QCD Light-Cone Sum Rules for charmless decays
- most recent developments -

Blaženka Melić
— Rudjer Bošković Institute, Zagreb —
⟨melic@thphys.irb.hr⟩

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- B-decays → testing of SM by overconstraining the CKM matrix parameters
- charmless B-decays → the most sensitive to New Physics
- isospin decomposition for $B \to \pi\pi$:

$$A(B^- \to \pi^-\pi^0) = \langle \pi^-\pi^0|H_{\rm eff}|B^-\rangle = \frac{3}{\sqrt{2}}A_2\,,$$
$$A(\bar{B}^0 \to \pi^+\pi^-) = \langle \pi^+\pi^-|H_{\rm eff}|\bar{B}^0\rangle = A_2 + A_0\,,$$
$$A(\bar{B}^0 \to \pi^0\pi^0) = \langle \pi^0\pi^0|H_{\rm eff}|\bar{B}^0\rangle = 2A_2 - A_0\,,$$

$$\frac{|A_0|}{|A_2|} = 1.33 \pm 0.31$$

-by naive factorization:

$$\frac{A_0}{A_2} = \frac{5}{4}\left(\frac{c_1(\mu) - c_2(\mu)/5}{c_1(\mu) + c_2(\mu)}\right) = \begin{cases} 1.92\,, \mu = m_b/2 \\ 1.68\,, \mu = m_b \\ 1.25\,, \mu = M_W\,. \end{cases}$$

- $B^- \to \pi^-\pi^0$ prediction agrees well with the data → A_2 amplitude may be estimated by naive factorization
- predictions for B^0 decays fail to reproduce tha data → A_0 amplitude misses contribution from non-emission topologies

▷ there are strong experimental indications for large nonfactorizable $\mathcal{O}(\Lambda_{QCD}/m_b)$ corrections in charmless B-decays :

▽ "charming penguin" (= 'LD charmed loops', '$D - \overline{D}$ rescattering') contributions (Ciuchini et al. '01)
- long-distance large $O(\Lambda/m_b)$ contributions and large strong phases

$$\frac{P^{charm}}{A^{TOT}} = (0.11 \pm 0.05) e^{i(-0.2 \pm 0.9)}$$
$$\frac{P^{GIM}}{A^{TOT}} = (0.43 \pm 0.14) e^{i(-0.2 \pm 0.7)}$$

(P^{GIM} includes also annihilation and corrections to emission topology)

▽ chirally-enhanced corrections could be large (nonfactor. hard-scattering corr.)

$$r_\chi^\pi = \frac{2m_\pi^2}{m_b(m_u + m_d)} = \mathcal{O}(1) \qquad !$$

▽ annihilation could be large

- CP-asymmetry : weak phase from a phase in CKM matrix & strong phases from CP-conserving strong amplitude:

$$|A|e^{i\delta} \sim \langle \bar{f}|\mathcal{H}_{weak}|\bar{B}\rangle = \sum_k \underbrace{C_k(\mu)}_{\text{pert. QCD}} \times \underbrace{\langle \bar{f}|\mathcal{O}_k(\mu)|\bar{B}\rangle}_{\text{non-pert. QCD}}$$

$$\mathcal{H}_{weak} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb}V^*_{pd} \left\{ C_1(\mu)\mathcal{O}_1^p + C_2(\mu)\mathcal{O}_2^p + \sum_{i=3,..,10} C_i(\mu)\mathcal{O}_i + C_{7\gamma}\mathcal{O}_{7\gamma} + C_{8g}\mathcal{O}_{8g} \right\}$$

- how to calculate hadronic matrix elements of the four-quark operators $\mathcal{O} = (\overline{q_1}_i\Gamma_\mu q_{2i})\left(\overline{q_3}_j\Gamma^\mu b_j\right) \quad \rightarrow \quad \langle \bar{f}|\mathcal{O}|\bar{B}\rangle$?

$$\text{i.e.}\quad \langle \pi\pi|\mathcal{O}_1|B\rangle = \underbrace{\langle \pi|\bar{d}\Gamma_\mu u|0\rangle\langle \pi|\bar{u}\Gamma^\mu b|B\rangle}_{\text{'naive' factorization}} \left[1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{QCD}/m_b)\right]$$

$$= i m_b^2 f_\pi F^+_{B\to\pi}(m_\pi^2)\left[1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{QCD}/m_b)\right]$$

- how large are $O(\Lambda_{QCD}/m_b)$ corrections ?

MODELS FOR CALCULATING MATRIX ELEMENTS:

- ▷ QCD FACTORIZATION APPROACH (Beneke, Buchalla, Neubert, Sachrajda '99)
- ▷ PQCD APPROACH (Keum, Sanda, Li '01)
- ▷ SCET(soft-collinear eff. theory) (Bauer, Fleming, Luke, Stewart '01)

- ▷ QCD LIGHT-CONE SUM RULE METHOD (Khodjamirian '01)
  - $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\Lambda/m_b)$ corrections are systematically calculable

$$\text{i.e.} \quad \langle \pi\pi|\mathcal{O}_1|B\rangle = \langle\pi|\bar{d}\Gamma_\mu u|0\rangle\langle\pi|\bar{u}\Gamma^\mu b|B\rangle\left[1 + O(\alpha_s) + O(\Lambda_{QCD}/m_b)\right]$$

$$= \underbrace{\underbrace{\underbrace{i m_b^2 f_\pi F^+_{B\to\pi}(m_\pi^2)}_{\text{naive factorization}} \left[1 + O(\alpha_s)\right.}_{\text{QCD factorization}} \left. + O(\Lambda_{QCD}/m_b)\right]}_{\text{QCD LCSR}}$$

see talk by P. Ball at this workshop

▷ LIGHT-CONE SUM RULES e.g. calculation of $B \to \pi$ form factor:

$$F_\mu(p,q) = i \int d^4x e^{-ipx} \langle \pi(q) | T\{\bar{u}\gamma_\mu b(x), m_b \bar{b} i\gamma_5 d(0)\} | 0 \rangle$$

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- $(p-q)^2 < 0$; $\rightarrow$ light-cone OPE (TWIST-expansion in terms of the LIGHT-CONE WAVE FUNCTIONS)

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MATCHING by applying quark-hadron duality

see talk by P. Ball at this workshop

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MATCHING by applying quark-hadron duality

- LIGHT-CONE WAVE FUNCTION e.g. pion-wave function:

-expansion at the light-cone $x^2 \to 0$ = TWIST-expansion
-leading term $\to$ twist-2 wave function:

$$\langle\pi(q)|\bar{u}(x)\gamma_\rho\gamma_5 d(0)|0\rangle_{x^2=0} = -iq_\mu \frac{f_\pi}{\sqrt{2}}\int_0^1 du e^{iuq\cdot x}\phi_\pi(u,\mu)$$

u = fraction of the momentum carried by a constituent $\to \phi_\pi^{asy} \sim u(1-u)$

$$F_\nu^{(\mathcal{O})}(p,q,k) = \int d^4x e^{-i(p-q)x} \int d^4y e^{i(p-k)y} \langle 0|T\{j_{\nu,5}^{(\pi)}(y)\mathcal{O}_i(0)j_5^{(B)}(x)\}|\pi(q)\rangle$$

- interpolating currents for a pion and a B meson:

$$j_{\nu,5}^{(\pi)} = \overline{u}\gamma_\nu\gamma_5 d \qquad\qquad j_5^{(B)} = m_b \overline{b} i\gamma_5 d$$

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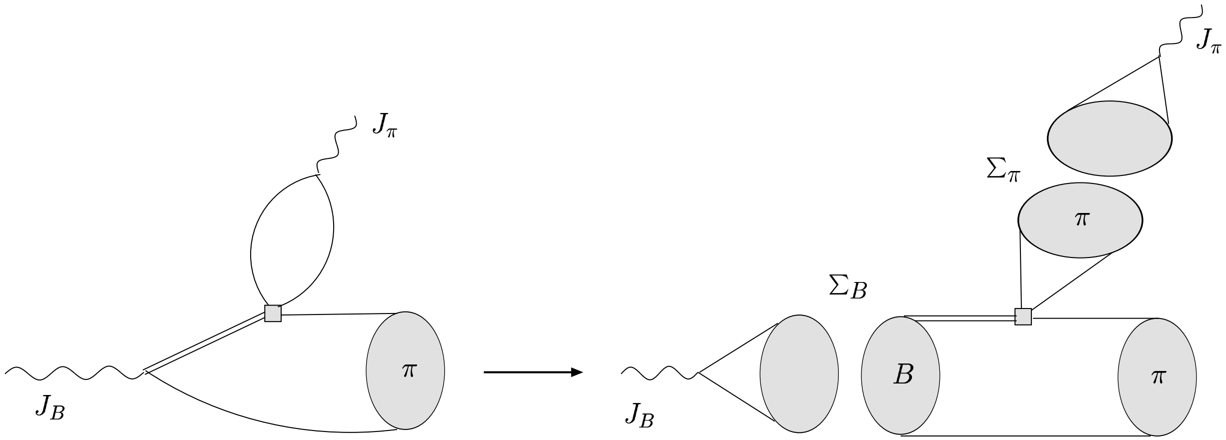
- interpolating currents for a pion and a B meson:

$$j_{\nu,5}^{(\pi)} = \bar{u}\gamma_\nu\gamma_5 d \qquad j_5^{(B)} = m_b \bar{b} i\gamma_5 d$$

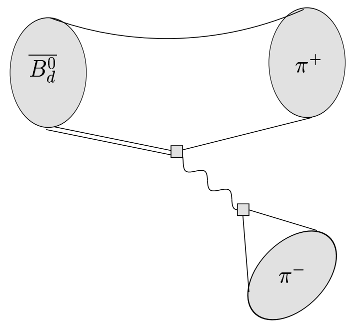
$$\sum_B \sum_\pi \frac{\langle 0|j_{\nu,5}^{(\pi)}|\pi\rangle\langle\pi|\langle\pi|\mathcal{O}_i(0)|B\rangle\langle B|j_5^{(B)}|0\rangle}{(m_\pi^2-(p-k)^2)(m_b^2-(p-q)^2)} = \int ds \int ds' \frac{Im_{s'} Im_s F_{QCD}(s,s')}{(s-(p-k)^2)(s'-(p-q)^2)}$$



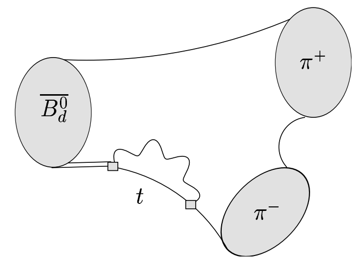
quark-hadron duality $\rightarrow \langle\pi\pi|\mathcal{O}|B\rangle$

ADVANTAGES OF LCSR APPLICATION FOR B-MESON DECAYS:

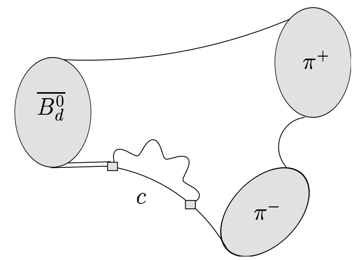
- no approximation like e.g. large m_b limit is necessary
- no heavy meson (B, D) wave function is needed
- theoretical uncertainty is reduced (parameters like f_B, $F_{B\to\pi}$ etc., are calculated inside the same model)



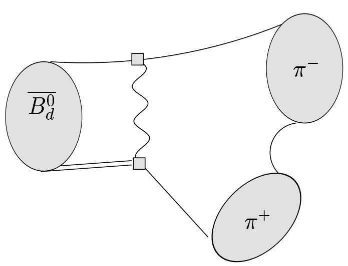
(a) emission



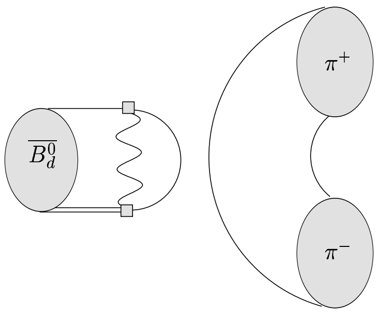
(b) penguin



(c) charming penguin

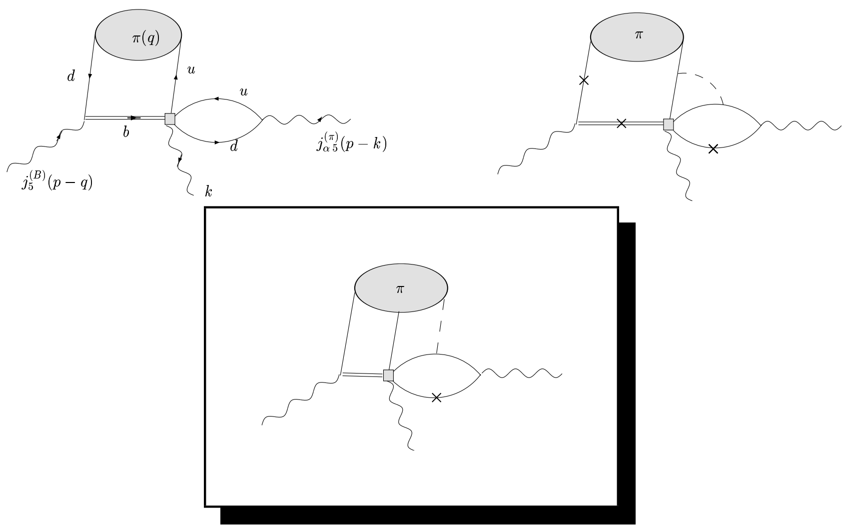


(d) annihila-tion



(e) penguin annihilation

▷ NONFACTORIZABLE $O(\Lambda/m_b)$ corrections from $\mathcal{O}_2 = \frac{1}{N_c}\mathcal{O}_1 + 2\,\tilde{\mathcal{O}}_1$ operator



color-octet operator $\tilde{\mathcal{O}}_1 = (\bar{d}\Gamma_\mu \frac{\lambda^a}{2} u)(\bar{u}\Gamma^\mu \frac{\lambda^a}{2} b)$ (Khodjamirian '01)

$$r_E^{(\pi\pi)} = \frac{\langle \pi^+\pi^- | \tilde{O}_1^u | \bar{B}^0 \rangle_E}{\langle \pi^+\pi^- | O_1^u | \bar{B}^0 \rangle_E}$$

- defined in terms of THREE-PARTICLE twist-3 and twist-4 distribution amplitudes:

$$\langle \pi | \bar{u} \sigma_{\mu\nu} \gamma_5 G_{\alpha\beta} d | 0 \rangle \sim \phi_{3\pi}(...) \qquad twist-3\ w.f.$$

$$\langle \pi | \bar{u} \gamma^\mu \tilde{G}_{\alpha\beta} d | 0 \rangle \sim (...)\tilde{\phi}_{||} + (...)\tilde{\phi}_{\perp} \qquad twist-4\ w.f.$$

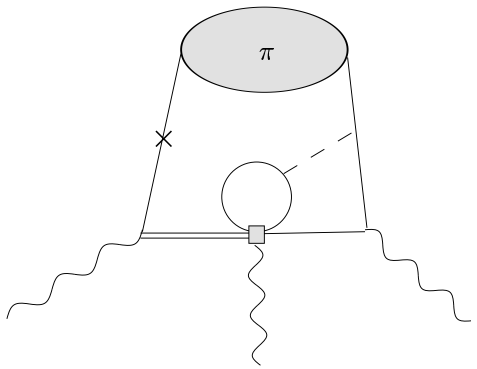
$O(\Lambda/m_b)$ corrections are small ($\sim 2\%$), but they are of the same order as $O(\alpha_s)$ corrections (preliminary, only twist2) (A. Khodjamirian, Th. Mannel, M. Melcher, BM, in preparation)

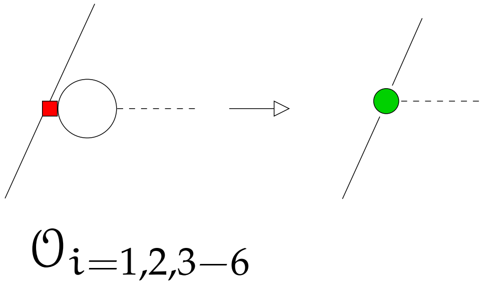
$$r_E^{(\pi\pi)} = \left[\left(1.8^{+0.5}_{-0.7}\right) \times 10^{-2} \right]_{\text{soft}} + \left[\left(-1.9^{+0.5}_{-0.1}\right) \times 10^{-2} + i \left(-3.6^{+1.0}_{-0.4}\right) \times 10^{-2} \right]_{\text{hard}}$$

▷ PENGUIN LOOP CONTRIBUTIONS - $r^{(\pi\pi)}_{P_{c,q}}$ and $r^{(\pi\pi)}_{8g}$ (Khodjamirian, Mannel, BM '03)

$$r^{(\pi\pi)}_{P_{c,q}} = \frac{\langle \pi^+\pi^- | \tilde{O}_2^{c,u} | \bar{B}^0 \rangle_{P_{c,q}}}{\langle \pi^+\pi^- | O_1^u | \bar{B}^0 \rangle_E} \qquad r^{(\pi\pi)}_{8g} = \frac{\langle \pi^+\pi^- | O_{8g} | \bar{B}^0 \rangle_{P_g}}{\langle \pi^+\pi^- | O_1^u | \bar{B}^0 \rangle_E}$$

"hard contributions" (QCD factorization & LCSR):



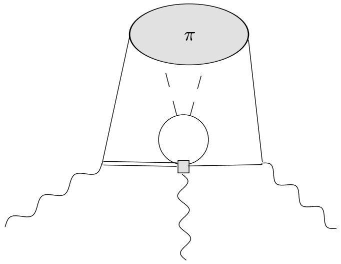


(Khodjamirian, Mannel, Urban '02)

$\mathcal{O}(\alpha_s)$ + $\mathcal{O}(\frac{\alpha_s}{m_b})$ + $\mathcal{O}(\alpha_s r_\chi)$ + $\mathcal{O}(\alpha_s \frac{r_\chi}{m_b})$

$$r_\chi = \frac{2m_\pi^2}{m_b(m_u+m_d)}$$

"soft contributions" (LCSR):



- small corrections $\mathcal{O}(\alpha_s\, r_\chi)$ and $\mathcal{O}(\frac{\alpha_s}{m_b^2})$

$$r_{P_q}^{(\pi\pi)} = \left[0.11^{+0.02}_{-0.36} + i\left(1.1^{+0.2}_{-0.1}\right)\right] \times 10^{-2}$$

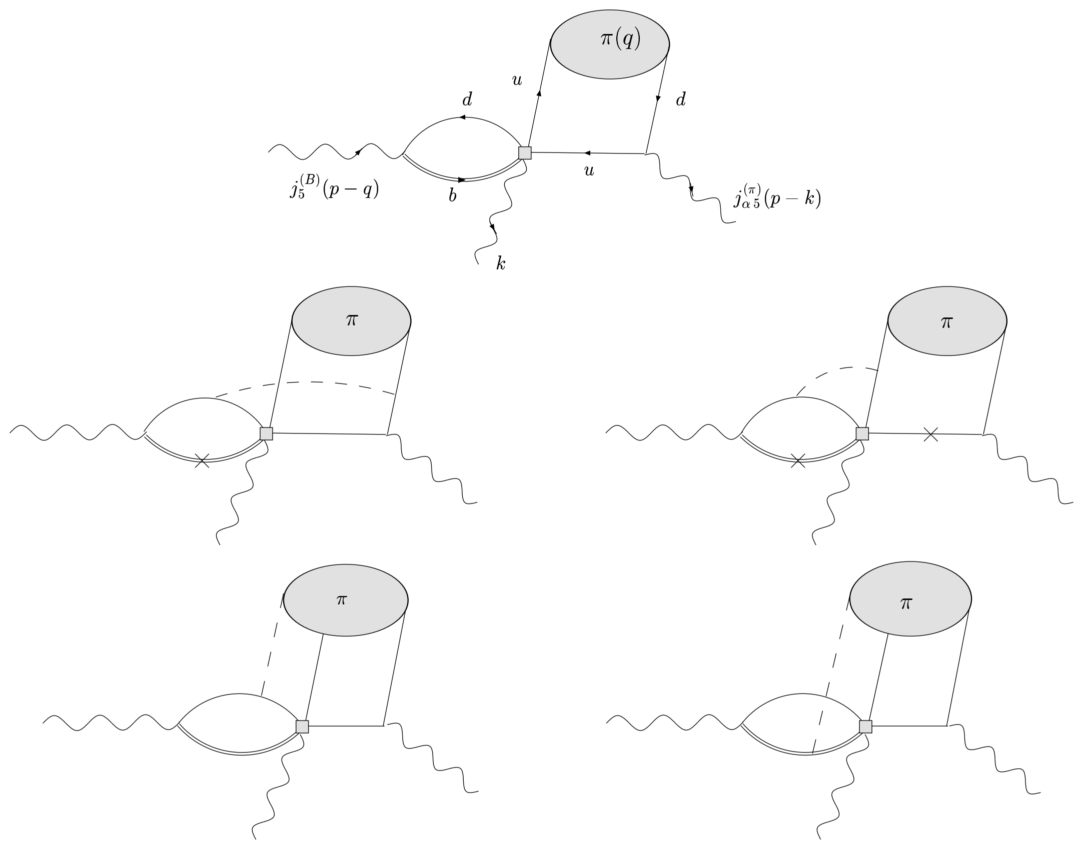
$$r_{P_c}^{(\pi\pi)} = \left[-0.18^{+0.06}_{-0.68} + i\left(-0.80^{+0.17}_{-0.08}\right)\right] \times 10^{-2}$$

$$r_{P_b}^{(\pi\pi)} = \left(0.93^{+0.09}_{-0.65}\right) \times 10^{-2}$$

$$r_{8g}^{(\pi\pi)} = -\left(3.8^{+1.3}_{-0.4}\right) \times 10^{-2}$$

and $\bar{r}_{P_{q,c,b}}^{(\pi\pi)} \simeq r_{P_{q,c,b}}^{(\pi\pi)} - (\alpha_s C_F)/(36\pi)$

▷ ANNIHILATION: $r_A^{(\pi\pi,(5,6))}$ and $R_A^{(\pi\pi,6)}$ (Khodjamirian, Mannel, Melcher, BM '05)



$$r_A^{(\pi\pi)} = \frac{\langle \pi^+\pi^- | \tilde{O}_2^u | \bar{B}^0 \rangle_A}{\langle \pi^+\pi^- | O_1^u | \bar{B}^0 \rangle_E} \qquad r_A^{(\pi\pi,(5,6))} = \frac{\langle \pi^+\pi^- | \tilde{O}^d_{(5,6)} | \bar{B}^0 \rangle_A}{\langle \pi^+\pi^- | O_1^u | \bar{B}^0 \rangle_E}$$

$$R_A^{(\pi\pi,6)} = \frac{\langle \pi^+\pi^- | O_6^d | \bar{B}^0 \rangle_A}{\langle \pi^+\pi^- | O_1^u | \bar{B}^0 \rangle_E} = -\frac{2 f_B F_\pi^S(m_B^2)}{m_b f_\pi f_{B\pi}^+(0)}$$

▷ all annihilation effects are finite at $m_b \to \infty$; no end-point divergences !

▷ $r_A^{(\pi\pi)}$ - both hard and soft contributions are calculated - finite with phase:

$$r_A^{(\pi\pi)} = \left[-0.67^{+0.47}_{-0.87} + i\left(3.6^{+0.5}_{-1.1}\right)\right] \times 10^{-3}$$

▷ $r_A^{(\pi\pi,(5,6))}$ - similar contributions like for $r_A^{(\pi\pi)}$ but highly suppressed by the small Wilson coefficients multiplying them

▷ $R_A^{(\pi\pi,6)}$ - large contribution, structure of a scalar pion form factor $\to O(\alpha_s^0)$ effect, but multiplied by a small coefficient $(c_6 + c_5/3) R_A^{(\pi\pi,6)} \sim c_1 r_A^{(\pi\pi)}$:

$$R_A^{(\pi\pi,6)} = 0.23^{+0.05}_{-0.08}$$

with $|V_{ub}| = (4.22 \pm 0.26) \cdot 10^{-3}$ and $\gamma = (58.6 \pm 10)^\circ$

$$BR(B^+ \to \pi^+\pi^0) = \left(6.6^{+1.8+0.8}_{-1.3-0.8}\right) \times 10^{-6} \qquad exp : (5.7 \pm 0.4) \times 10^{-6}$$
$$BR(B^0 \to \pi^+\pi^-) = \left(9.7^{+2.3+1.2}_{-1.9-1.2}\right) \times 10^{-6} \qquad exp : (5.2 \pm 0.2) \times 10^{-6}$$
$$BR(B^0 \to \pi^0\pi^0) = \left(0.25^{+0.12+0.07}_{-0.08-0.06}\right) \times 10^{-6} \qquad exp : (1.3 \pm 0.2) \times 10^{-6}$$

CP-asymmetry:

	$a_{CP}^{dir}(B^+ \to \pi^+\pi^0)$	$a_{CP}^{dir}(B^0 \to \pi^+\pi^-)$	$a_{CP}^{dir}(B^0 \to \pi^0\pi^0)$
Average	0.04 ± 0.05	-0.39 ± 0.07	0.36 ± 0.33
This work	0	$-0.04 \pm 0.01 \pm 0.01$	$0.65^{+0.13+0.10}_{-0.26-0.09}$

A. Khodjamirian, Th. Mannel, N. Offen, hep-ph/0611193; A. Khodjamirian, Th. Mannel, N. Offen, hep-ph/0504091; T.Hurth, F. DeFazio, T. Feldmann, ...

$$F_{\mu,\nu}(p,q) = i\int d^4x e^{ipx} \langle 0|T\{\overline{q}_2\gamma_\mu q_1(x), \overline{q}_1\gamma_\nu b(0)\}|\overline{B}(p_b)\rangle$$

IDEA:

- use B meson DA's (Grozin, Neubert) in the correlation function instead of π, ρ, K, K^* DAs
- new LCSR for $B \to P, V$ form factors as an independent check of conventional LCSR

ADVANTAGES:

- universal B-meson DAs for all B-to-light channels; decay products are interpolated by currents
- SU(3) breaking is control able, realized via threshold parameter, strange quark mass and decay constants
- applied to the calculation of $B \to \pi, K$ and $B \to \rho, K^*$ form factors:

$$f^+_{B\pi}(0) = \frac{f_B}{f_\pi}\int_0^{s_0^\pi/m_B} d\omega e^{-m_B\omega/M^2}\phi^B_-(\omega) \qquad\qquad \phi^B_-(0) = \lambda_B^{-1}$$

-λ_B is independently calculated from HQET two-point sum rules
- results are in agreement with those obtained by traditional LSCRs

- ▷ factorization is unable to reproduce observed BR's of $B \to \pi\pi$ and ($B \to K\pi$ decays)
  - $B^0 \to \pi\pi$ data favour large nonfactorizable effects
  - $B^0 \to \pi\pi$ data point to large a_{CP}^{mix} and a_{CP}^{dir} asymmetries
- ▷ CHARMED PENGUIN effects in $B \to \pi\pi$ are found to be small (Khodjamirian, Mannel, Urban'02 and Khodjamirian, Mannel, BM '03)
- ▷ ANNIHILATION effects in $B \to \pi\pi$ are found to be small (Khodajmirian, Mannel, Melcher, BM, hep-ph/0505049)
- ▷ CONC: $\mathcal{O}(1/m_b)$ corrections in charmless B decays are small $\to$ LCSR results are close to those obtained by QCDF and SCET
- ▷ the discrepancy between $B \to \pi\pi$ observables calculated in the factorization limit and the current experimental data remains - missing $\Delta I = 1/2$ effects, i.e part of $I = 0$ two-pion amplitude
- ▷ the unsatisfactory theoretical understanding of nonfactorizable power-suppressed effects in charmless nonleptonic B-decays prevent identification of New Physics effects in these decays