R} f_2(t_0) \ , \\
CS &\equiv \cos(2\beta + \gamma) \sin\delta = \frac{1+R^2}{2b(t_0)\,R} f_3(t_0) \ .
\end{aligned}
$$

# Quick Error Estimate

- However, $R$ determined in this way has a large error:

$$\sigma(R) = \sqrt{\frac{x_d^2(2+x_d^2)}{16R^2}\,\frac{1}{\epsilon(\mathcal{B}^r+\mathcal{B}^w)N_B}} \approx 0.013,$$

which is more than half its value, if we take

$N_B = 400\text{M}$, [$B$-$B$bar pairs]

$\varepsilon = 0.793$, [tagging efficiency of BABAR (hep-ex/0609033)]

$x_d = 0.776$, [HFAG 2006]

$B^r \equiv BR(B^0 \to D^{*-}\pi^+) = 2.85 \times 10^{-3}$, and

$B^w \equiv B^r\, x_d^2/(2+x_d^2) \simeq 0.23\, B^r$.

- Therefore, $R$ is too small to be determined using this method.

[Dunietz, PLB **427**, 179 (1998); London, Sinha, and Sinha, PRL **85**, 1807 (2000)]

# Isospin Estimate

- The primary reason $R$ cannot be easily obtained from the ratio of $B^0 \to D^{*+}\pi^-$ and $B^0 \to D^{*-}\pi^+$ decay rates is that the large $B^0 \to B^0$-bar mixing amplitude in the former (WS) overwhelms the smaller direct tree contribution. [See Baak's talk]
- Instead, one can consider the isospin-related decay $B \to D^{*\pm}\pi^0$. [Dunietz, PLB **427**, 179 (1998)]
- Using this method, we obtain

$$R = \sqrt{2\frac{\tau_{B^0}}{\tau_{B^+}}\frac{N_2}{N_1}}$$

# of $B^\pm \to D^{*\pm}\pi^0$ (→ $N_2$)

# of $B^0 \to D^{*+}\pi^-$ (→ $N_1$)

$$\sigma(R) = \sqrt{2\frac{\tau_{B^0}}{\tau_{B^+}}}\frac{1}{2\sqrt{N_1}}\sqrt{1+\frac{N_2}{N_1}} \approx \frac{1}{\sqrt{2N_1}} \simeq 7\times 10^{-4}\ .$$

for 400M $B$-$B$bar pairs, tremendously improved from before and less than 5% of its value.

# Estimate of $r$

- In our analysis, we will take $R = 0.022\, r$, where $r$ is estimated as follows:
- Assuming factorization, one has for the ratio

$$r(D^*\pi) = \frac{f_{D^*}\, F_1^{B\pi}(m_{D^*}^2)}{f_\pi\, A_0^{BD^*}(m_\pi^2)}\,. \qquad \left[ r(D\pi) = \frac{f_D\,(m_B^2 - m_\pi^2)\, F_0^{B\pi}(m_D^2)}{f_\pi\,(m_B^2 - m_D^2)\, F_0^{BD}(m_\pi^2)}\,. \right]$$

- These ratios are determined to be close to unity. [$r(D^*\pi) = 0.97$ and $r(D\pi) = 1.09$ using LF form factors.] We will assume them to be 1 in the analysis. [Cheng, Chua, and Hwang, PRD **69**, 074025 (2004)]

# Questions

- What about the error analysis for

  $SC \equiv \sin(2\beta+\gamma)\cos\delta$ and $CS \equiv \cos(2\beta+\gamma)\sin\delta$?

- Is there an optimal condition of the minimal vertex separation $t_0$ for measuring these quantities?

- Can we determine $2\beta+\gamma$ without ambiguity?

# Uncertainties with Perfect Time Resolution and No Mistagging

- The uncertainties on *SC* and *CS* are (expecting $SC \simeq 1$)

$$\sigma(SC) \approx \frac{1}{2b(t_0)\,R}\sqrt{\frac{2(1+k)\,b^2(t_0)+e^{\Gamma t_0}}{\epsilon(1+k)\,\mathcal{B}^r N^B}}\,,$$

$$\sigma(CS) \approx \frac{1}{2b(t_0)\,R}\sqrt{\frac{e^{\Gamma t_0}}{\epsilon(1+k)\,\mathcal{B}^r N^B}}\,.$$

- Inverting the above relations, one can compute how many *B*-*B*bar pairs are required to achieve the accuracies $\sigma_0(SC,CS)$ for *SC* and *CS*:

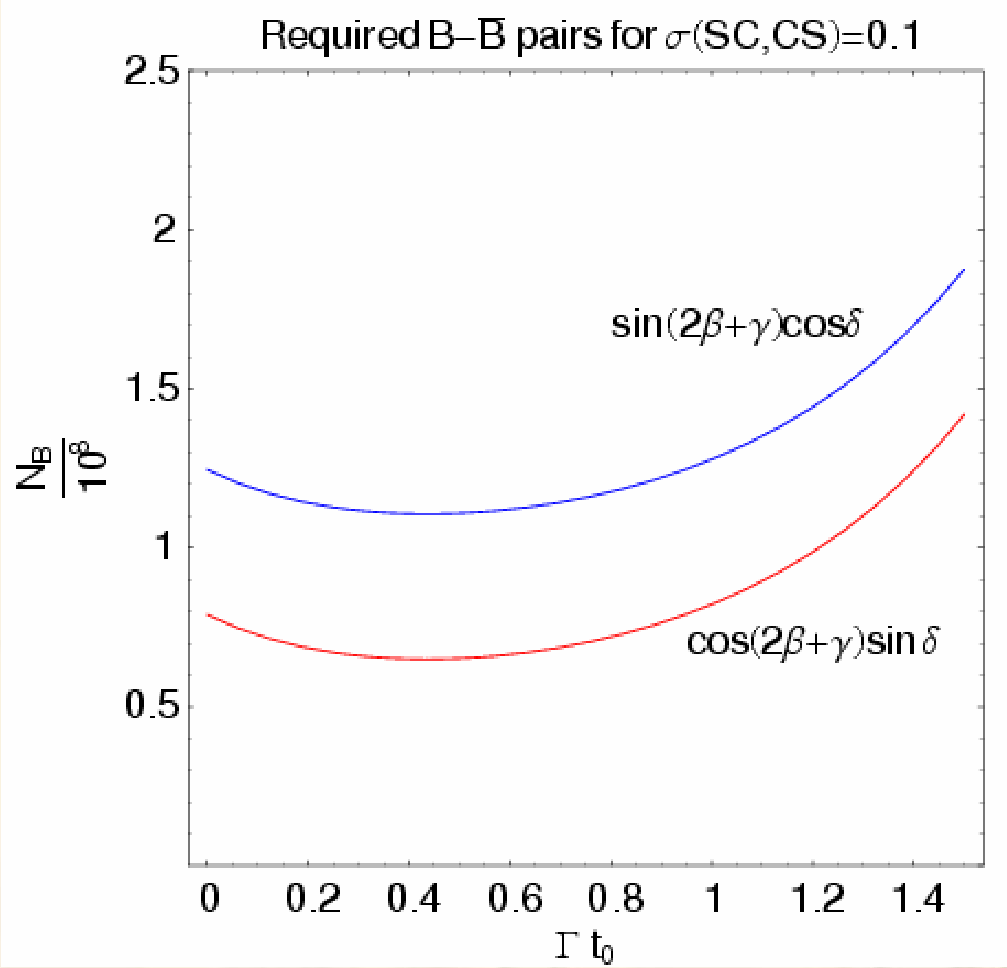
$$N_B \approx \frac{2(1+k)\,b^2(t_0)+e^{\Gamma t_0}}{4\epsilon(1+k)\,b^2(t_0)R^2\mathcal{B}^r\sigma_0^2(SC)},$$

$$N_B \approx \frac{e^{\Gamma t_0}}{4\epsilon(1+k)\,b^2(t_0)R^2\mathcal{B}^r\sigma_0^2(CS)}.$$

- We shall take $\sigma_0(SC,CS) = 0.1$ as an example in the following analysis.

# Optimal Condition

- Number of produced *B*-*B*bar pairs needed to achieve an uncertainty of 0.1 in measurements of $\sin(2\beta+\gamma)\cos\delta$ and $\cos(2\beta+\gamma)\sin\delta$ vs. minimum vertex separation $t_0$.
- Perfect time resolution and no mistagging are assumed here.
- We have an optimal condition $t_0 \simeq 0.44/\Gamma$ for BABAR detector ($\varepsilon = 0.793$), for example.



# Finite-Time Resolution

- The observed numbers of decays should be further convoluted with the resolution function (with $\sigma$ = resolution of time separation between vertices):

$$R(t_0) \equiv \int_{-\infty}^{+\infty} d\mu \frac{1}{\sqrt{2\pi}\sigma} e^{-(\mu-t_0)^2/2\sigma^2} .$$

- For example, $N_{\pm}^{\tau}(t_0) = N_0 \, R(t_0) \otimes N_{\pm}^{\tau}(\mu)$ .

- In this case, one encounters the error function along with others:

$$\begin{aligned}
\Phi(x) &\equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-z^2} dz , \\
J_1 &\equiv \int_{-\infty}^{+\infty} e^{-|\tilde{t}|} \, d\tilde{t} = 2 , \\
J_2 &\equiv \int_{-\infty}^{+\infty} d\tilde{\mu} \; \Phi\left(\frac{\tilde{\mu} - \Gamma t_0}{\sqrt{2}\sigma\Gamma}\right) e^{-|\tilde{\mu}|} , \\
J_3 &\equiv \int_{-\infty}^{+\infty} d\tilde{\mu} \; \Phi\left(\frac{\tilde{\mu} - \Gamma t_0}{\sqrt{2}\sigma\Gamma}\right) e^{-|\tilde{\mu}|} \sin x_d \tilde{\mu} .
\end{aligned}$$

$$\Rightarrow \quad SC, CS = \frac{J_1 + J_2}{2J_3} \frac{1 + R^2}{R} f_{2,3}(t_0) .$$

# Optimal Condition

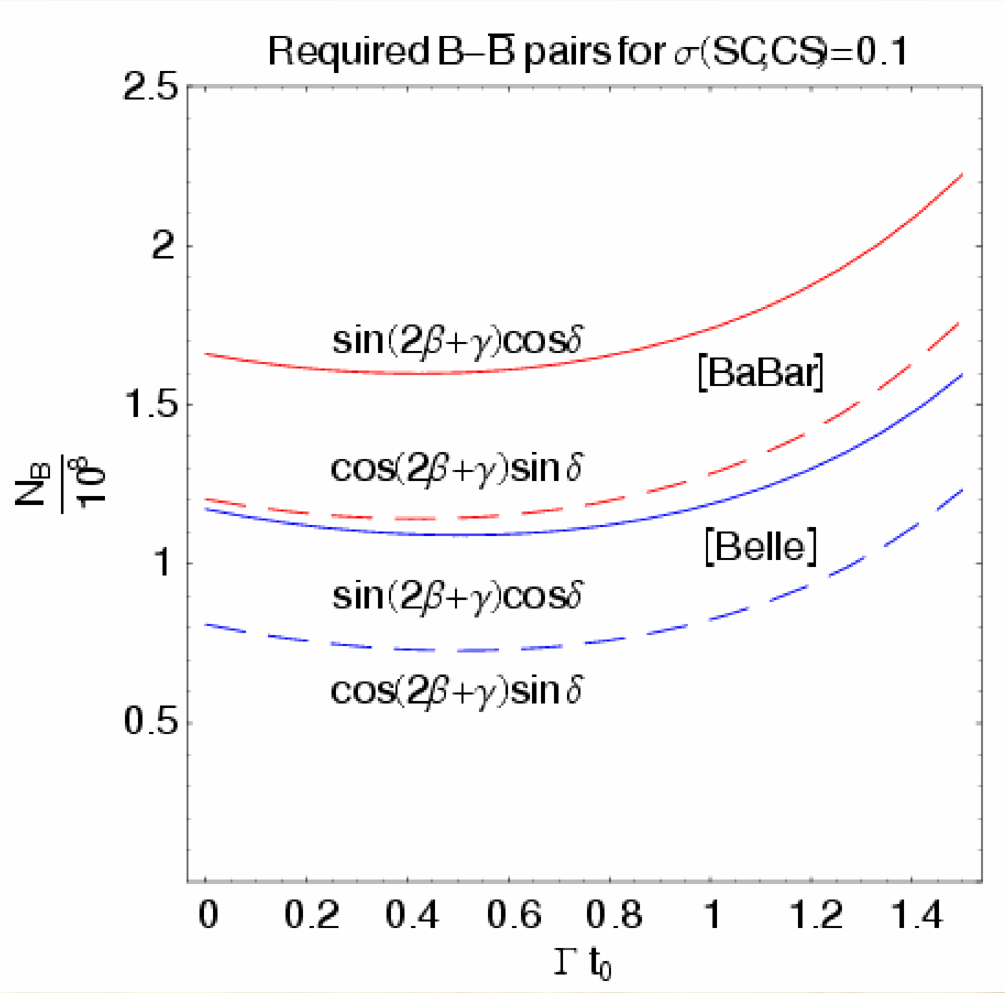
- Finite-time resolution with no mistagging.
- BABAR: [PRL 87, 091801 (2001); hep-ex/0609033]

$$\varepsilon = 0.793,$$
$$\sigma\Gamma = 0.69,$$
$$\Rightarrow \quad t_0 = 0.42/\Gamma.$$

- Belle: [hep-ex/0403022]

$$\varepsilon = 0.998,$$
$$\sigma\Gamma = 0.5,$$
$$\Rightarrow \quad t_0 = 0.50/\Gamma.$$

- The optimal vertex separation $t_0$ depends on $\sigma$ and required precision on *SC* and *CS*.



# Mistagging Effect

- Suppose $w$ = effective mistagging probability, related to the effective efficiency by $Q = \varepsilon(1-2w)^2$, assuming the simplified single tagging option.
- The number of apparent RS decays, for example, becomes

$$N_{\pm}^{r\prime}(t_0) = (1-w)N_{\pm}^{r}(t_0) + w\overline{N_{\pm}^{w}}(t_0) \ .$$

and

$$SC, CS = \frac{1}{1-2w}\frac{J_1+J_2}{2J_3}\frac{1+R^2}{R} f'_{2,3}(t_0) \ .$$

where the primed quantities denote the unprimed ones but with mistagging taken into account. [e.g., $f_{1,2,3}' = (1-2w) f_{1,2,3}$.]

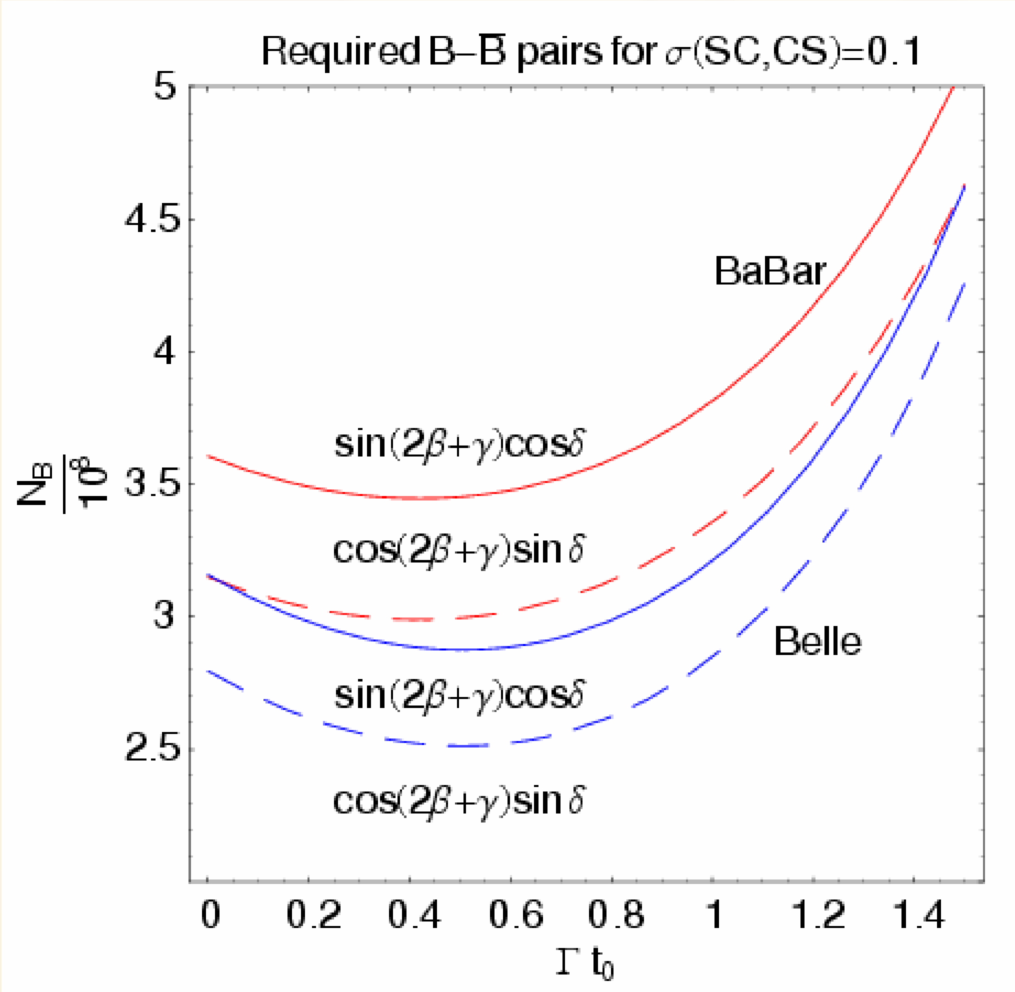
# Optimal Condition

- Finite-time resolution with mistagging.
- BABAR: [PRL 87, 091801 (2001); hep-ex/0609033]

$$\varepsilon = 0.793,$$
$$\sigma\Gamma = 0.69,$$
$$w = 0.191,$$
$$\Rightarrow \quad t_0 = 0.42/\Gamma.$$

- Belle: [hep-ex/0403022]

$$\varepsilon = 0.998,$$
$$\sigma\Gamma = 0.5,$$
$$w = 0.231,$$
$$\Rightarrow \quad t_0 = 0.50/\Gamma.$$



# Four-Fold Ambiguity

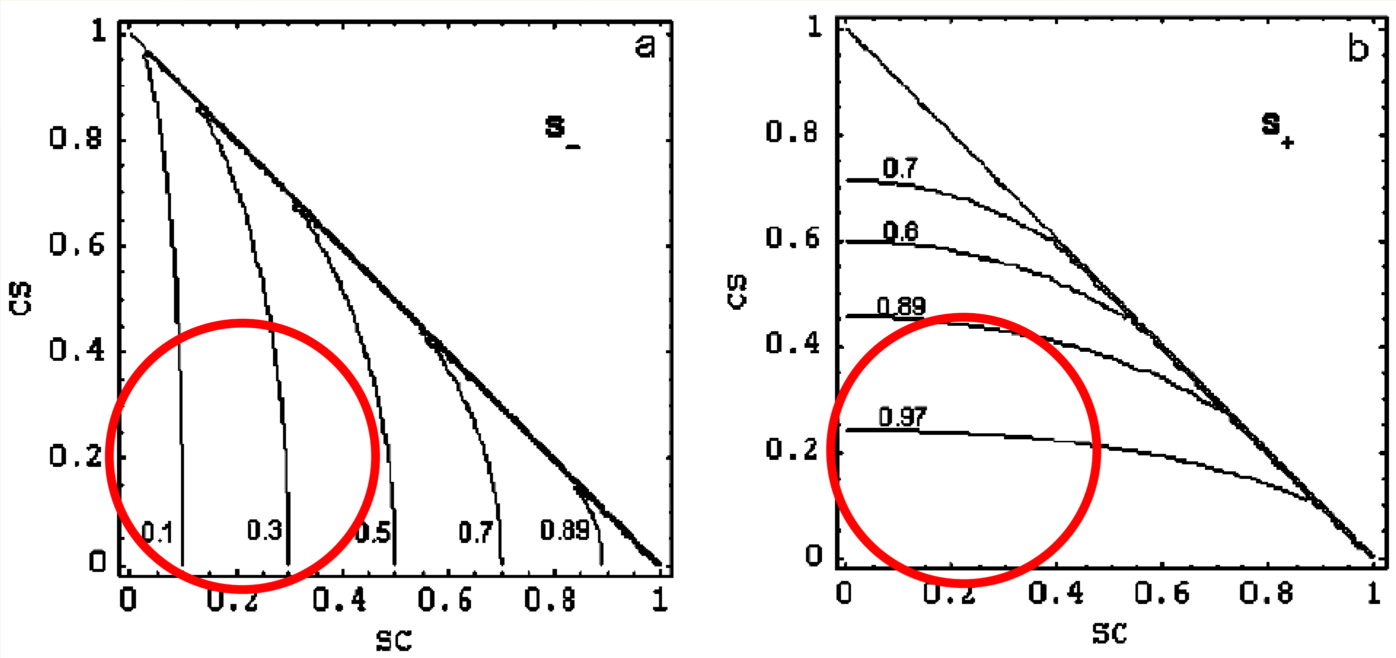
- Simple trigonometry gives

$$\left.\begin{array}{c}\sin^2(2\beta+\gamma)\\ \cos^2\delta\end{array}\right\} = s_\pm^2 \equiv \frac{1}{2}\left(1 + SC^2 - CS^2 \pm \sqrt{\lambda(1, SC^2, CS^2)}\right) ,$$

$$\lambda(x,y,z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$

- There is a four-fold ambiguity in determining the value of $2\beta+\gamma$. [However, only $\sin(2\beta+\gamma) > 0$ is consistent with other constraints.]
- It is suggested to consider several similar decays: $B\rightarrow D^{(*)}\pi$, $B\rightarrow D^{(*)}\rho$, and $B\rightarrow D^{(*)}a_1$, where the strong phases may be very different from each other.

- Contours of $s_+$ and $s_-$ on the *SC*-*CS* plane.




- The ambiguity can be removed if (*SC*,*CS*) is measured to be at the lower left region, where $s_+$ is unambiguously greater than $s_-$. In this case, $\sin(2\beta+\gamma)$ can only be associated with $s_+$ if it is to be in agreement with other constraints.

# Conclusions

- We have explored the optimal conditions for measurements of $\sin(2\beta+\gamma)\cos\delta$ and $\cos(2\beta+\gamma)\sin\delta$ from Cabibbo-favored and doubly Cabibbo-suppressed $B\to D^{(*)}\pi$ decays in the time-integrated approach.
- It is more advantageous to only consider events with vertex separation greater than about half of the *B* meson lifetime instead of all.
- The optimal vertex separation depends on the time resolution parameter $\sigma$ and the required precisions on *SC* and *CS*.
- To reach an accuracy of 0.1, one requires about $(2.5 \sim 3.5) \times 10^8$ *B*-*B*bar pairs, which also depends upon the tagging efficiency $\varepsilon$.
- We comment on a four-fold ambiguity in determining $2\beta+\gamma$. This can be readily resolved if $\cos\delta$ is not too large and other information is taken into account.

Thank You