γ from $B \to DK$: Theoretical review

Jure Zupan

IJS & Univ. of Ljubljana

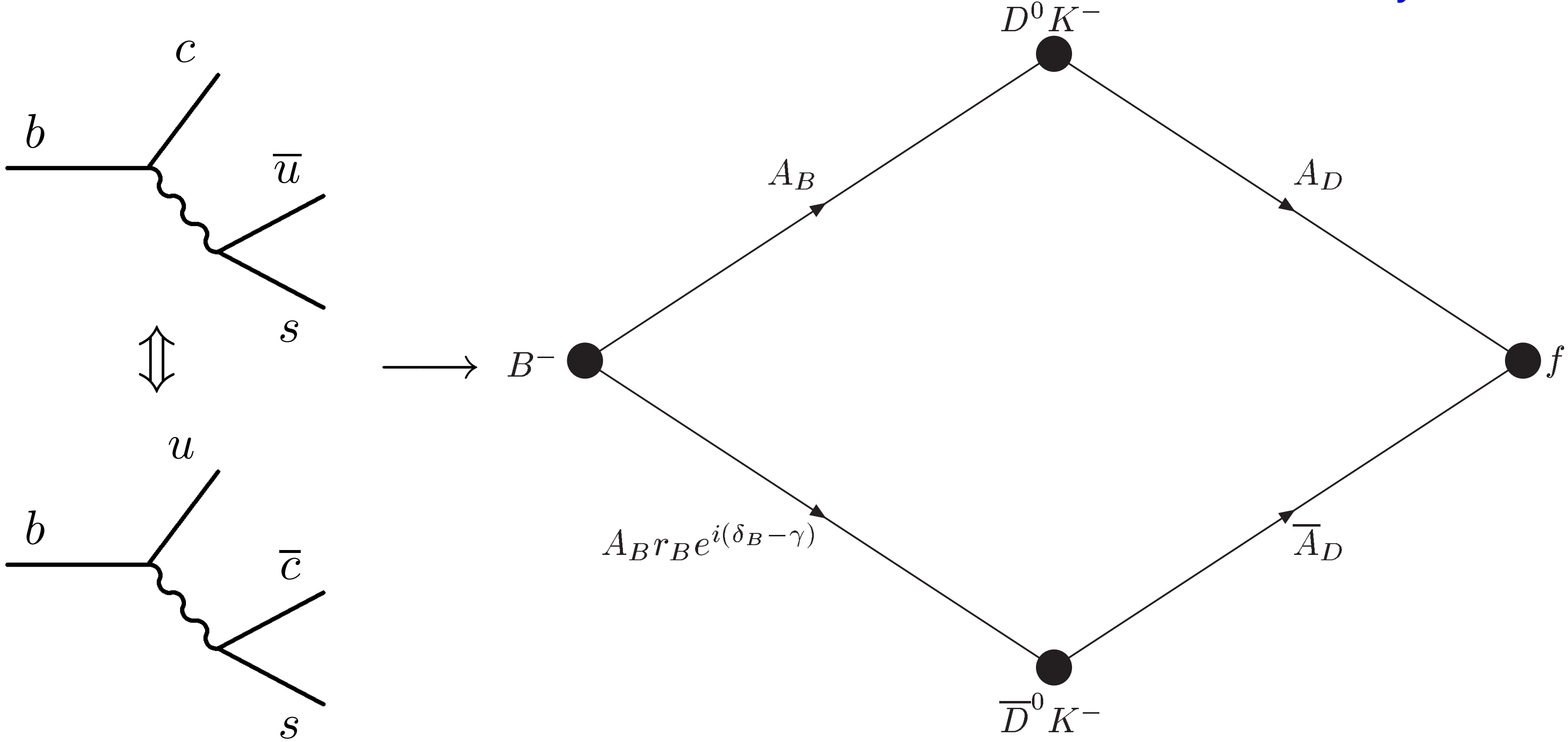
Outline

- short overview
- Dalitz plot analysis
- charm factory input
- theory errors
  - the effect of $D - \bar{D}$ mixing
- conclusions

Obtaining γ

- use interference between $b \to c\bar{u}s$ and $b \to u\bar{c}s$

Gronau, Wyler, 1991



- no penguin contributions

Different methods

methods can be grouped by the choice of final state f

- CP- eigenstate (e.g. $K_S\pi^0$) Gronau, London, Wyler (1991)
- flavor state (e.g. $K^+\pi^-$) Atwood, Dunietz, Soni (1997)
- singly Cabibbo suppressed (e.g. $K^{*+}K^-$) Grossman, Ligeti, Soffer (2002)
- many-body final state (e.g. $K_S\pi^+\pi^-$) Giri, Grossman, Soffer, JZ (2003) Poluektov et al. [Belle] (2004)

other extensions:

- many body B final states (e.g. $B^+ \to DK^+\pi^0$) Aleksan, Petersen, Soffer (2002)
- use D^{0*} in addition to D^0 Bondar, Gershon (2004)
- use self tagging D^{0**} Sinha (2004)
- neutral B decays (time dependent and time-integrated) many refs.

Combining methods

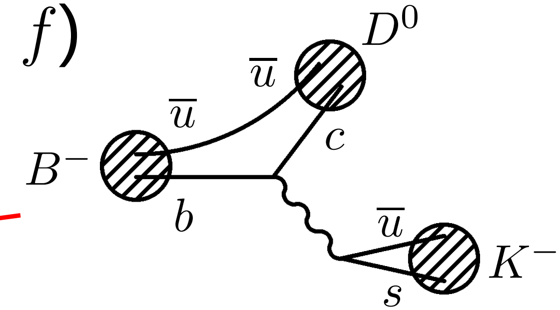
- all methods are statistic dominated at present
- for different D channels the B system parameters are common

combining different D channels buys
you more than just statistics

Counting the parameters

as an example consider the decay chain

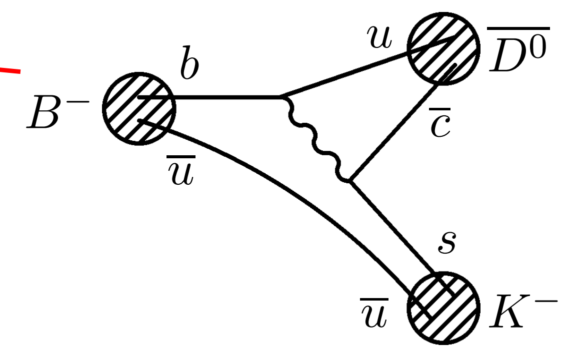
$$B^\pm \to DK^\pm \to (f)_D K^\pm \ (f \neq \bar{f})$$



the amplitudes for B decays

$$A(B^- \to D^0 K^-) \equiv A_B$$

$$A(B^- \to \overline{D}^0 K^-) \equiv A_B r_B e^{i(\delta_B - \gamma)}$$



color suppression + CKM $\Rightarrow r_B \sim 0.1$

neglecting CP violation in $D^0 \to f$ decay:

$$A(D^0 \to f) = A(\bar{D}^0 \to \bar{f}) = |A_f|$$

$$A(\bar{D}^0 \to f) = A(D^0 \to \bar{f}) = |\bar{A}_f| e^{i\delta_f}$$

Counting the parameters

- unknowns:
  - common to all decays: γ
  - $3N_B$ from the B system: A_B, r_B, δ_B for each $B \to DK, D^*K, DK^*$
  - N_f from the D system: δ_f ($|A_f|$, $|\bar{A}_f|$ measured)
    - $D^* \to D_f \pi^0(\gamma)$ have same $\delta_f(+\pi)$ strong phase
      Bondar, Gershon (2004)
- measurables (from B decays):
  - $B^\pm \to f_D K^\pm$, $B^\pm \to \bar{f}_D K^\pm$ decay widths
- $3N_B + N_f + 1$ unknowns vs. $4N_f N_B$ measurements
  - solvable for $N_B \geq 1$, $N_f \geq 2$
- slightly different counting if some $f = f_{CP}$ or semilept.

three body D decays

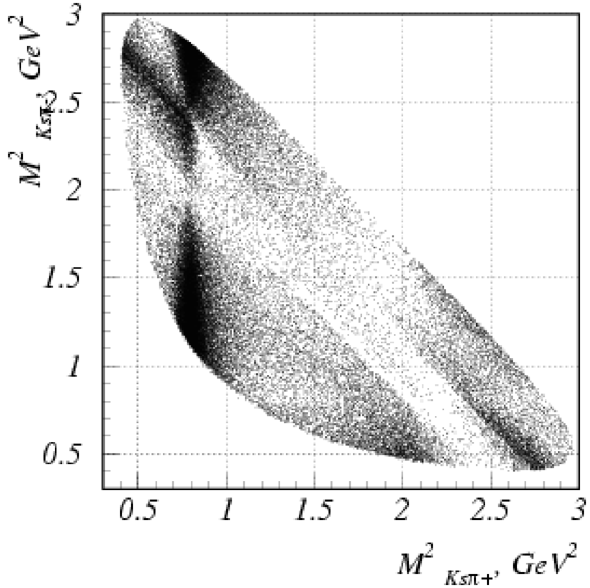
- the extreme case: $B^\pm \to [K_S\pi^+\pi^-]_D K^\pm$, where a continous set of "channels" in three-body final state

$$A_f \to A_D(s_{12}, s_{13}) \equiv A_{12,13}\, e^{i\delta_{12,13}}$$
$$\equiv A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3))$$

- even better: with some model dependence one can measure also the variation of the phase $\delta_{12,13}$ over the Dalitz plot $\Rightarrow A_f$ completely known (!)
- with higher statistics a model independent treatment also possible

Modeling A_D

- model A_D with a fit to a sum of Breit-Wigners
cf. talks by A. Poluektov, F. Martinez-Vidal



Belle, hep-ex/0308043

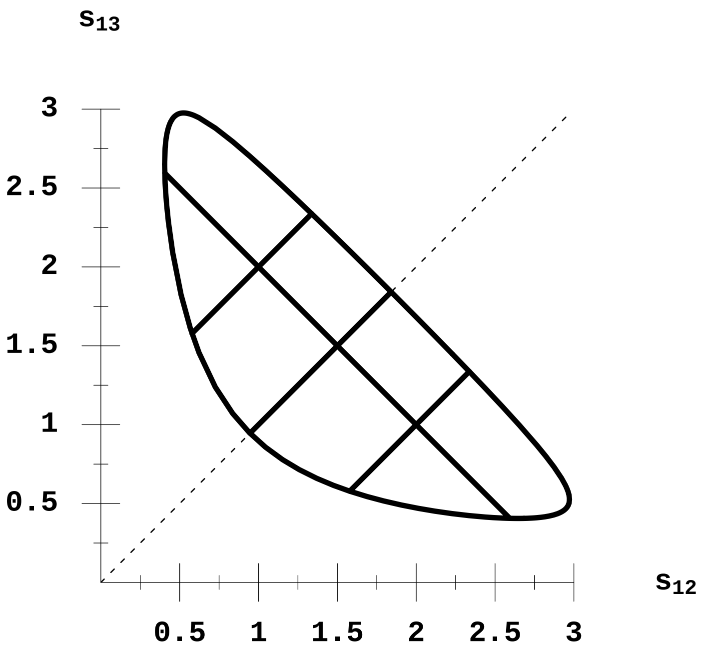
- model fit to high statistics D^0 tagged decay data
- in $B^\pm \to (K_S\pi^-\pi^+)_D K^\pm$ only r_B, δ_B and γ to be fit

- Q: what is the modelling error?
- Alternative descriptions/models (K-matrix)
cf. talk by S. Descotes-Genon

Model independent method

Giri, Grossman, Soffer, JZ, 2003

- at present the modeling error on γ is estimated to $\sim 10^\circ$
- partition Dalitz plot in bins



$s_{12} = m^2_{K_s\pi^-}$, $s_{13} = m^2_{K_s\pi^+}$

- unknowns:

$$c_i \equiv \int_i dp\, A_{12,13}\, A_{13,12} \cos(\delta_{12,13} - \delta_{13,\ldots}$$

$$s_i \equiv \int_i dp\, A_{12,13}\, A_{13,12} \sin(\delta_{12,13} - \delta_{13,\ldots}$$

- using CP : $c_{\bar{i}} = c_i, s_{\bar{i}} = -s_i$
- as in two-body decays enough measurables
- MC feasibility study: optimal strategy $\Rightarrow$ only 30% stat. worse than model dep. method

Bondar, Poluektov 2005
cf . talk by A. Poluektov

Input from charm factory

cf. talk by D. Asner

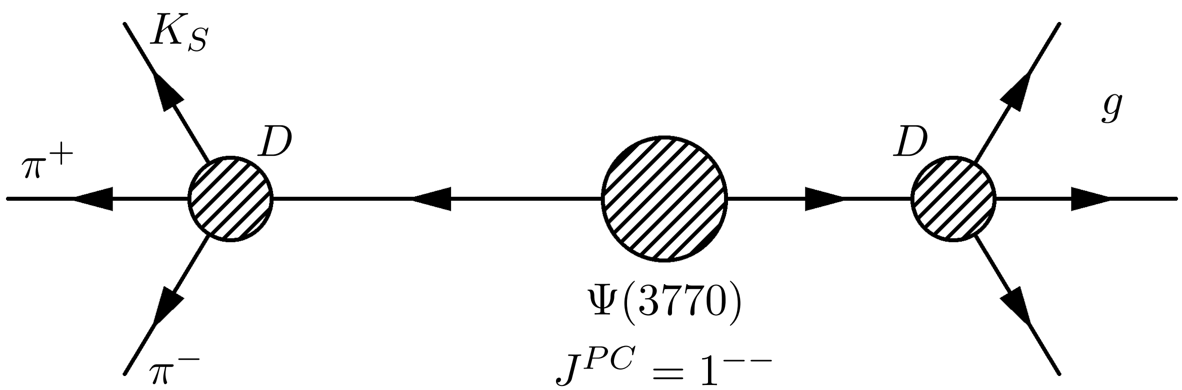
- optimal strategy includes data from charm factory
- charm factory can have a big impact
- working at $\psi(3770) \to D\bar{D}$ can measure $\arg(A_D)$

$\Rightarrow A_D$ completely known model indep.

- with this input one would be measuring from B decays only B-system parameters (like in the model-dependent Dalitz analysis)

Measuring phases @ charm factory

- entangled states: consider the following decay



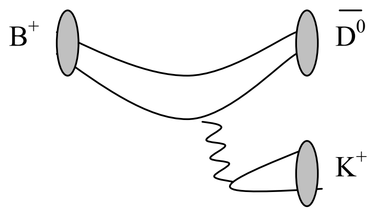
ith bin of $K_S\pi^+\pi^-$ and jth bin of g

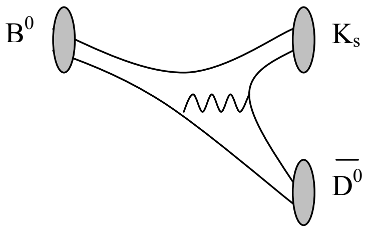
$$\Gamma_{i,j} \propto T_i T^g_{\bar{j}} + T_{\bar{i}} T^g_j - 2(c_i c^g_j + s_i s^g_j)$$

- if g a CP even (odd) eigenstate, $s^g_j = 0$, $T^g_j = T^g_{\bar{j}} = \pm c^g_j$, no sensitivity to s_i
- if $g = K_S\pi^+\pi^-$ and $j = i$ ($j = \bar{i}$) one measures s_i^2

Increasing statistics

- include as many D decay modes as possible
- include more B decay modes
  - $B^{\pm} \to D^* K^{*\pm} \Rightarrow$ would need to measure polarizations $\Rightarrow$ hard!
  - $B^0 \to D K_S$
- at first glance neutral B decays less attractive
  - have smaller decay rates: $A_n \sim \frac{1}{3} A_c$

$A_c =$ 

$A_n =$ 

  - time dependent measurements?

Gronau & London, 1990
Kayser & London, 1999

Using neutral B decays

- but: the statistical error on γ scales roughly as the smallest amplitude
- both in B^+ and B^0 these are color suppressed, using isospin (and negl. annih.):

$$A(B^+ \to D^0 K^+) \simeq \sqrt{2} A(B^0 \to D^0 K_S)$$

or

$$A_c r_c \simeq \sqrt{2} A_n r_n$$

- time integrated rates (untagged rates) of B^0 alone are sufficient to determine γ
  Gronau, Grossman, Shumaher, Soffer, J.Z., (2004)
- in DCS decays $B - \bar{B}$ mixing can induce few deg. errors
  Gronau, Grossman, Ze'ev, J.Z., unpublished

Almost there?

- BaBar analysis of $B^0 \to DK^{*0}$

hep-ex/0604016
cf. talk by V. Sordini

- finds $r_n^* < 0.4$ @ 90% CL
- from isospin

$$r_n^* = \frac{A_c^*}{A_n^*} r_c^* = \sqrt{\frac{\Gamma(B^+ \to \bar{D}^0 K^{*+})}{\Gamma(B^0 \to \bar{D}^0 K^{*0})}} r_c^* \Rightarrow$$

$$r_n^* = (4.0 \pm 0.5) r_c^* = 0.4^{+0.3}_{-0.4}$$

(using $r_c^* = 0.11^{+0.08}_{-0.11}$ from S. t'Jampens's talk at Beauty 06)

Theory errors

- $D - \bar{D}$ mixing the largest uncertainty (still very small!)
- in SM $D - \bar{D}$ mixing is CP conserving to very good approximation, $\theta \sim O(10^{-4})$
- the effect on γ is $O(x^2, y^2)$ Grossman, Soffer, JZ, 2005

  here $x \equiv \frac{\Delta m_D}{\Gamma_D}, y \equiv \frac{\Delta \Gamma_D}{2\Gamma_D}$, with $x \sim y \sim O(10^{-2})$

Effect of $D - \bar{D}$ mixing

- time-integr. decay rates measured in tagged D decays

$$\Gamma_f = \int dt |\mathcal{A}_f(t)|^2, \qquad \bar{\Gamma}_f = \int dt |\bar{\mathcal{A}}_f(t)|^2,$$

where $\mathcal{A}_f(t) = \mathcal{A}(D^0(t) \to f)$, $\bar{\mathcal{A}}_f(t) = \mathcal{A}(\bar{D}^0(t) \to f)$

- these are exactly the same as in B decay rate

$$\Gamma(B^+ \to f_D K^+) \propto \bar{\Gamma}_f + r_B^2 \Gamma_f + 2 r_B \mathcal{R}e\Big(e^{i(\delta_B + \gamma)} \int dt \mathcal{A}_f(t) \bar{\mathcal{A}}_f(t)^* \Big)$$

- the effect of $D - \bar{D}$ mixing on the γ measurement only in the interference term

$$\int dt \mathcal{A}_f(t) \bar{\mathcal{A}}_f(t)^* \equiv \sqrt{\Gamma_f \bar{\Gamma}_f} e^{i\tilde{\delta}_f} e^{-\epsilon_f}$$

- $\tilde{\delta}_f$ is pure strong phase, does not effect γ extraction
- ϵ_f is dilution parameter

$D - \bar{D}$ mixing

- the dilution due to $D - \bar{D}$ mixing

$$\epsilon_f = \frac{1}{8}(x^2 + y^2)\left(\frac{1}{r_f^2} + r_f^2\right) - \frac{1}{4}\left(x^2 \cos 2\delta_f + y^2 \sin 2\delta_f\right)$$

  where $r_f = |A(\bar{D}^0 \to f)/A(D^0 \to f)|$
- it gives the approximate magnitude of the shift $\Delta\gamma$ in the determination of γ
- the leading term in ϵ_f is proportional to $(x^2 + y^2)/r_f^2$, $\Delta\gamma$ is larger for cases where r_f is smaller
- for DCS decays the shift $\Delta\gamma \lesssim 1^\circ$, other much smaller

Theory errors in the far future

- the effect of $D - \bar{D}$ mixing can be included once x, y are measured
- in model independent Dalitz plot analysis no changes needed
  - here one can fit for both δ_f and ϵ_f (since this equivalent to c_i, s_i)
- then the theory error is coming from
  - higher electroweak corrections
  - CP violation in D decays $\Rightarrow \Delta\gamma \sim O(x\theta, y\theta)$
- the error is conservatively $\Delta\gamma < 10^{-5}$
- will be statistics dominated for a long time

Conclusions

- γ extraction from $B^\pm \to DK^\pm$ offers the theoretically cleanest measurement of SM CKM phase
- for the foreseeable future the determination will be statistics dominated

Backup slides

Master formulae

- a set of $4k$ equations
- the k equations for i bins

$$\hat{\Gamma}_i^- \equiv \int_i d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) =$$
$$T_i + r_B^2 T_{\bar{i}} + 2r_B[\cos(\delta_B - \gamma)c_i + \sin(\delta_B - \gamma)s_i]$$

eqs. for $\hat{\Gamma}_{\bar{i}}^-$, $\hat{\Gamma}_i^+$, $\hat{\Gamma}_{\bar{i}}^+$ obtained by $12 \leftrightarrow 13$ and/or $\gamma \leftrightarrow -\gamma$

- $2k+3$ unknowns: $c_i,\ s_i,\ r_B,\ \delta_B,\ \gamma$

solvable for $k \geq 2$

Comment on use of D^*

Bondar, Gershon (2004)

- define CP eigenstates

$$D^{(*)}_{CP=\pm} = \frac{1}{\sqrt{2}}\left(D^{0(*)} \pm \bar{D}^{0(*)}\right)$$

$$\begin{array}{l} \text{CP}(\pi^0) = -1 \\ \text{CP}(\gamma) = +1 \end{array} \Rightarrow \qquad \begin{array}{l} D^*_\pm \to D_\pm \pi^0 \\ D^*_\pm \to D_\mp \gamma \end{array}$$

- introduces a sign flip

$$Br(B^\pm \to D^*[D_f\pi^0]K^\pm) = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D \pm \gamma)$$

$$Br(B^\pm \to D^*[D_f\gamma]K^\pm) = r_B^2 + r_D^2 - 2 r_B r_D \cos(\delta_B + \delta_D \pm \gamma)$$

$$A_D = r_D e^{-i\delta_D} \bar{A}_D$$

Decay width

reduced partial decay width

$$d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) = \Big(A_{12,13}^2 + r_B^2\, A_{13,12}^2 + 2r_B \mathcal{R}e\big[A_D(s_{12},s_{13})\, A_D^*(s_{13},s_{12})\, e^{-i(\delta_B-\gamma)}\big]\Big) dp$$

Untagged decays

$$\langle\Gamma(B \to f_D K_n)\rangle \equiv \Gamma(B^0 \to f_D K_S) + \Gamma(\bar{B}^0 \to f_D K_S)$$
$$= A_f^2 \left[X_n(1 + r_f^2) + 2Y_n r_f \cos(\delta_f + \gamma)\right]$$
$$\langle\Gamma(B \to \bar{f}_D K_n)\rangle \equiv \Gamma(B^0 \to \bar{f}_D K_S) + \Gamma(\bar{B}^0 \to \bar{f}_D K_S)$$
$$= A_f^2 \left[X_n(1 + r_f^2) + 2Y_n r_f \cos(\delta_f - \gamma)\right]$$

where

$$X_n \equiv A_n^2(1 + r_n^2)\ , \qquad Y_n \equiv 2A_n^2 r_n \cos\delta_n$$

Four hadronic unknowns: X_n, Y_n, δ_f and a weak phase γ