Dalitz plot, resonances and K-matrix (the good, the bad and the ugly)

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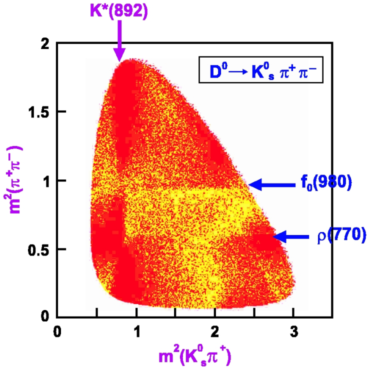
Dalitz plots

Analysis $H \longrightarrow abc$ of obvious interest in this WG



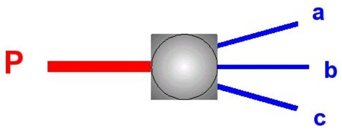
Dalitz plot according to

$$m^2_{ab} = (P - p_c)^2 \quad m^2_{bc} = (P - p_a)^2$$

$$d\Gamma \propto |\langle abc|T|H\rangle|^2 \, d(m^2_{ab}) d(m^2_{bc})$$

- Flat if no dynamics involved
- Bands corresponding to resonances
- Interferences between resonances in different channels

Isobar model



What to do with $\langle abc|T|H\rangle$?

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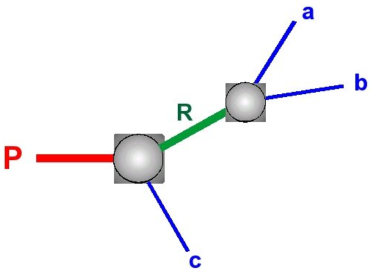

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3-body rescattering neglected,

2-body scattering only

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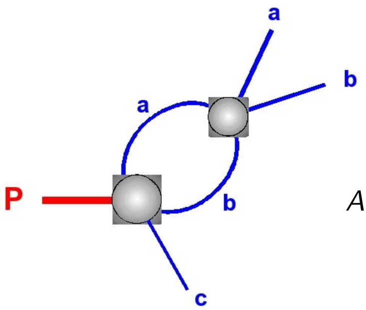

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$$A(B \to abc) = \alpha_a^{bc} T_{bc\to bc} + \alpha_b^{ac} T_{ac\to ac} + \alpha_c^{ab} T_{ab\to ab}$$

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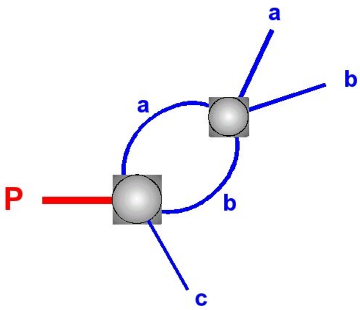

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Not necessarily elastic scattering !

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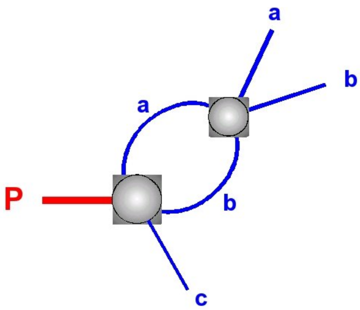

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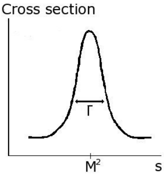
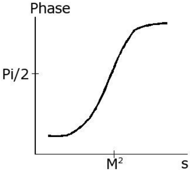
How to model this dynamics ? Mainly driven by resonances

Breit Wigner

Each partial wave is often approximated as a sum of Breit-Wigner's





$$T = \sum_l (2l+1) P_l(cos\theta) \sum_k c_k BW(s; m_k, \Gamma_k)$$

$$BW(s; m, \Gamma) = \frac{m\Gamma}{m^2 - s - im\Gamma}$$

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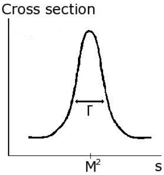

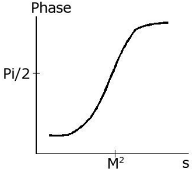

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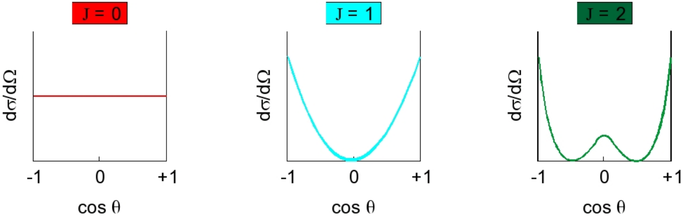
with spin related to zeroes in angular analysis



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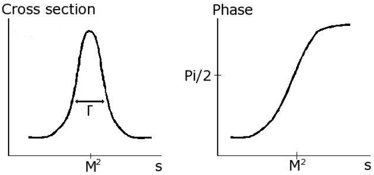

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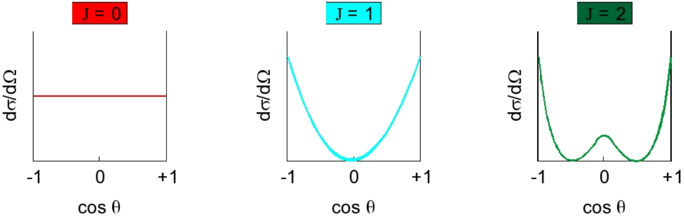

But BW valid only close to a peak, for narrow non-overlapping resonances

Unitarity

S-matrix restricted to 1 partial wave and 2-body channels $i = ab, a'b' \dots$

$$S = 1 + 2i\sqrt{\rho}T\sqrt{\rho}$$

with ρ two-body phase-space diagonal matrix

$$\begin{aligned} \rho_{ij}(s) &= \delta_{ij}\rho_i(s)\theta(s-(m_a+m_b)^2) \\ \rho_i(s) &= \frac{1}{s}\sqrt{(s-(m_a+m_b)^2)(s-(m_a-m_b)^2)} \end{aligned}$$

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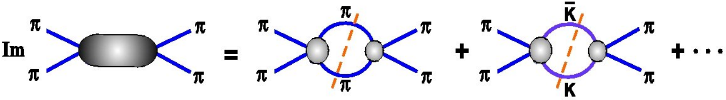
$$\begin{aligned} S^\dagger S = 1 \quad &\Longrightarrow \quad T - T^\dagger = 2iT^\dagger \rho T \\ &\Longrightarrow \quad (T^\dagger)^{-1} - T^{-1} = 2i\rho \Longrightarrow T^{-1} - (T^{-1})^* = -2i\rho \end{aligned}$$

T symmetric by time-reversal invariance of strong interactions

Unitarity and K-matrix



Since $\mathrm{Im}\,(T^{-1}) = -\rho$, let us define $T^{-1} = K^{-1} - G$

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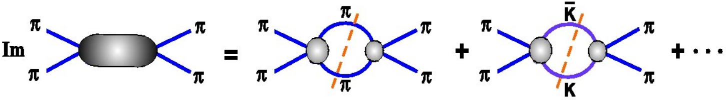

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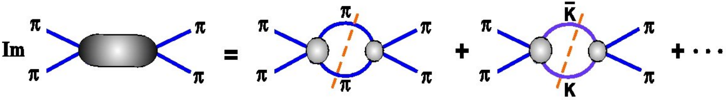

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NB : Some arbitrariness in G

$$G^{(1)}_{ij}(s) = i\frac{\delta_{ij}}{s}\sqrt{(s-(m_a+m_b)^2)(s-(m_a-m_b)^2)} \quad (\text{only if } s \geq 0)$$

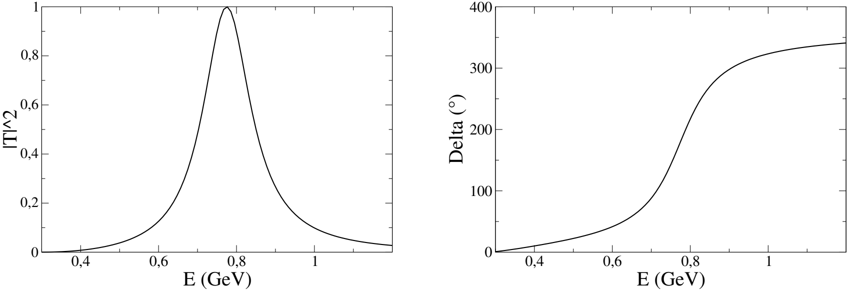
$$G^{(2)}_{ij}(s) = \delta_{ij}\int_{(m_a+m_b)^2}^{\infty} \frac{dx}{x-s}\frac{s}{x}\rho_i(x) \qquad (\text{OK for all } s)$$

K-matrix : 1 channel and 1 resonance

In the most simple case, with $G = i\rho$

$$K = \frac{m\Gamma(s)}{\rho(s)(m^2 - s)} \Longrightarrow T = \frac{m\Gamma(s)}{\rho(s)(m^2 - s - im\Gamma(s))}$$

one recovers a Breit Wigner (for instance P-wave ρ)



(unitarity in one channel : $|\rho T|^2 \leq 1$)

K-matrix : 1 channel and 2 resonances

Let us consider two resonances in two frameworks

- Sum of two BW's: $T = \frac{m_a \Gamma_a(s)}{\rho(s)(m_a^2 - s - i m_a \Gamma_a(s))} + \frac{m_b \Gamma_b(s)}{\rho(s)(m_b^2 - s - i m_b \Gamma_b(s))}$
- K matrix formalism: $K = \frac{m_a \Gamma_a(s)}{\rho(s)(m_a^2 - s)} + \frac{m_a \Gamma_a(s)}{\rho(s)(m_b^2 - s)}$

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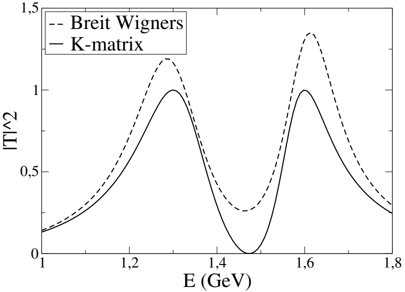
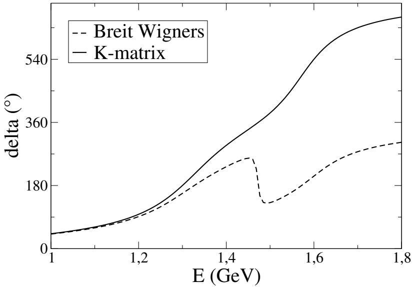


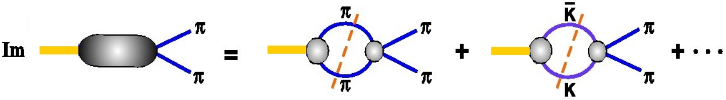


- BW not unitary if resonances are close !
- Issue of interferences between the peaks

Form factors and Watson theorem

If we consider form factors, for instance π, K electromagnetic form factors

$$F - F^* = 2iT^*\rho F \qquad F_i \equiv \text{ FF for channel } i$$



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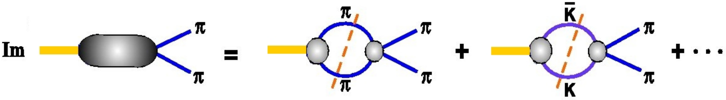

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Elastic region (sidebands) : $T_{11} = \frac{e^{i\delta(s)}-1}{2i\rho_1} \qquad \text{Im } F_1 = T_{11}^*\rho_1 F_1$

- Same phase occur in elastic scattering and low-energy form factor
- Determined through dispersive methods (Roy equations) for
  - $\pi\pi$ (Ananthanarayan, Colangelo, Gasser, Leutwyler 2001)
  - πK (Büttiker, SDG, Moussallam 2004)

Resonances

- Unstable particles quantum mechanics $\Psi(t) = \Psi(0)\exp[-imt - \Gamma t/2]$
- From bound state to resonance : $\frac{1}{s-m^2} \longrightarrow \frac{1}{s-(m-i\Gamma/2)^2}$
- Complex poles of the S-matrix in the s-plane

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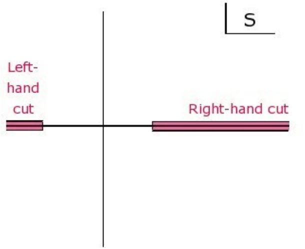

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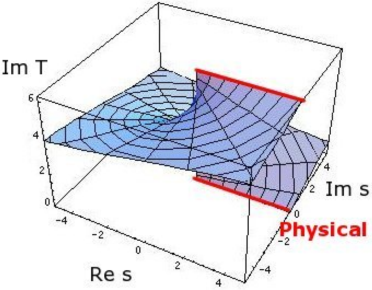

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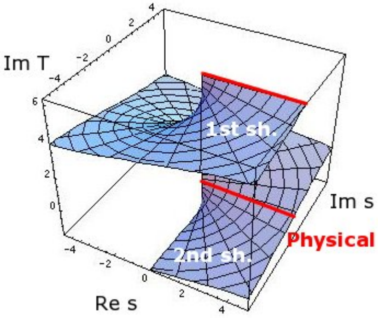

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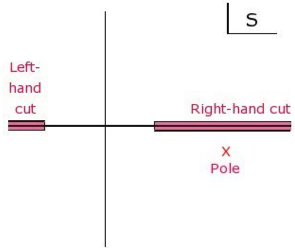

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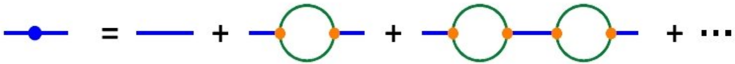
Troublesome scalar resonances



Shift in propagator from self-energy $M_\rho^2 \to M_\rho^2 + \mathrm{Re}\ \Sigma_{\pi\pi}(s) - i\ \mathrm{Im}\ \Sigma_{\pi\pi}(s)$

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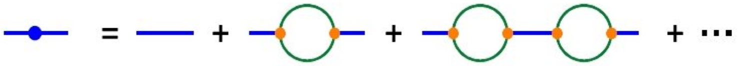

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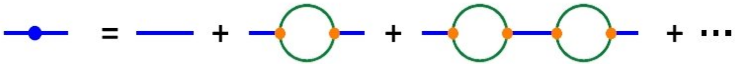

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Lowest scalars

- From dispersive analyses of $\pi\pi$, πK scattering (Roy equations)
- Deep in the complex plane, with $\sqrt{s_{pole}} = M - i\Gamma/2$

$M_\sigma = 441^{+16}_{-8}$ $\quad \Gamma_\sigma = 544^{+18}_{-25}$ MeV (Caprini,Colangelo,Leutwyler 2005)
$M_\kappa = 658 \pm 13$ $\quad \Gamma_\kappa = 557 \pm 24$ MeV (SDG,Moussallam 2006)

Resonances in K-matrix approach

Usual structure $$K_{ij} = \sum_\alpha \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_\alpha^2 - s} + \sum_n c_{ij}^{(n)} s^n$$

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Resonances in $T = K(1 - GK)^{-1}$ from two origins

- Pole in K: "normal" resonance related to α's
- Singular $1 - GK$: "molecular" resonance related to c's

Pole structure of T more complex than that of K

$\Longrightarrow$Some care needed to "isolate" resonances

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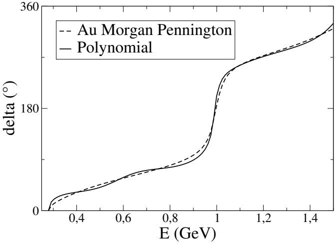

$\delta_{\pi\pi}$

Some arbitrariness in functional form of K

$\pi\pi$ and $K\bar{K}$ S-wave

Au Morgan Pennington vs. pure polynomial

Similar features,

including $f_0(980)$ pole(s)

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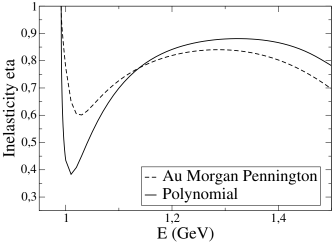

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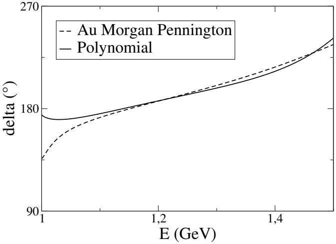

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Crossing symmetry (1)

Up to now, projections on partial waves in the s-channel :

$$T(s,t,u) \propto \sum_l (2l+1) P_l(cos\theta) T_l(s)$$

and consequences of unitarity for $T_l(s)$ (imaginary part above threshold)

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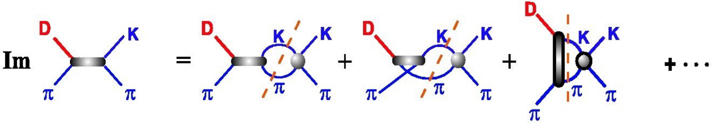
Actually unitarity constraints in s- t- and u-channels



Contrary to the case of form factors,

kinematics of 3-body decays implies all three cuts are relevant !

Complicated analytic structure for the decay amplitude A

Crossing symmetry (2)

Often simpler separation assumed, such as

$$A(s,t,u) = A_s(s,t) + A_t(t,s) + A_u(u,s)$$

where A_s has a cut only in s but is analytic in $t\ldots$

But unitarity holds for the whole amplitude, not separate terms

$$Disc_s A(s,t,u) \propto \int \; d\Omega \; \rho(s) T^*(s,t') A(s,t'')$$

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- Inhomogenous terms in unitarity relations for partial waves

$$Im A_s = T^* \rho A_s + (\int A_t, \int A_u)$$

- "Violation" of Watson theorem, even without 3-body interactions

(Caprini 2006)

Better than nothing, but clearly an approximation
in addition to isobar model and functional structure of K

Conclusions

K-matrix approach for two-body scattering from unitarity

- Overcomes Breit Wigner for non-textbook resonances
- Easy determination of resonances, universal elements of the dynamics
- But delicate task of isolating the contribution of each one
- Arbitrariness in functional form of K

Extension to 3-body decays delicate

- Isobar model (no 3-particle rescattering)
- Many channels to consider (sidebands easier)
- Incomplete analytic structure (inhomogeneities in unitarity relations)

Some interesting examples, not treated due to lack of time:

- $\pi\pi$ S-wave (Au,Morgan,Pennington 1987, Anisovich,Sarantsev 2003)
- D and D_s three-body decays (Focus Coll 2003, Oller 2004)