

Study of $B \rightarrow K h h$ Decays at Belle

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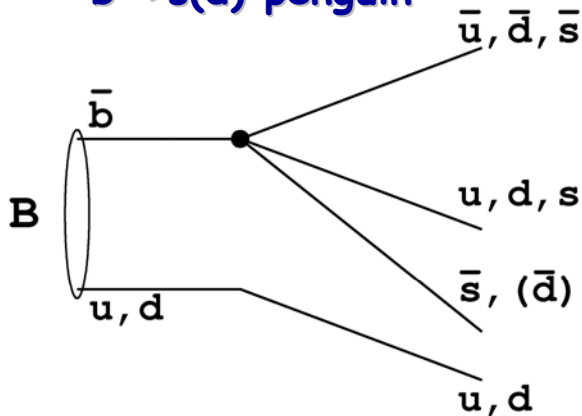
- @ Introduction
- @ Three-Body Charmless Decays
- @ Dalitz Analysis Results
- @ Search for DCPV in $B^{\pm} \rightarrow K^{\pm} \pi^+ \pi^-$
- @ Summary



Classification

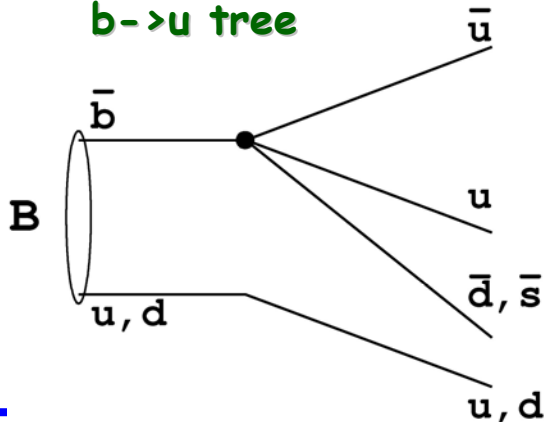
The dominant contributions to various 3-body final states:

b → s(d) penguin



b → s penguin transition contributes only to final states with odd number of kaons (s-quarks): $K\pi\pi$, KKK

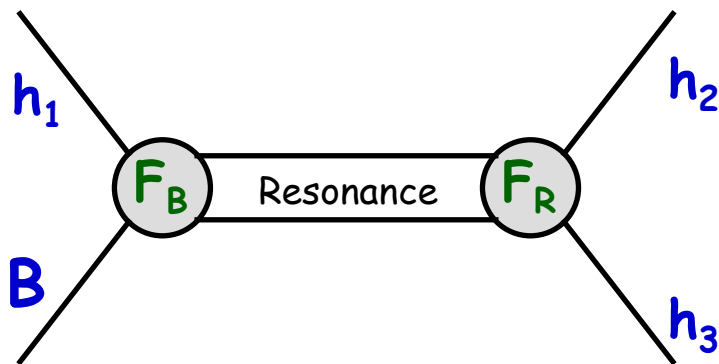
b → u tree



b → u tree and b → d penguin transitions contribute mainly to final states with even number of kaons (s-quarks): $\pi\pi\pi$, $KK\pi$. Contribution to states with odd number of kaons is Cabibbo suppressed

"wrong flavor" final states such as $K^+K^+\pi^-$ and $K^-\pi^+\pi^+$ are expected to be exceedingly small (10^{-11}) in the SM
 → good place to search for NP

Dalitz Analysis Basics



In multi-body decays additional degrees of freedom appears. In the simplest case of the three-body $B \rightarrow h_1 h_2 h_3$ decay, where h_i are all spin-0 particles there are two additional degrees of freedom:

$$s_{ij} = m^2(h_i h_j) - \text{three combinations}$$

From energy-momentum conservation:

$$s_{12} + s_{13} + s_{23} = M_B^2 + m_1^2 + m_2^2 + m_3^2$$

In the general case there is a set of intermediate resonances possible ...

Dalitz Analysis Basics

$$M(h_1 h_2 h_3) = \sum a_j A_j e^{i\delta_j}$$

- ✦ BW_J - Breit-Wigner function
- ✦ $\Gamma_R^{(J)}(s)$ - s -dependent width
- ✦ T_J - Angular correlations of the decay products

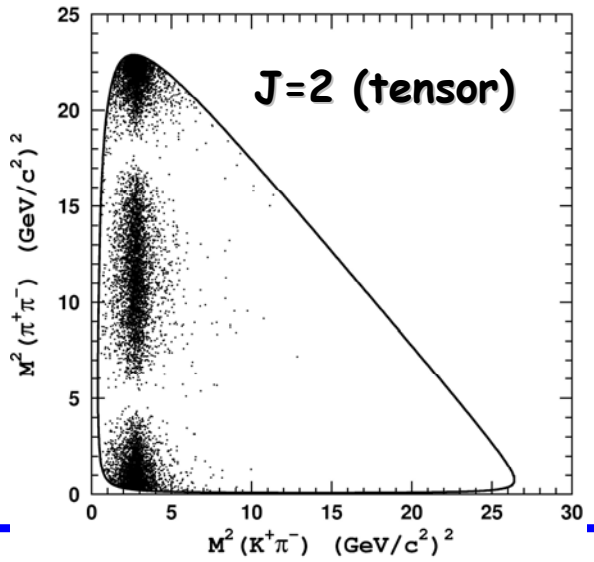
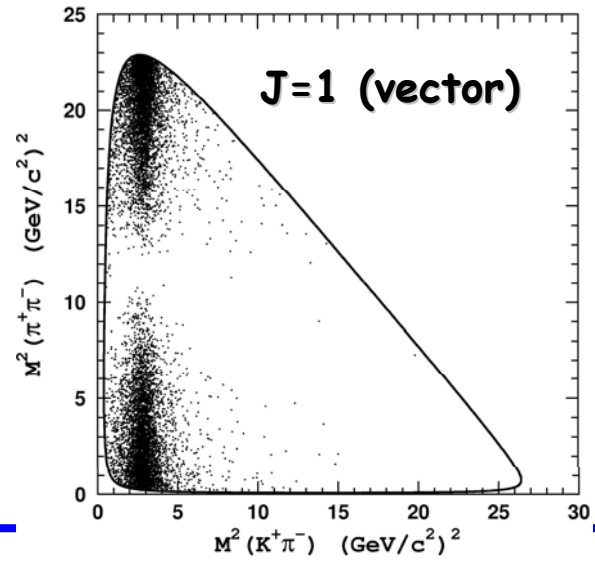
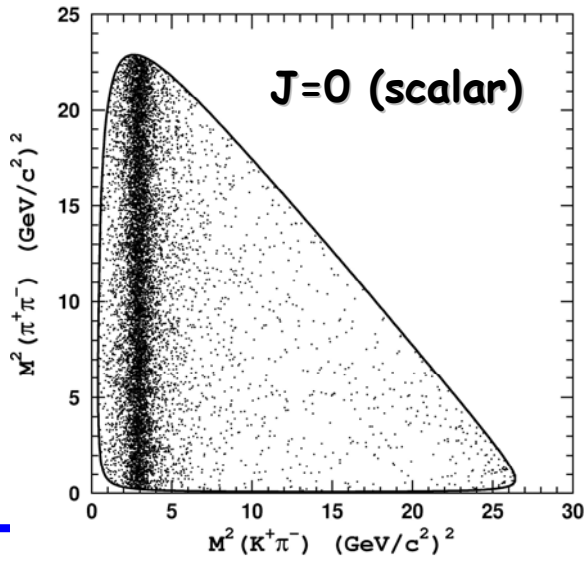
$$A_J = F_B F^{(J)}_R B W_J T_J$$

$$B W^J = \frac{1}{M_R^2 - s - i M_R \Gamma_R^{(J)}(s)}$$

$$\Gamma_R^{(J)}(s) = \Gamma_R F_R^2 \frac{M_R}{s^{1/2}} \left(\frac{p_s}{p_0} \right)^{2J+1}$$

$$\frac{d\Gamma}{ds_1 ds_2} \sim |M|^2$$

Simplest case of a single quasi-two-body channel



$B^{+(0)} \rightarrow K^{+(0)} \pi^+ \pi^-$

Large background from other B decays
 → impose 2-body vetoes:

- ✗ $B^- \rightarrow D^0(K^- \pi^+) \pi^-$: $|M(K^+ \pi^-) - M_D| > 100 \text{ MeV}$
- ✗ $B^- \rightarrow J/\psi(\mu^+ \mu^-) K^-$: $|M^*(\pi^+ \pi^-) - M_{J/\psi}| > 70 \text{ MeV}$
- ✗ $B^- \rightarrow \Psi'(\mu^+ \mu^-) K^-$: $|M^*(\pi^+ \pi^-) - M_{\Psi'}| > 50 \text{ MeV}$
- ✗ $B^- \rightarrow D^0(\pi^+ \pi^-) K^-$: $|M(\pi^+ \pi^-) - M_D| > 15 \text{ MeV}$

Miss-ID D-veto:

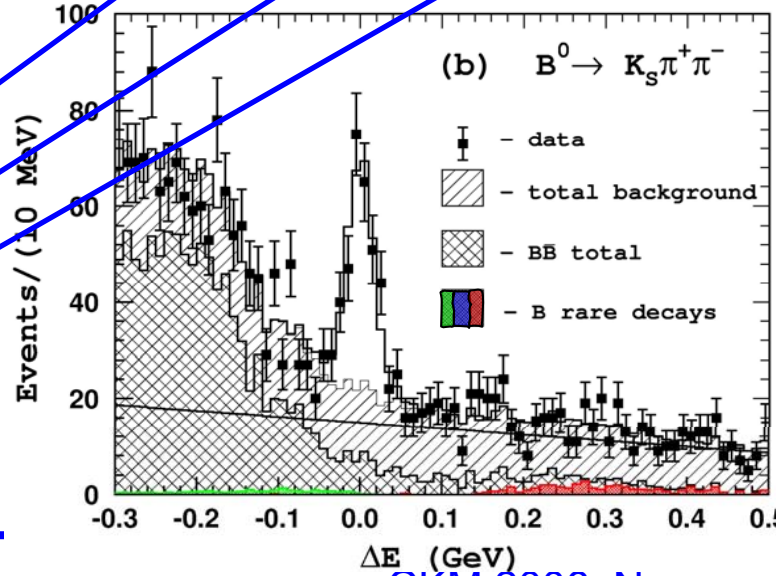
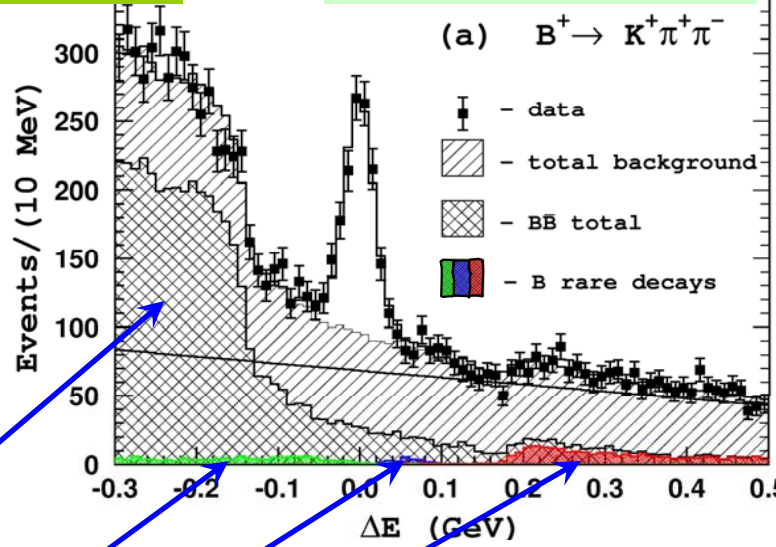
- ✗ $|M(h^+ h^-) - M_D| > 15 \text{ MeV}$

Remaining non-peaking BB background is well described by MC simulation

- ✗ $B^- \rightarrow D(K^- \pi^+ \pi^0) \pi^-$
 - ✗ $B^- \rightarrow D(K^- \mu^+ \nu) \pi^-$
 - ✗ $B^- \rightarrow D^*(K^- \mu^+ \nu(\pi^0 \text{ or } \gamma)) \pi^-$
-
- ✗ $B^- \rightarrow \eta'(\pi^+ \pi^- \gamma) K^-$
 - ✗ $B^- \rightarrow \pi^+ \pi^- \pi^-$
 - ✗ $B^0 \rightarrow K^+ \pi^-$

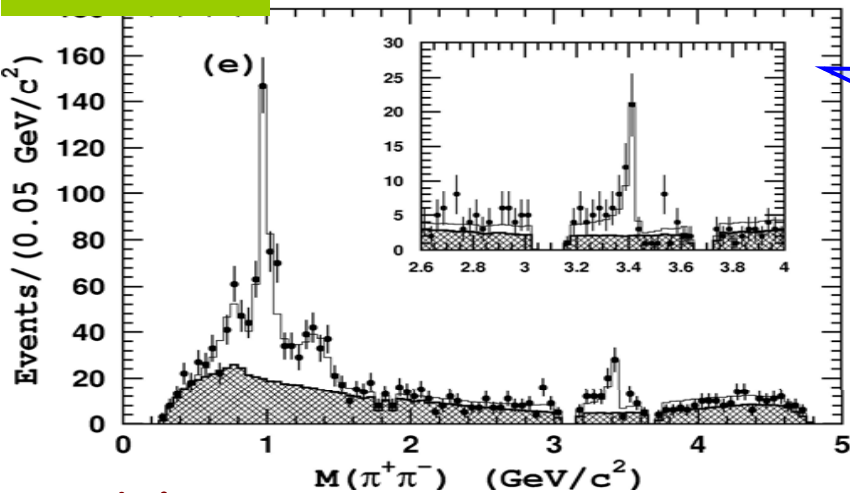
$L = 78 \text{ fb}^{-1}$

$|M_{bc} - M_B| < 7.5 \text{ MeV}$



$B^+ \rightarrow K^+ \pi^+ \pi^-$: Fitting The Signal

$L=140 \text{ fb}^{-1}$



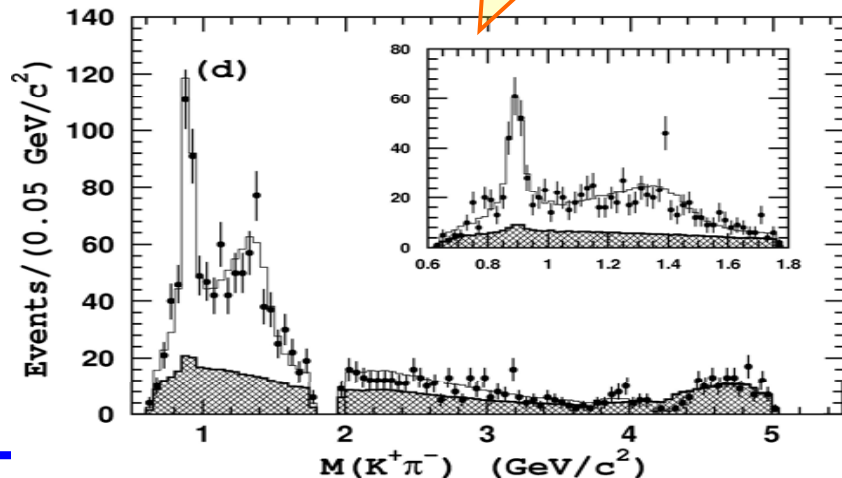
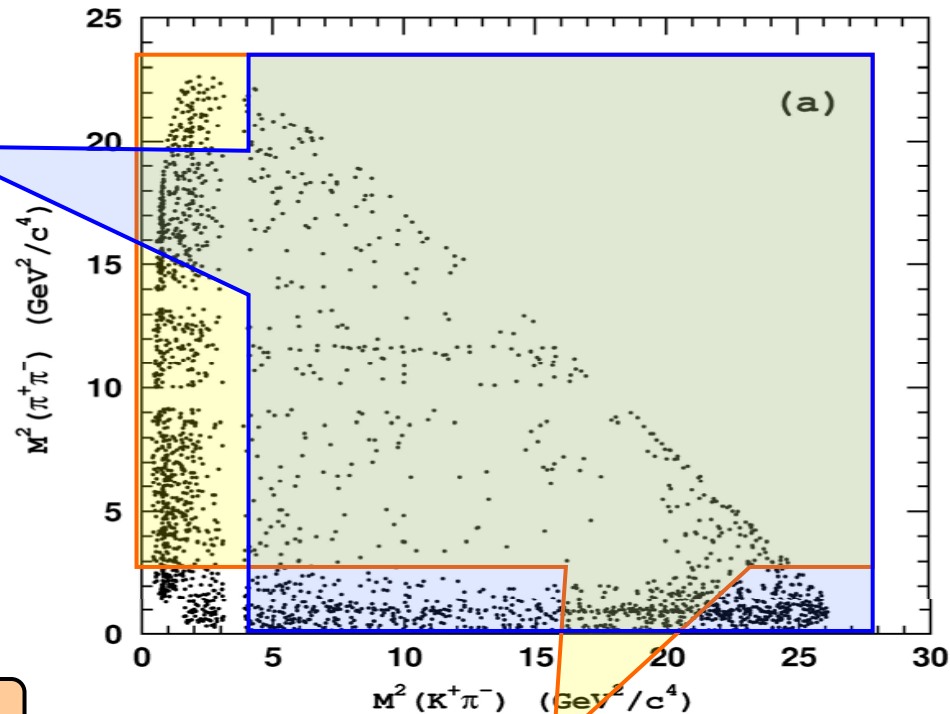
Model $K\pi\pi - A_J$:

$$S_{AJ}(K\pi\pi) = A_1(K^*(892)) + A_0(K^*_0(1430))$$

$$+ A_1(\rho(770)) + A_0(f_0(980))$$

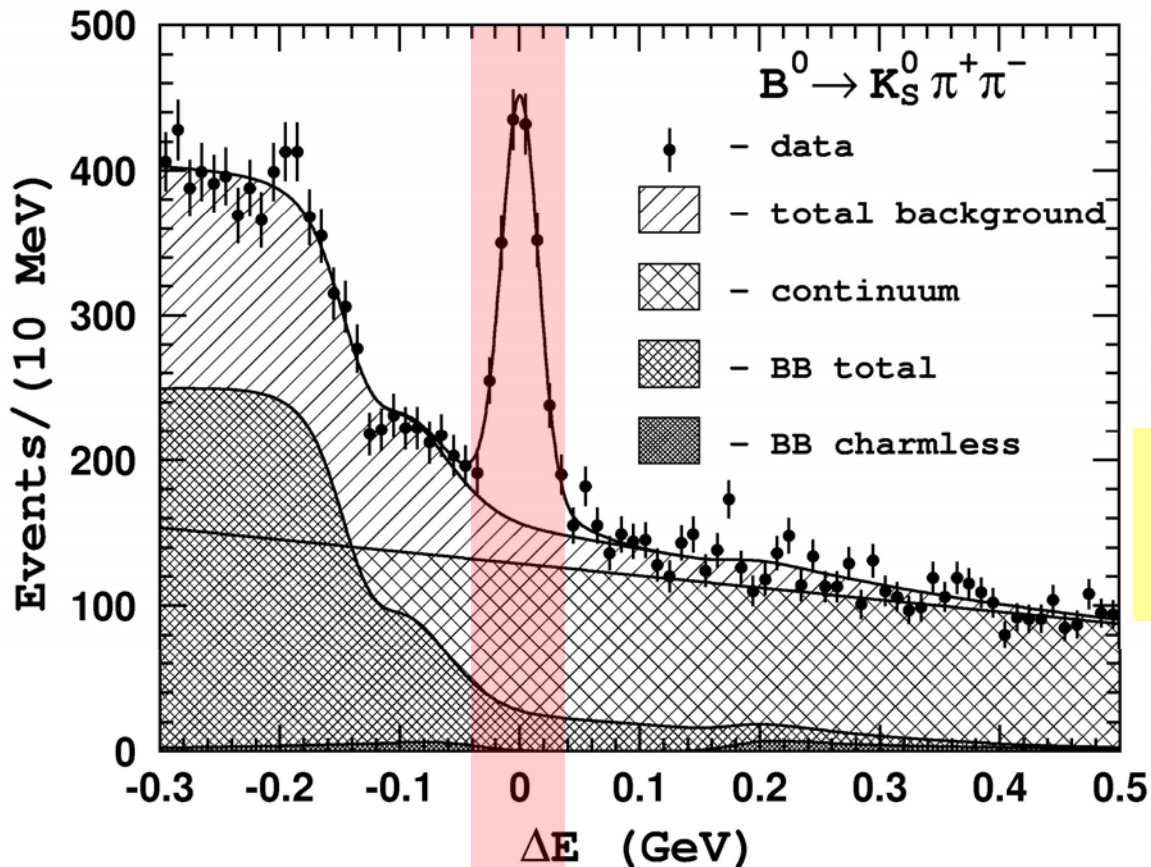
$$+ A_J(f_\chi(1300)) + A_0(\chi_{c0}) + A_{NR}$$

fit the data with different spin assumptions



$B^0 \rightarrow K_S \pi^+ \pi^-$: Signal Yield

$L = 353 \text{ fb}^{-1}$



Use the same model to fit the signal as for the $B^+ \rightarrow K^+ \pi^+ \pi^-$ decay:

$$M \sim A(K^*(892)\pi) + A(K^*_0(1430)\pi) + A(\rho(770)K) + A(f_0(980)K) + A(f_\chi(1300)K) + A(\chi_{c0}K) + A_{NR}$$

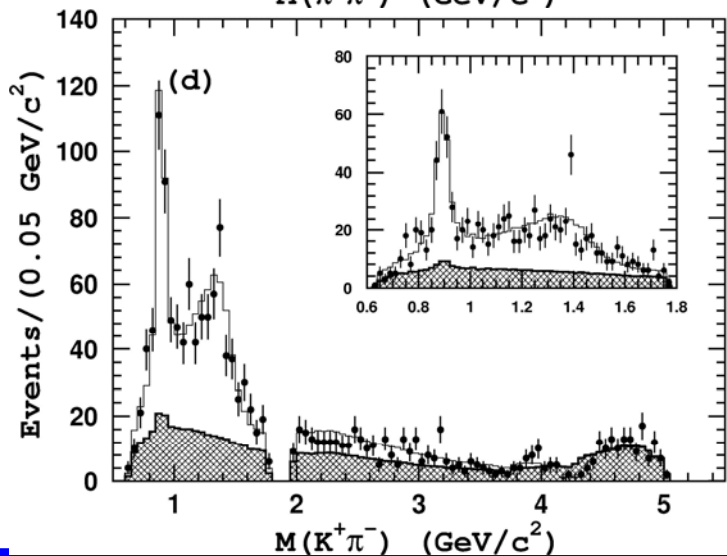
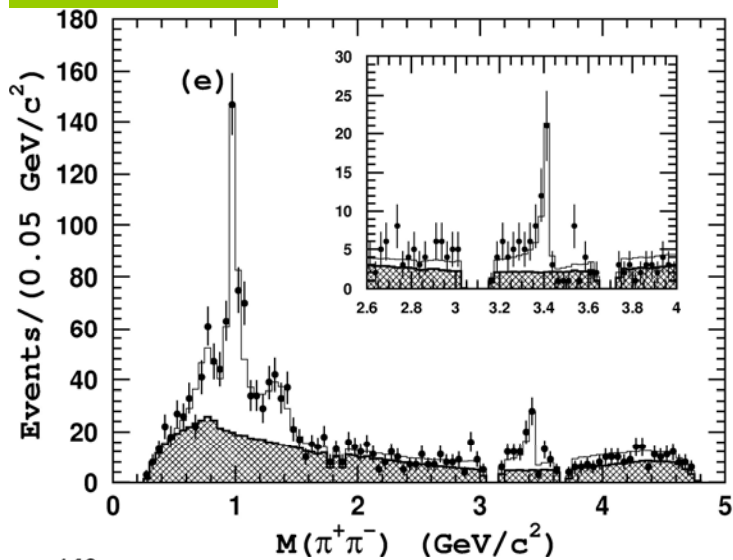
$$N_{sig}(K_S \pi^+ \pi^-) = 1246 \pm 63$$

S/B ~ 1

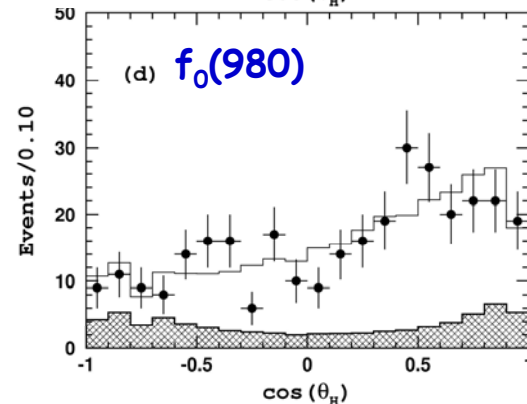
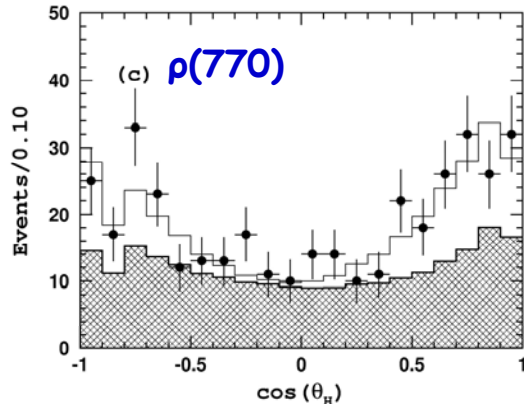
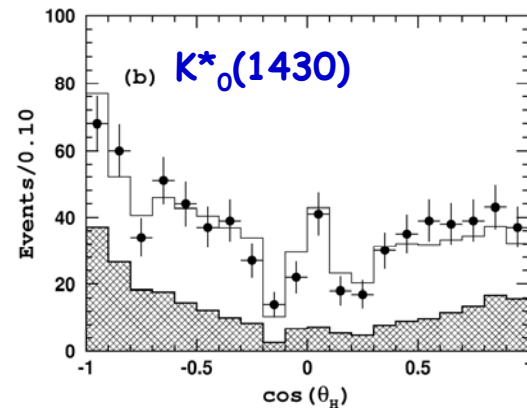
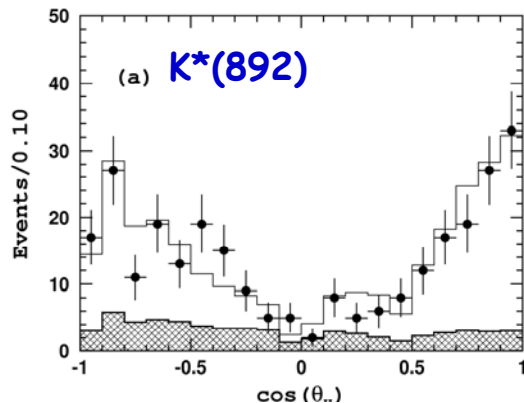
No flavor tagging applied - measuring average fractions

$B^+ \rightarrow K^+ \pi^+ \pi^-$: Fitting The Signal

$L=140 \text{ fb}^{-1}$

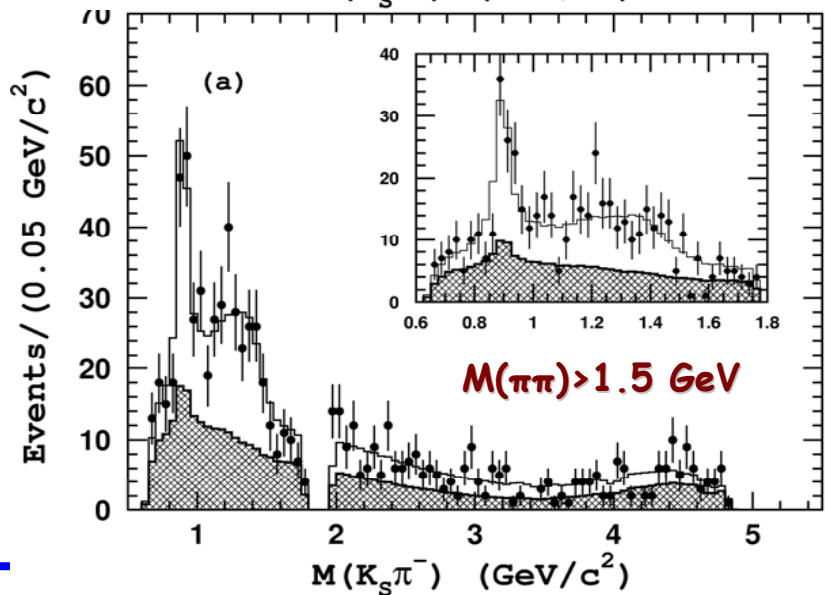
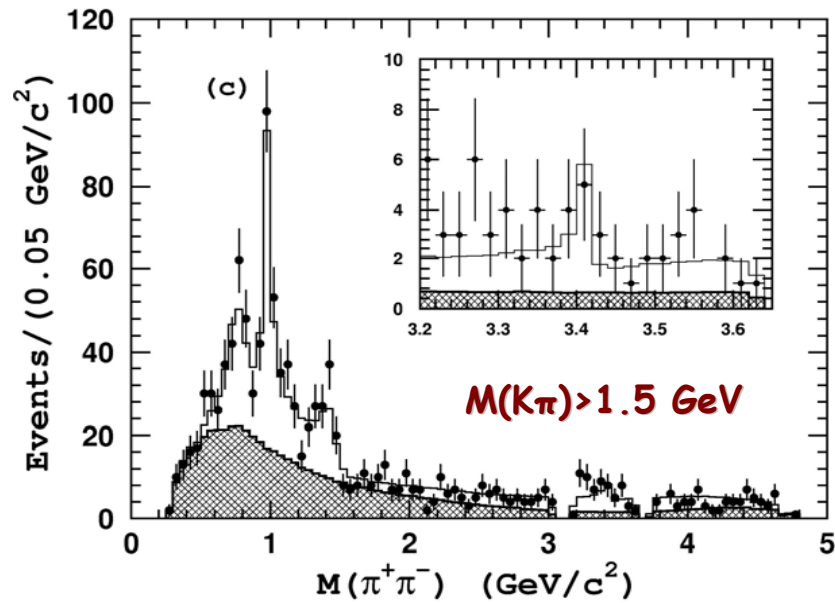
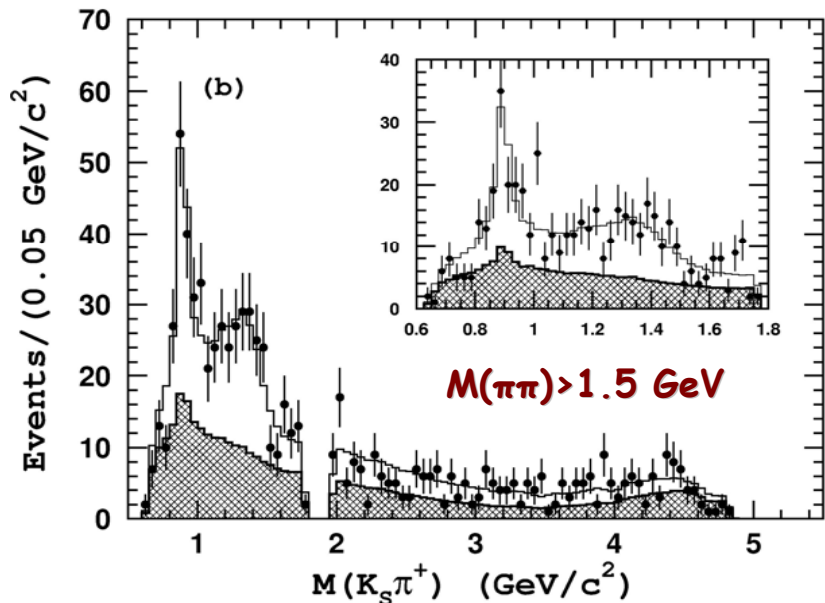


Helicity angle distributions:

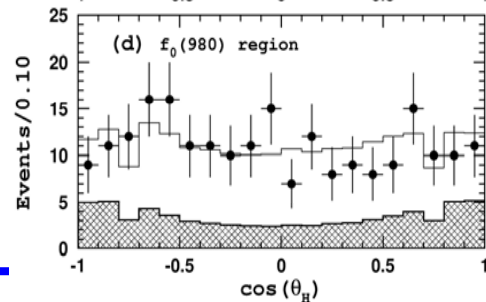
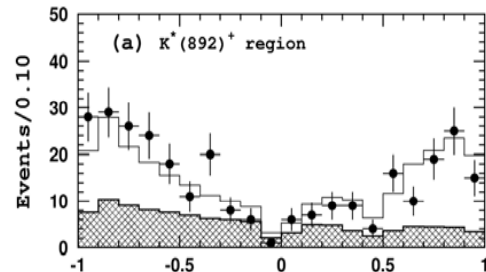
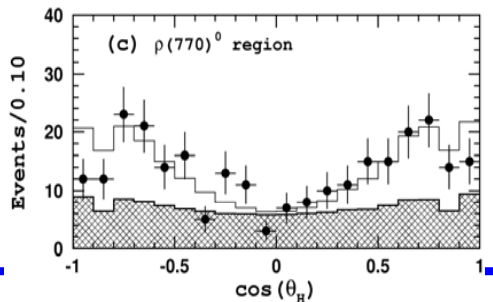


Spin of the $f_x(1300)$ is not well defined; scalar and vector hypotheses give equally good description of the data. More data is required ...

$B^0 \rightarrow K_S \pi^+ \pi^-$: Fitting the Signal



Helicity angle
Distributions



Summary of two-body BF's

L=140 fb⁻¹

Mode	$\mathcal{B}(B^+ \rightarrow Rh^+) \times \mathcal{B}(R \rightarrow h^+h^-) \times 10^6$	$\mathcal{B}(B^+ \rightarrow Rh^+) \times 10^6$
$K^+\pi^+\pi^-$ charmless total	—	$46.6 \pm 2.1 \pm 4.3$
$K^*(892)^0\pi^+, K^*(892)^0 \rightarrow K^+\pi^-$	$6.55 \pm 0.60 \pm 0.60^{+0.38}_{-0.57}$	$9.83 \pm 0.90 \pm 0.90^{+0.57}_{-0.86}$
$K_0^*(1430)\pi^+, K_0^*(1430) \rightarrow K^+\pi^-$	$27.9 \pm 1.8 \pm 2.6^{+8.5}_{-5.4}$ ($5.12 \pm 1.36 \pm 0.49^{+1.91}_{-0.51}$)	$45.0 \pm 2.9 \pm 6.2^{+13.7}_{-8.7}$ ($8.26 \pm 2.20 \pm 1.19^{+3.08}_{-0.82}$)
$K^*(1410)\pi^+, K^*(1410) \rightarrow K^+\pi^-$	< 2.0	—
$K^*(1680)\pi^+, K^*(1680) \rightarrow K^+\pi^-$	< 3.1	—
$K_2^*(1430)\pi^+, K_2^*(1430) \rightarrow K^+\pi^-$	< 2.3	—
$\rho^0(770)K^+, \rho^0(770) \rightarrow \pi^+\pi^-$	$4.78 \pm 0.75 \pm 0.44^{+0.91}_{-0.87}$	$4.78 \pm 0.75 \pm 0.44^{+0.91}_{-0.87}$
$f_0(980)K^+, f_0(980) \rightarrow \pi^+\pi^-$	$7.55 \pm 1.24 \pm 0.69^{+1.48}_{-0.96}$	—
$f_2(1270)K^+, f_2(1270) \rightarrow \pi^+\pi^-$	< 1.3	—
Non-resonant	—	$17.3 \pm 1.7 \pm 1.6^{+17.1}_{-7.8}$
$K^+K^+K^-$ charmless total	—	$30.6 \pm 1.2 \pm 2.3$
$\phi K^+, \phi \rightarrow K^+K^-$	$4.72 \pm 0.45 \pm 0.35^{+0.39}_{-0.22}$	$9.60 \pm 0.92 \pm 0.71^{+0.78}_{-0.46}$
$\phi(1680)K^+, \phi(1680) \rightarrow K^+K^-$	< 0.8	—
$f_0(980)K^+, f_0(980) \rightarrow K^+K^-$	< 2.9	—
$f_2'(1525)K^+, f_2'(1525) \rightarrow K^+K^-$	< 2.1	—
$a_2(1320)K^+, a_2(1320) \rightarrow K^+K^-$	< 1.1	—
Non-resonant	—	$24.0 \pm 1.5 \pm 1.8^{+1.9}_{-5.7}$
$\chi_{c0}K^+, \chi_{c0} \rightarrow \pi^+\pi^-$	$1.37 \pm 0.28 \pm 0.12^{+0.34}_{-0.35}$	—
$\chi_{c0}K^+, \chi_{c0} \rightarrow K^+K^-$	$0.86 \pm 0.26 \pm 0.06^{+0.20}_{-0.05}$ ($2.58 \pm 0.43 \pm 0.19^{+0.20}_{-0.05}$)	—
$\chi_{c0}K^+$ combined	—	$196 \pm 35 \pm 33^{+197}_{-26}$

Summary of two-body BF's II

$L = 353 \text{ fb}^{-1}$

Mode	$\mathcal{B}(B \rightarrow Rh) \times \mathcal{B}(R \rightarrow hh) \times 10^6$	$\mathcal{B}(B \rightarrow Rh) \times 10^6$
$K_S^0 \pi^+ \pi^-$ charmless total		$47.5 \pm 2.4 \pm 3.7$
$K^*(892)^+ \pi^-, K^*(892)^+ \rightarrow K^0 \pi^+$	$5.61 \pm 0.72 \pm 0.43_{-0.29}^{+0.43}$	$8.42 \pm 1.08 \pm 0.65_{-0.43}^{+0.64}$
$K_0^*(1430)^+ \pi^-, K_0^*(1430)^+ \rightarrow K^0 \pi^+$	$30.8 \pm 2.4 \pm 2.4_{-3.0}^{+0.8}$	$49.7 \pm 3.8 \pm 3.8_{-4.8}^{+1.2}$
$K^*(1410)^+ \pi^-, K^*(1410)^+ \rightarrow K^0 \pi^+$	< 3.8	—
$K^*(1680)^+ \pi^-, K^*(1680)^+ \rightarrow K^0 \pi^+$	< 2.6	—
$K_2^*(1430)^+ \pi^-, K_2^*(1430)^+ \rightarrow K^0 \pi^+$	< 2.1	—
$\rho(770)^0 K^0, \rho(770)^0 \rightarrow \pi^+ \pi^-$	$6.13 \pm 0.95 \pm 0.47_{-1.05}^{+1.00}$	$6.13 \pm 0.95 \pm 0.47_{-1.05}^{+1.00}$
$f_0(980) K^0, f_0(980) \rightarrow \pi^+ \pi^-$	$7.60 \pm 1.66 \pm 0.59_{-0.67}^{+0.48}$	—
$f_2(1270) K^0, f_2(1270) \rightarrow \pi^+ \pi^-$	< 1.4	—
Non-resonant		$19.9 \pm 2.5 \pm 1.5_{-1.2}^{+0.7}$
$\chi_{c0} K^0, \chi_{c0} \rightarrow \pi^+ \pi^-$	< 0.56	< 113

BELLE-CONF-577, hep-ex/0509047

DCPV in $B^{\pm} \rightarrow K^{\pm} \pi^+ \pi^-$

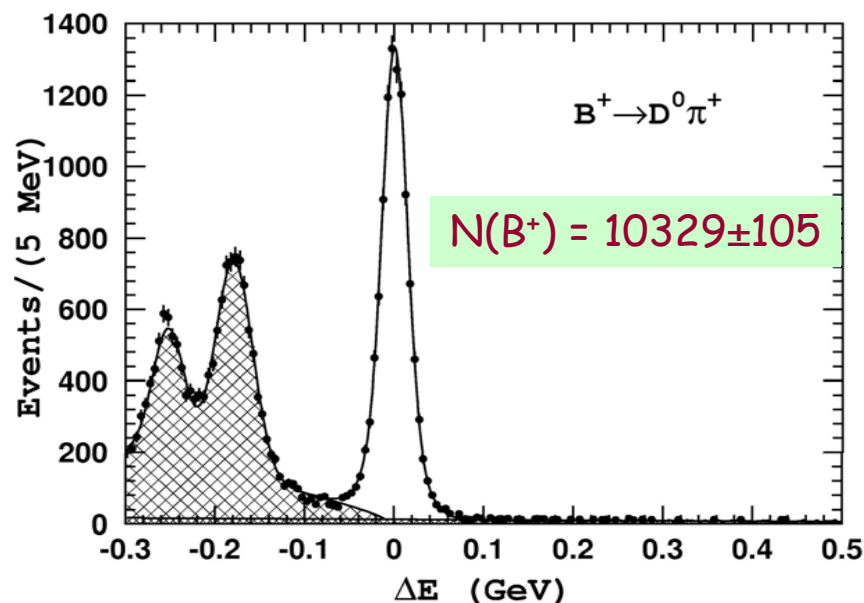
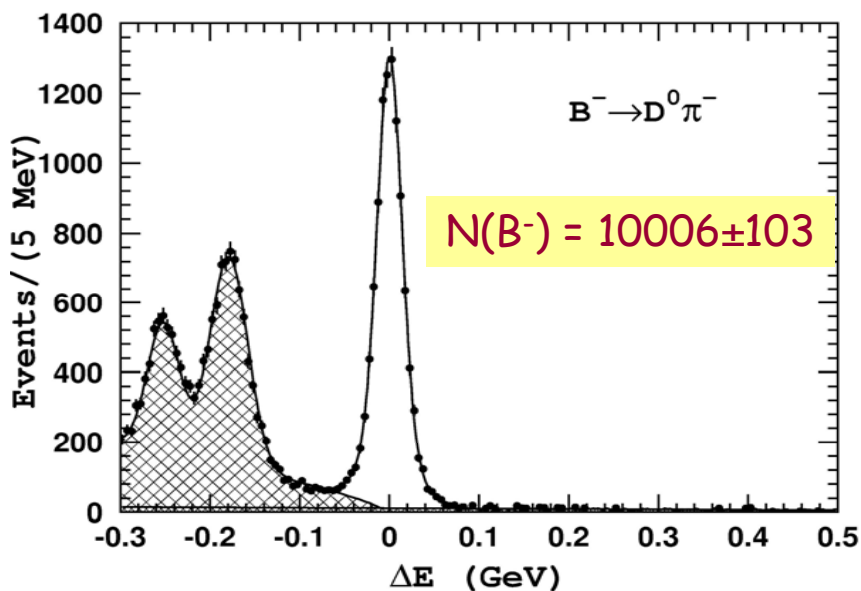
Null Asymmetry Tests

$$A_{CP} = \frac{\Gamma(B^- \rightarrow f) - \Gamma(B^+ \rightarrow f)}{\Gamma(B^- \rightarrow f) + \Gamma(B^+ \rightarrow f)}$$

- ⊕ continuum background events
- ⊕ BB background events

$$A_{CP}(qq) = -0.9 \pm 1.1 \%$$

$$A_{CP}(BB) = -1.1 \pm 1.8 \%$$



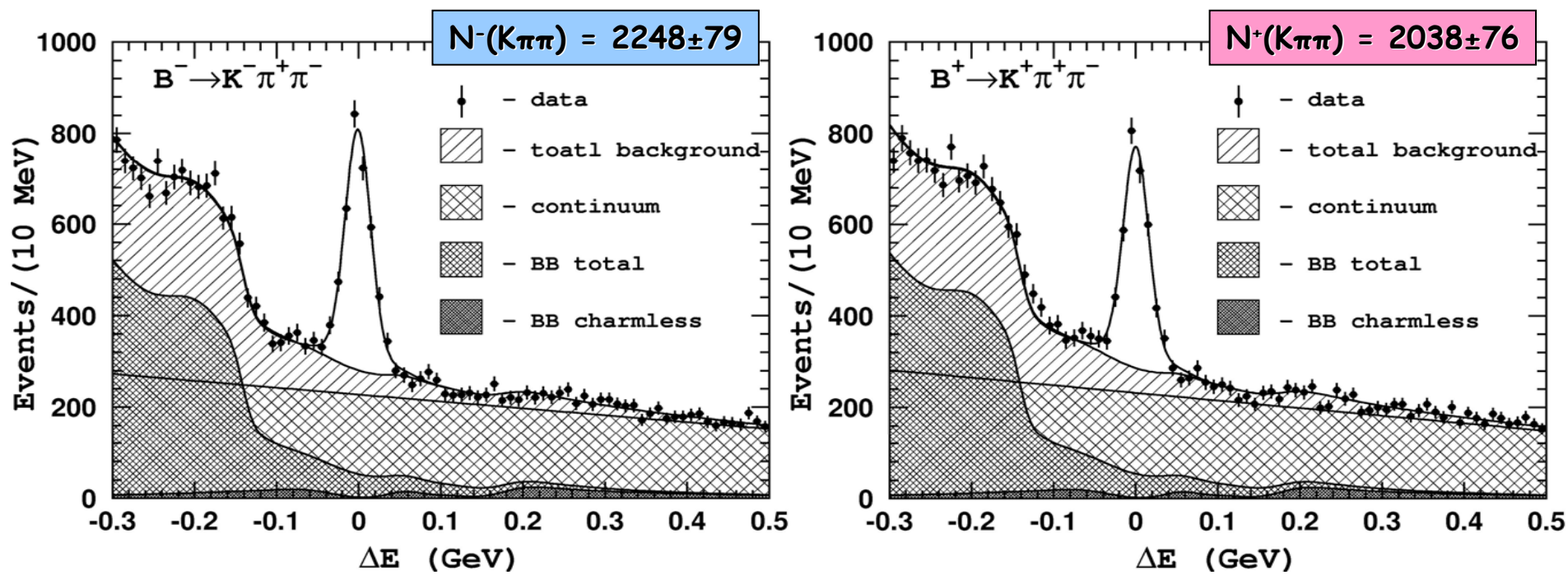
- ⊕ $B \rightarrow D(K\pi)\pi$

$$A_{CP}(D\pi) = -1.6 \pm 0.7 \%$$

Considered as systematic uncertainty

DCPV in $B^{\pm} \rightarrow K^{\pm} \pi^+ \pi^-$

$L = 353 \text{ fb}^{-1}$

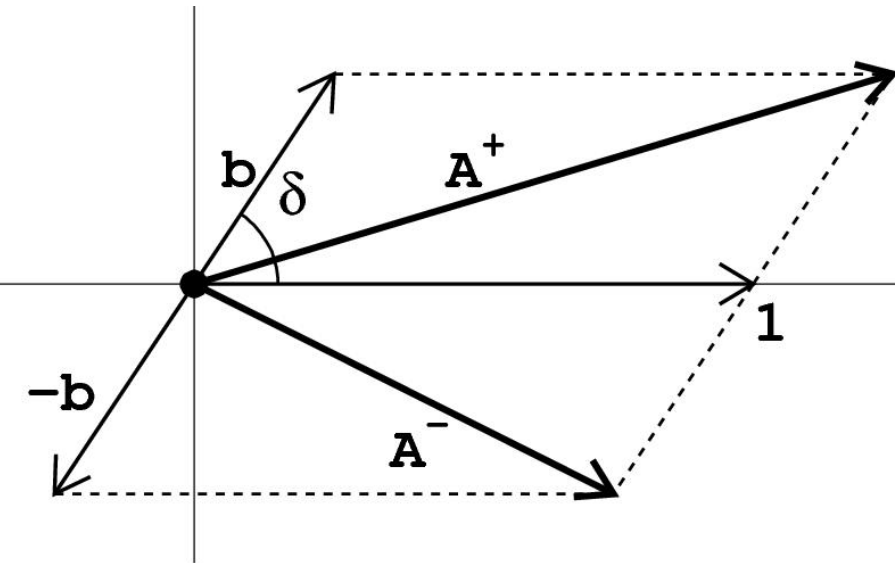


$$A_{CP}(K^{\pm} \pi^+ \pi^-) = \frac{N(K^- \pi^- \pi^+) - N(K^+ \pi^+ \pi^-)}{N(K^- \pi^- \pi^+) + N(K^+ \pi^+ \pi^-)} = (+4.9 \pm 2.6 \pm 2.0)\%$$

Dalitz analysis is then performed with an amplitude sensitive to the charge of the B meson ...

DCPV in $B^\pm \rightarrow K^\pm \pi^+ \pi^-$

Parameterization



basic
parameterization

$$ae^{i\varphi_j} \longrightarrow ae^{i\varphi_j} (1 + \eta b_j e^{i\delta_j})$$

parameterization
with DCPV



-1 for B^-
+1 for B^+

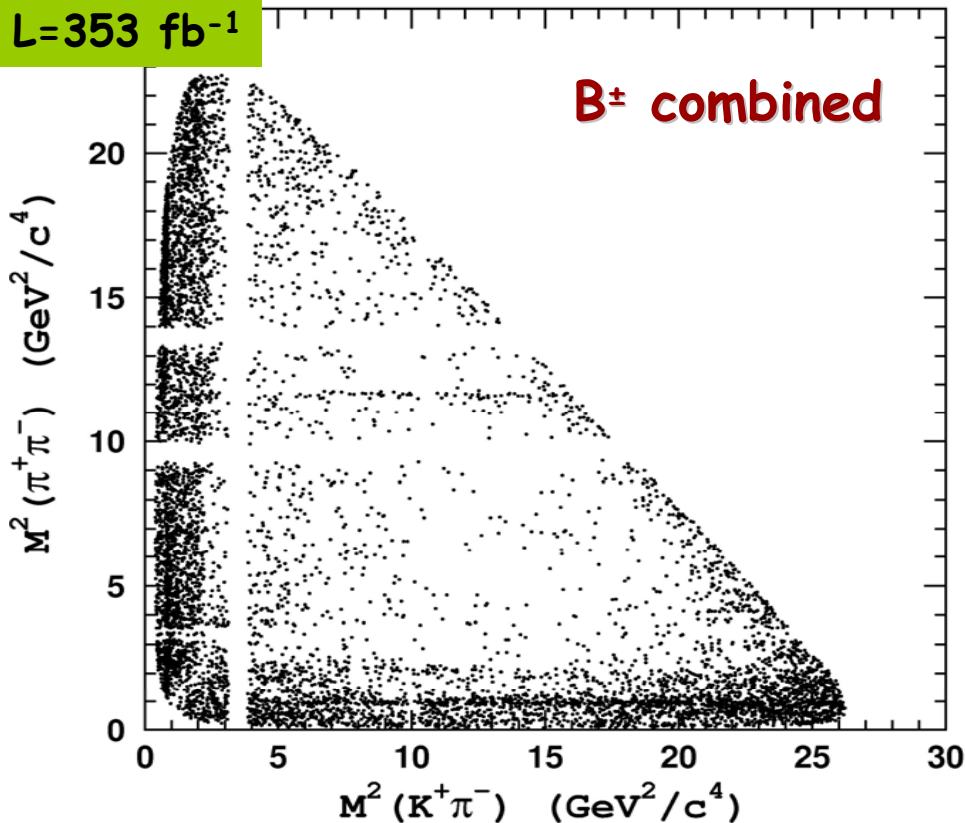
Charge asymmetry A_{CP} is given by

$$A_{CP}^j = -\frac{2b_j \cos \delta_j}{1 + b_j^2}$$

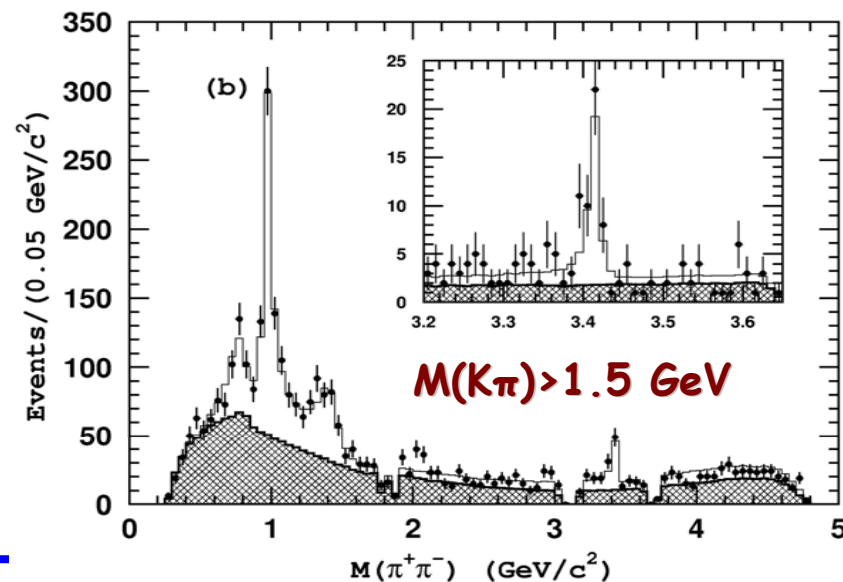
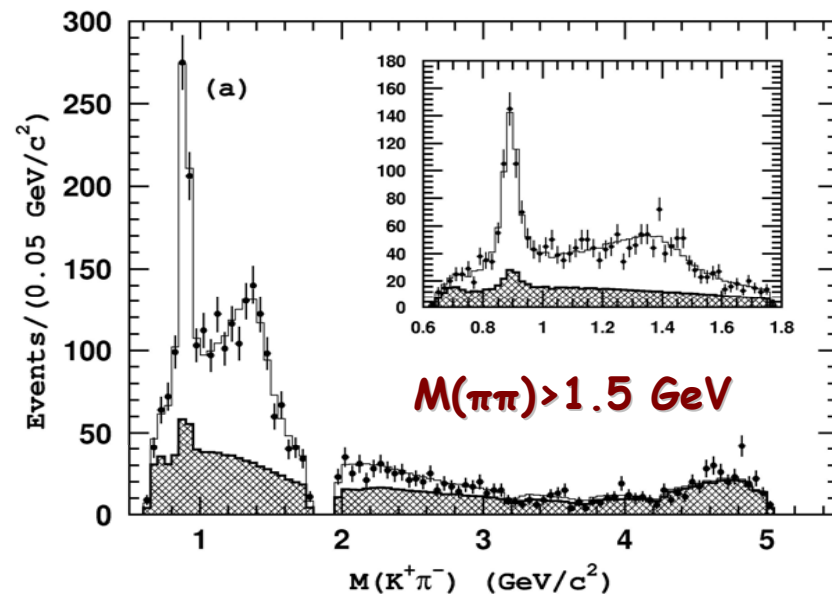
Note that CP violation can be observed even in the case of $A_{CP}=0$ (that is $\delta=90^\circ$ but $b \neq 0$) - possible only in three-body decays!

DCPV with $B^\pm \rightarrow K^\pm \pi^+ \pi^-$

$L = 353 \text{ fb}^{-1}$



- ⊗ $K^*(892)^0 \pi^+$
- ⊗ $K^*_0(1430)^0 \pi^+$
- ⊗ Non-resonant
- ⊗ $\rho(770)^0 K^+$
- ⊗ $\omega(782) K^+$
- ⊗ $f_0(980) K^+$
- ⊗ $f_0(1370) K^+$
- ⊗ $f_2(1270) K^+$
- ⊗ $\chi_{c0} K^+$

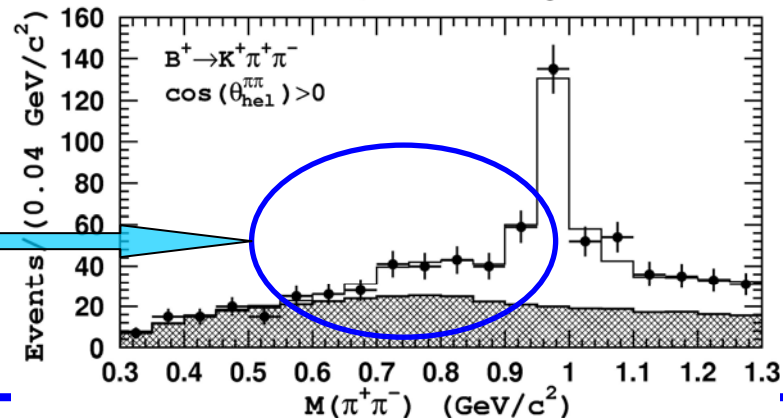
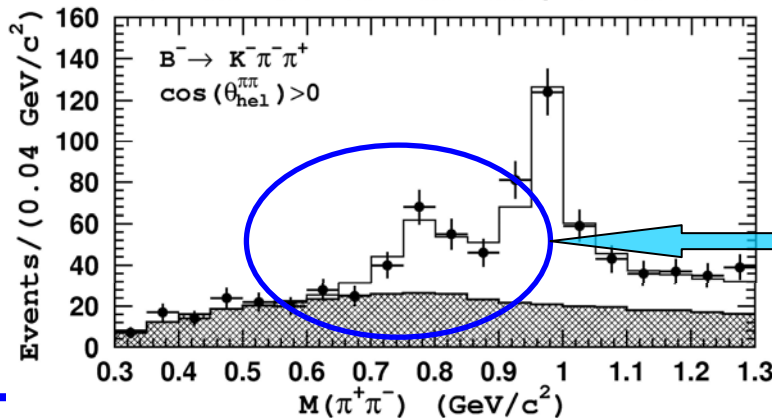
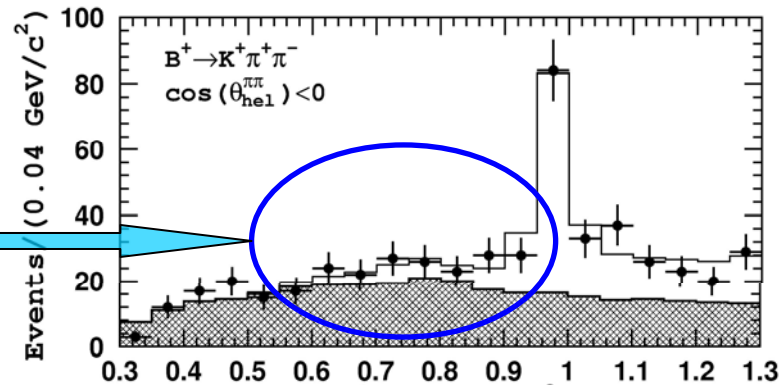
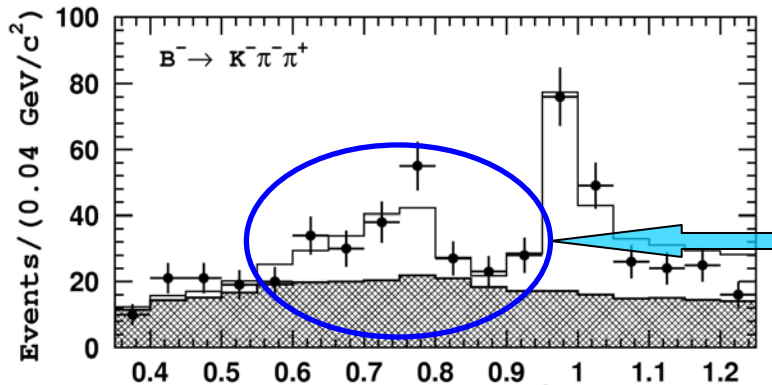
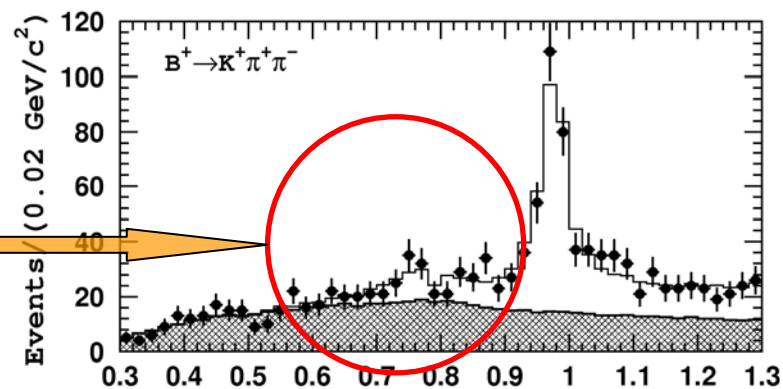
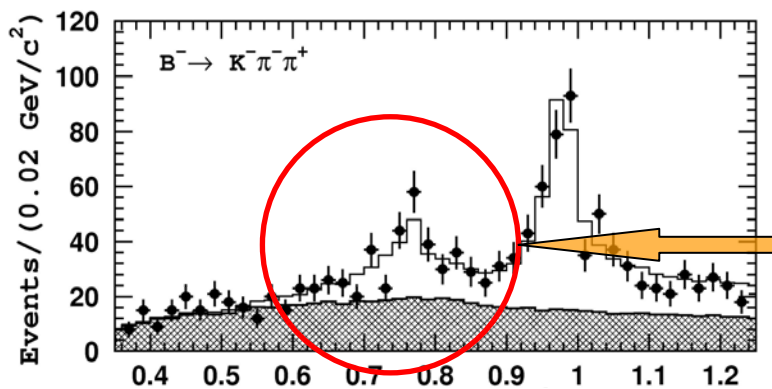


DCPV in $B^{\pm} \rightarrow K^{\pm} \pi^+ \pi^-$: Results

Channel	CP averaged fraction, %	δ , degrees	b	φ , degrees	A_{CP} , %	Significance, σ
$K^*(892)^0 \pi^{\pm}$	$13.0 \pm 0.8^{+0.5}_{-0.7}$	0 (fixed)	$0.078 \pm 0.033^{+0.012}_{-0.003}$	$-18 \pm 44^{+5}_{-13}$	$-14.9 \pm 6.4^{+0.8}_{-0.8}$	2.6
$K_0(1430)^0 \pi^{\pm}$	$65.5 \pm 1.5^{+2.2}_{-3.9}$	$55 \pm 4^{+1}_{-5}$	$0.069 \pm 0.031^{+0.010}_{-0.008}$	$-123 \pm 16^{+4}_{-5}$	$+7.5 \pm 3.8^{+2.0}_{-0.9}$	2.7
$\rho(770)^0 K^{\pm}$	$7.85 \pm 0.93^{+0.64}_{-0.59}$	$-21 \pm 14^{+14}_{-19}$	$0.28 \pm 0.11^{+0.07}_{-0.09}$	$-125 \pm 32^{+10}_{-85}$	$+30 \pm 11^{+11}_{-4}$	3.9
$\omega(782) K^{\pm}$	$0.15 \pm 0.12^{+0.03}_{-0.02}$	$100 \pm 31^{+38}_{-21}$	0 (fixed)	—	—	—
$f_0(980) K^{\pm}$	$17.7 \pm 1.6^{+1.1}_{-3.3}$	$67 \pm 11^{+10}_{-11}$	$0.30 \pm 0.19^{+0.05}_{-0.10}$	$-82 \pm 8^{+2}_{-2}$	$-7.7 \pm 6.5^{+4.1}_{-1.6}$	1.6
$f_2(1270) K^{\pm}$	$1.52 \pm 0.35^{+0.22}_{-0.37}$	$140 \pm 11^{+18}_{-7}$	$0.37 \pm 0.17^{+0.11}_{-0.03}$	$-24 \pm 29^{+14}_{-20}$	$-59 \pm 22^{+3}_{-3}$	2.7
$f_X(1300) K^{\pm}$	$4.14 \pm 0.81^{+0.31}_{-0.30}$	$-141 \pm 10^{+8}_{-9}$	$0.12 \pm 0.17^{+0.04}_{-0.07}$	$-77 \pm 56^{+88}_{-43}$	$-5.4 \pm 16.5^{+10.3}_{-2.4}$	1.0
Non-Res.	$34.0 \pm 2.2^{+2.1}_{-1.8}$	$\delta_1^{\text{nr}} = -11 \pm 5^{+3}_{-3}$ $\delta_2^{\text{nr}} = 185 \pm 20^{+62}_{-19}$	0 (fixed)	—	—	—
$\chi_{c0} K^{\pm}$	$1.12 \pm 0.12^{+0.24}_{-0.08}$	$-118 \pm 24^{+37}_{-38}$	$0.15 \pm 0.35^{+0.08}_{-0.07}$	$-77 \pm 94^{+154}_{-11}$	$-6.5 \pm 19.6^{+2.9}_{-1.4}$	0.7

PRL 96, 251803 (2006)

DCPV in $B^{\pm} \rightarrow \rho(770)^0 K^{\pm}$



Summary

- Analysis of B meson decays to three-body charmless hadronic final states have been done:
 - branching fractions for 6 channels are measured;
 - upper limits for 5 more channels are set;
- Dalitz analysis for three three-body modes have been performed:
 - branching fractions for >20 quasi-two-body channels are measured;
- First evidence for CP violation in charged meson decays is observed in $B^{\pm} \rightarrow \rho(770)^0 K^{\pm}$

Three-Body Branching Fractions

3-Body Mode	Belle	BaBar	CLEO
$K^+\pi^+\pi^-$	$53.6 \pm 3.1 \pm 5.1$	$61.4 \pm 2.4 \pm 4.5$	-
$K^0\pi^+\pi^-$	$45.4 \pm 5.2 \pm 5.9$	$43.0 \pm 2.3 \pm 2.3$	$50^{+10}_{-9} \pm 7$
$K^+K^+K^-$	$32.8 \pm 1.8 \pm 2.8$	$29.6 \pm 2.1 \pm 1.6$	-
$K^0K^+K^-$	$28.3 \pm 3.3 \pm 4.0$	$23.8 \pm 2.0 \pm 1.6$	-
$K_S K_S K^+$	$13.4 \pm 1.9 \pm 1.5$	$10.7 \pm 1.2 \pm 1.0$	-
$K_S K_S K_S$	$4.2^{+1.6}_{-1.3} \pm 0.8$	$6.5 \pm 0.8 \pm 0.8$	-
$K^+K^-\pi^+$	<13	<6.3	-
$K^0K^-\pi^+$	<18	-	<21
$K_S K_S \pi^+$	<3.2	-	-
$K^-\pi^+\pi^+$	<4.5	<1.8	-
$K^+K^+\pi^-$	<2.4	<1.3	-

in units of 10^{-6}

$B^+ \rightarrow K^+ \pi^+ \pi^-$

Large background from other B decays
 → impose 2-body vetoes:

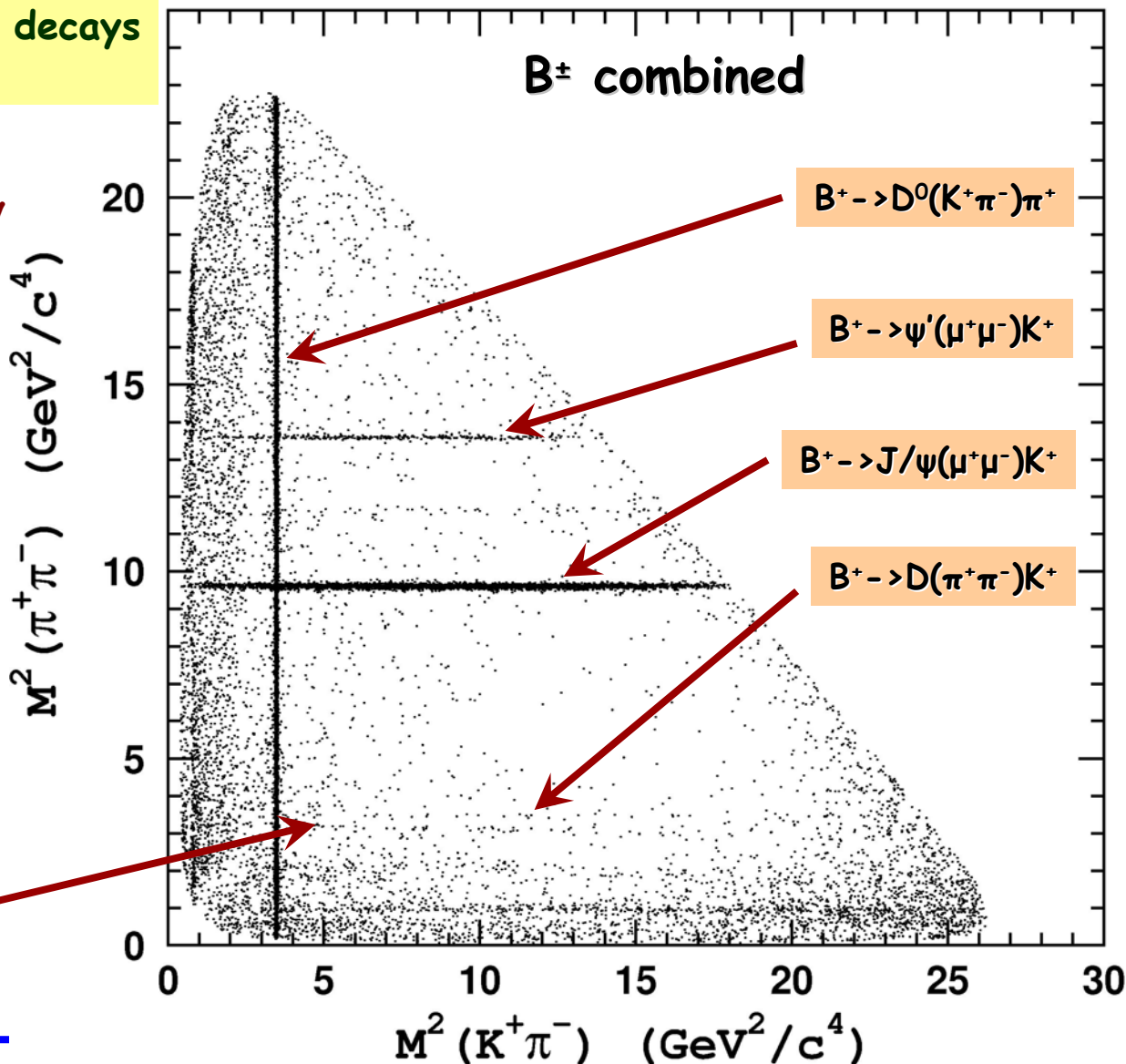
- ↗ $|M(K^+ \pi^-) - M_D| > 100 \text{ MeV}$
- ↗ $|M^*(\pi^+ \pi^-) - M_{J/\psi}| > 70 \text{ MeV}$
- ↗ $|M^*(\pi^+ \pi^-) - M_{\psi'}| > 50 \text{ MeV}$
- ↗ $|M(\pi^+ \pi^-) - M_D| > 15 \text{ MeV}$

+

Miss-ID D-veto:

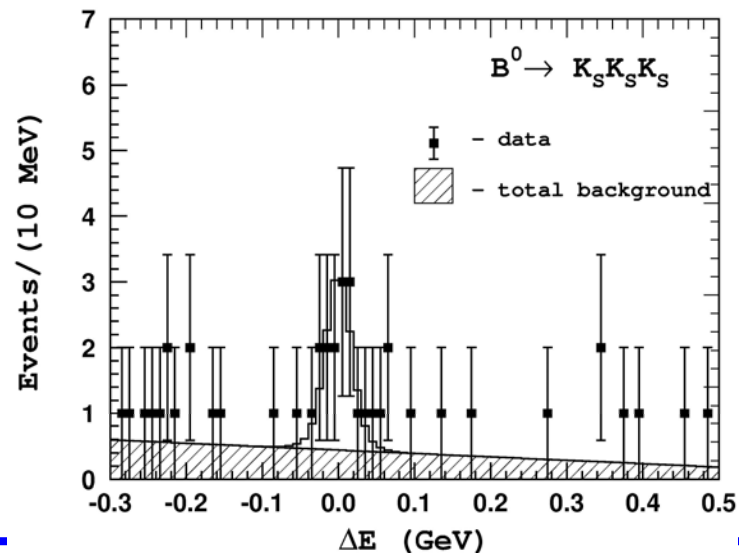
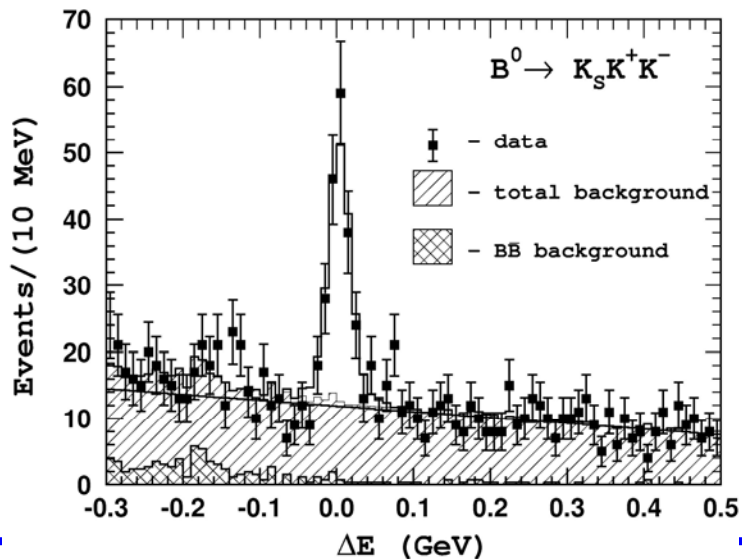
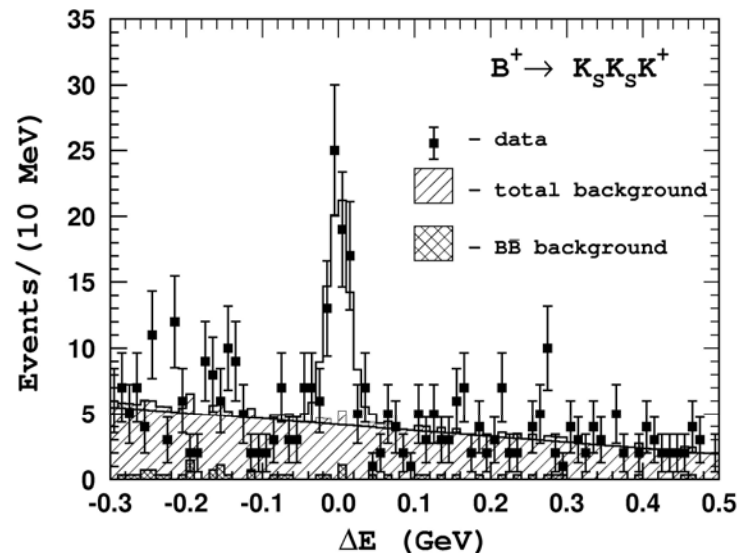
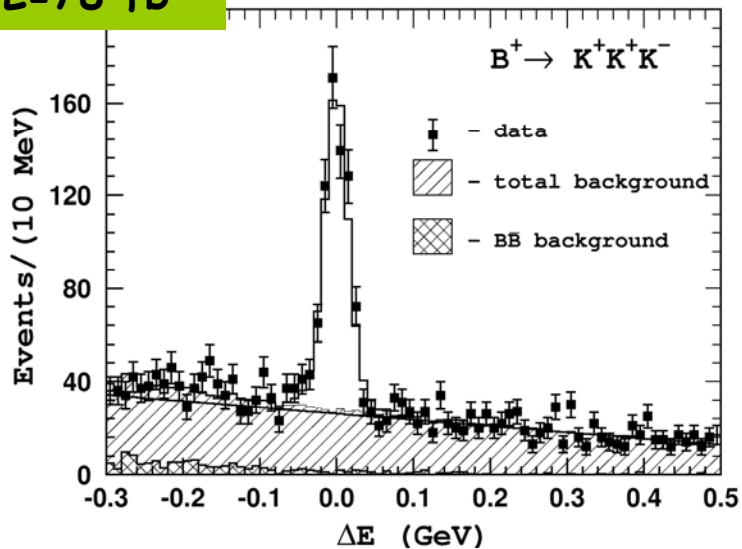
if any $h^+ h^-$ combination is consistent with $D \rightarrow K\pi$

- ↗ $|M(h^+ h^-) - M_D| > 15 \text{ MeV}$



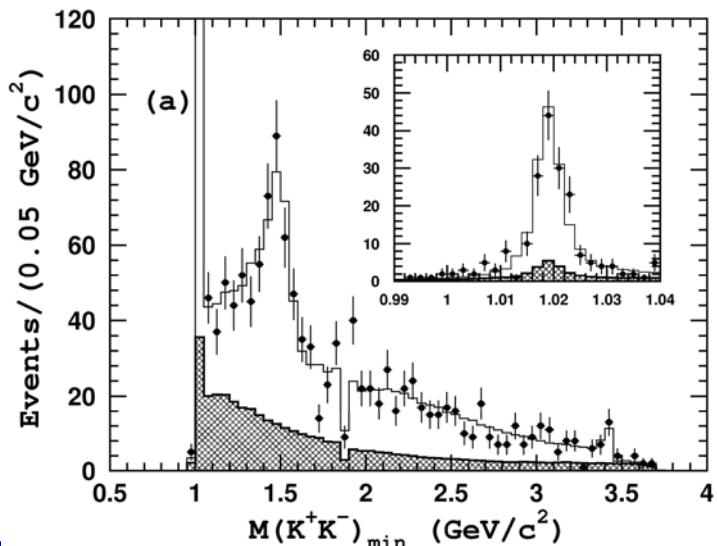
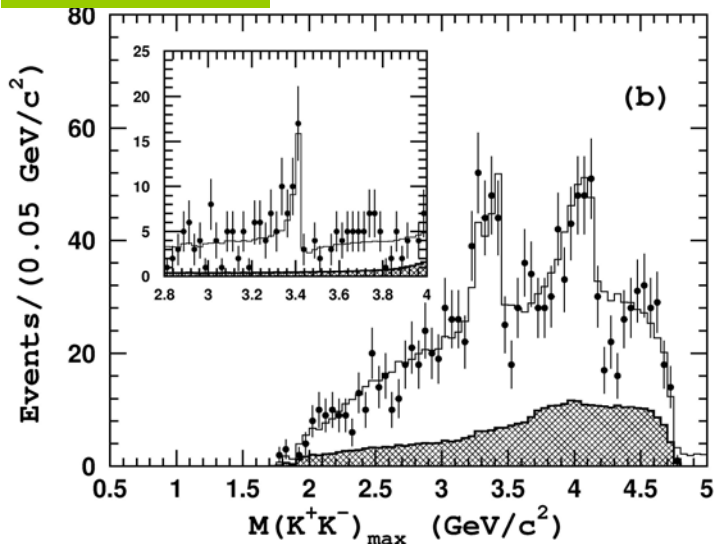
B → KKK

L=78 fb⁻¹



$B^+ \rightarrow K^+ K^+ K^-$: Fitting The Signal

$L=140 \text{ fb}^{-1}$



Model KKK- B_J :

$$S_{BJ}(KKK) = S_{AJ}(KKK) + A_{NR}$$

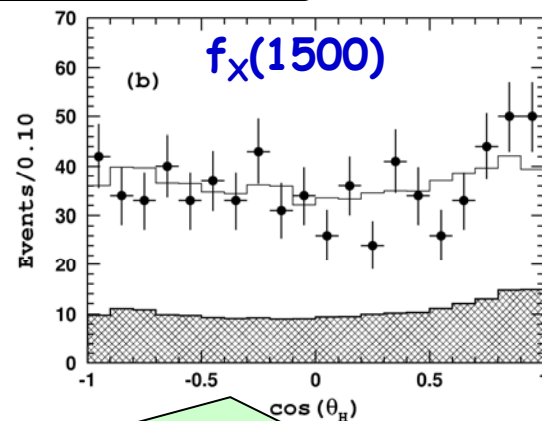
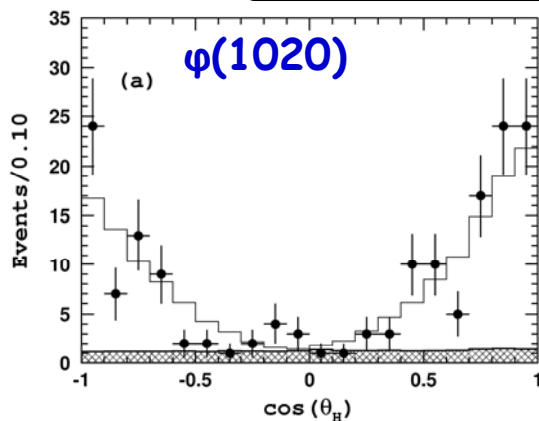
A_{NR} Parameterizations used:

$$A_{NR}(s_{13}, s_{23}) = a_1 e^{i\delta}$$

$$A_{NR}(s_{13}, s_{23}) = a_1 [(1/s_{13})^\beta + (1/s_{23})^\beta] e^{i\delta}$$

$$A_{NR}(s_{13}, s_{23}) = a_1 (e^{-\beta s_{13}} + e^{-\beta s_{23}}) e^{i\delta}$$

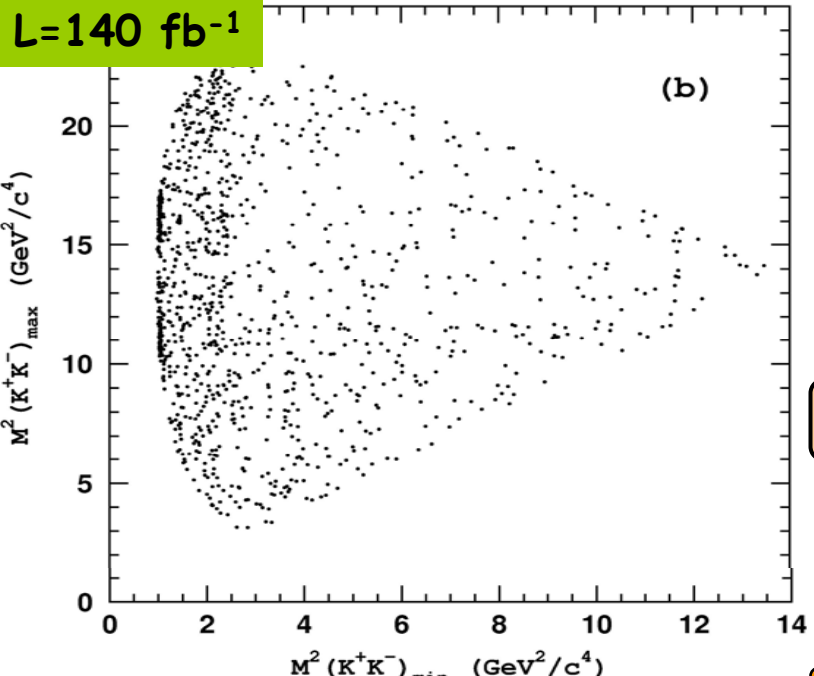
Helicity angle distributions:



$f_X(1500)$ is best fit with the scalar hypothesis

$B^+ \rightarrow K^+ K^+ K^-$: Fitting The Signal

$L=140 \text{ fb}^{-1}$

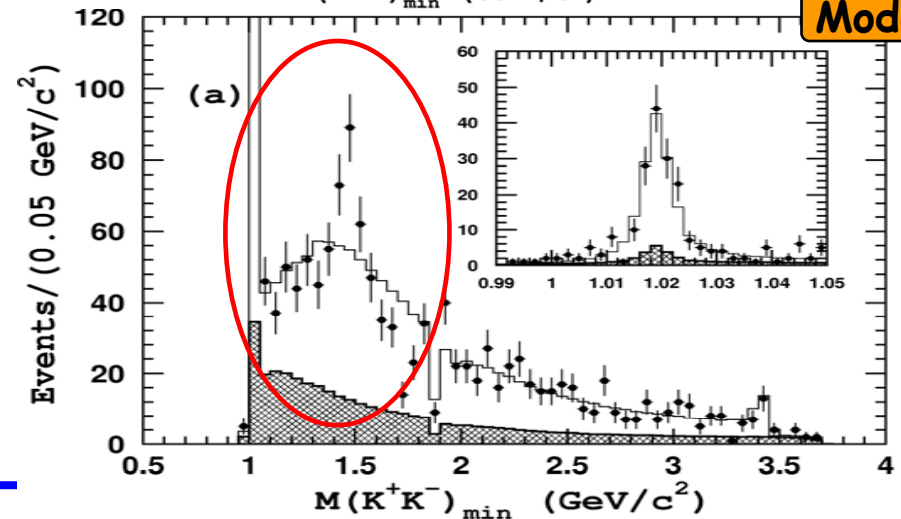


Two identical kaons in the final state -> amplitude must be symmetrized with respect to $K^+_1 \leftrightarrow K^+_2$

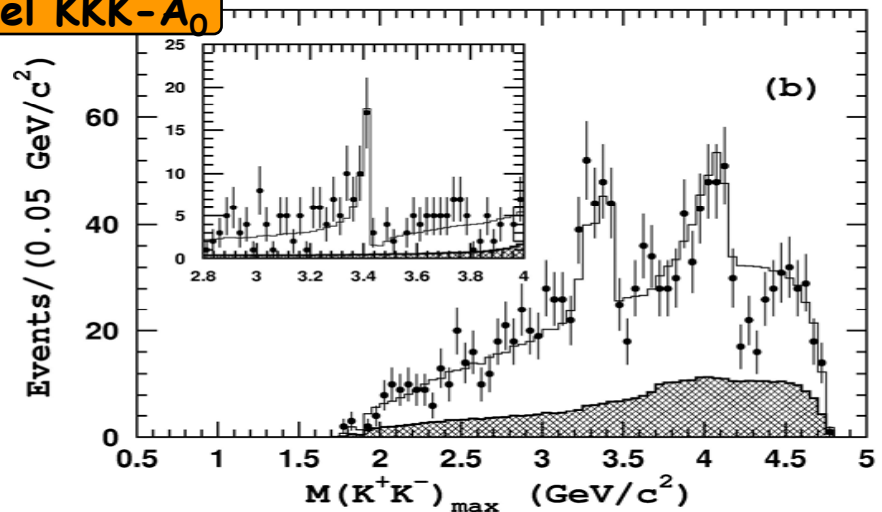
Model KKK- A_J :

$$S_{A_J}(\text{KKK}) = A_1(\phi(1020)) + A_J(f_X(1500)) + A_0(\chi_{c0})$$

fit the data with different spin assumptions

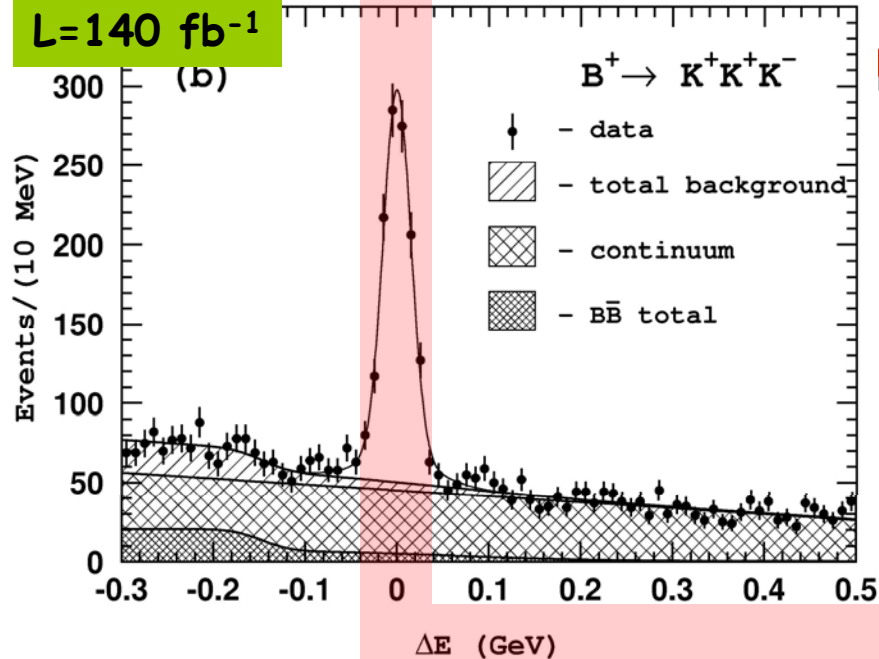


Model KKK- A_0



$B^+ \rightarrow K^+ K^+ K^-$: Three-body Signal

$L=140 \text{ fb}^{-1}$



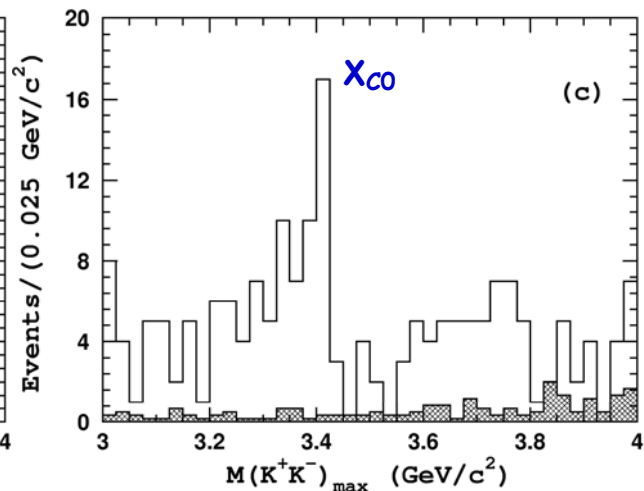
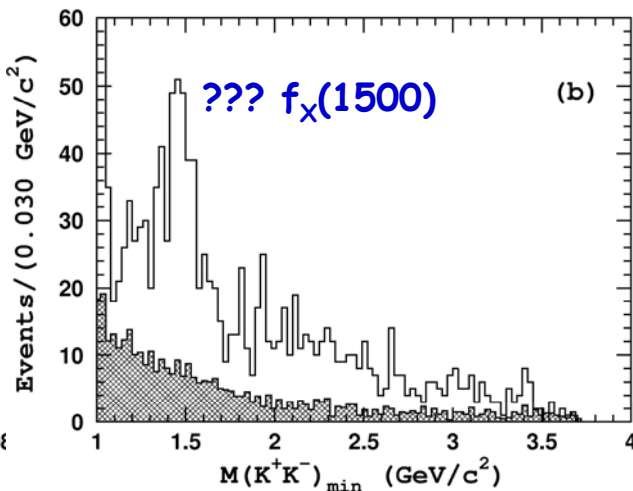
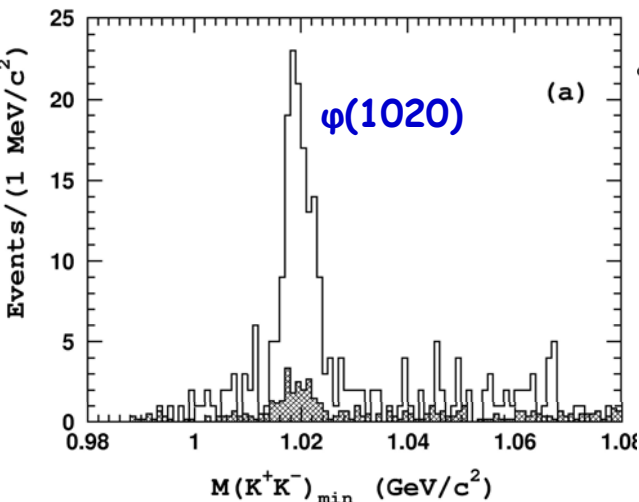
Vetoed B backgrounds:

- $\times B^+ \rightarrow D^0 K^+; D^0 \rightarrow K^+ \pi^-$: $\pm 15 \text{ MeV}$
- $\times B^+ \rightarrow D^0 h^+; D^0 \rightarrow K^+ K^-$: $\pm 15 \text{ MeV}$
- $\times B^+ \rightarrow J/\psi K^+; J/\psi \rightarrow K^+ K^-$: $\pm 15 \text{ MeV}$

$B^+ \rightarrow K^+ K^+ K^-$ charmless signal yield:

$N_{\text{signal}} = 1089 \pm 41$

$S/B \sim 2.5$

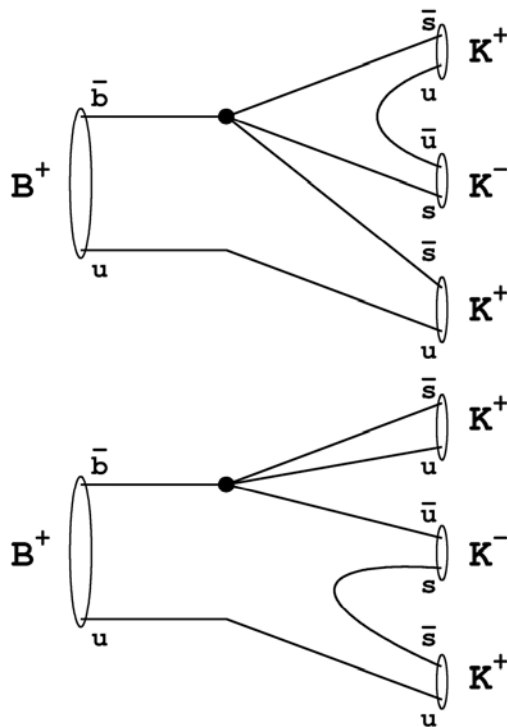


$B^0 \rightarrow K^+ K^- K_S$: $b \rightarrow u$ Contribution

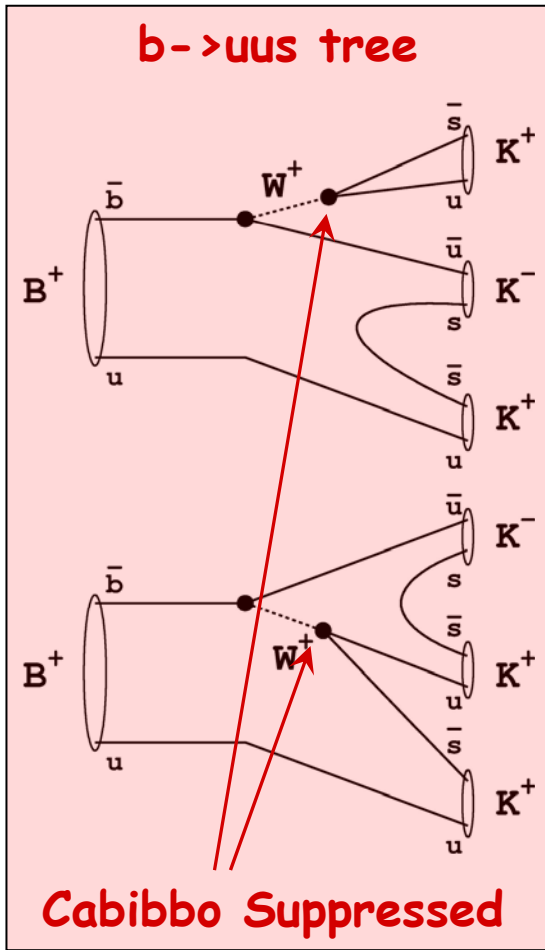
$B^+ \rightarrow K^+ K^+ K^-$

$B^+ \rightarrow K^+ K^- \pi^+$

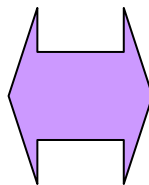
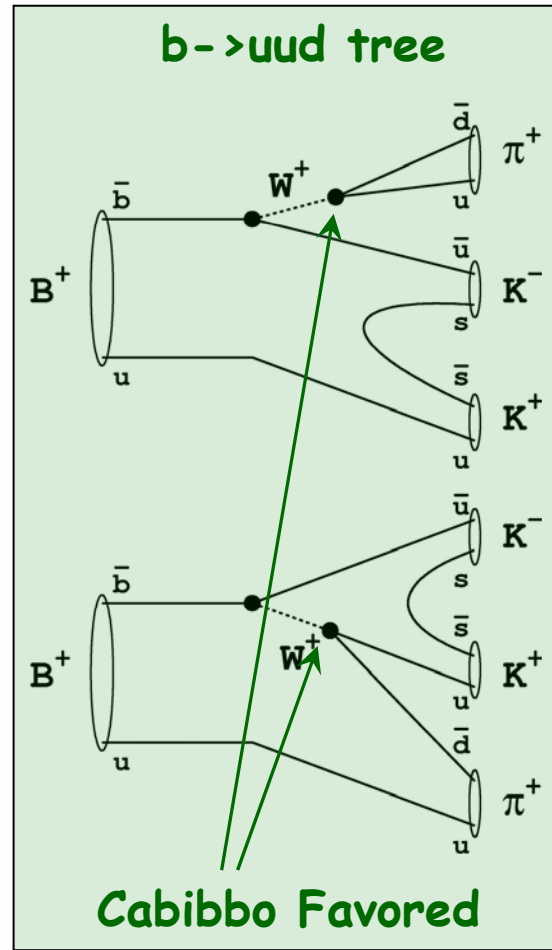
$b \rightarrow sqq$ penguin



$b \rightarrow uus$ tree



$b \rightarrow uud$ tree



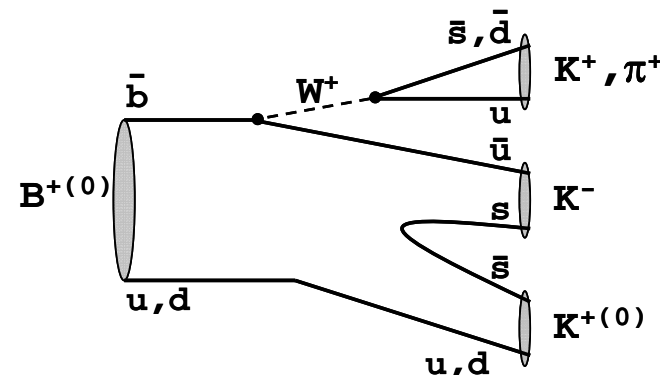
$B \rightarrow KK\pi$ provides a sensitive probe for the $b \rightarrow u$ contribution in $B \rightarrow KKK$



$B^0 \rightarrow K^+ K^- K_S$: $b \rightarrow u$ Contribution-cont'd

We can estimate the $b \rightarrow u$ contribution as:

$$\frac{|\mathcal{A}_{b \rightarrow u}^{KKK}|^2}{|\mathcal{A}_{\text{total}}^{KKK}|^2} \simeq \frac{\mathcal{B}(B^+ \rightarrow K^+ K^- \pi^+)}{\mathcal{B}(B^+ \rightarrow K^+ K^- K^+)} \times \tan^2 \theta_C \left(\frac{f_K}{f_\pi} \right)^2$$



With current experimental results:

- < 3% in BF
- < 15% in amplitude

BaBar: hep-ex/0407013

$b \rightarrow u$ tree contribution would probably result in quasi-two-body final states with a resonant state in $K_S K^-$ system (such as $a_0 \pi / K$) while $b \rightarrow s$ (as observed) proceed via resonances in $K^+ K^-$ system \rightarrow the interference term might be suppressed and $b \rightarrow u$ contribution is significantly smaller than 15%

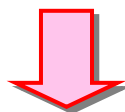
More analysis (experimental & theoretical) is required

Mode	Yield	ϵ (%)	$\prod B_i$ (%)	Signif.	$\mathcal{B}(10^{-6})$	UL(10^{-6})
$a_0^-(\eta_{\gamma\gamma})\pi^+$	18_{-10}^{+11}	18.8	39.4	1.3	$2.3_{-1.5}^{+1.7} \pm 0.9$	
$a_0^-(\eta_{3\pi})\pi^+$	15_{-8}^{+9}	15.5	22.6	1.6	$3.9_{-2.5}^{+2.9} \pm 1.0$	
$a_0^- \pi^+$				2.0	$2.8_{-1.3}^{+1.5} \pm 0.7$	< 5.1
$a_0^-(\eta_{\gamma\gamma})K^+$	2_{-4}^{+6}	17.9	39.4	0.1	$0.0_{-0.6}^{+0.9} \pm 0.3$	
$a_0^-(\eta_{3\pi})K^+$	13_{-6}^{+8}	14.9	22.6	1.1	$3.1_{-2.1}^{+2.5} \pm 1.9$	
$a_0^- K^+$				0.4	$0.4_{-0.8}^{+1.0} \pm 0.2$	< 2.1
$a_0^-(\eta_{\gamma\gamma})\bar{K}^0$	-12_{-6}^{+8}	21.4	13.5	0.0	$-3.7_{-2.3}^{+2.9} \pm 0.9$	
$a_0^-(\eta_{3\pi})\bar{K}^0$	0_{-5}^{+7}	15.8	7.9	0.5	$2.7_{-4.4}^{+6.1} \pm 1.9$	
$a_0^- \bar{K}^0$				0.6	$-1.5_{-1.8}^{+2.4} \pm 0.8$	< 3.9
$a_0^0(\eta_{\gamma\gamma})\pi^+$	17_{-9}^{+11}	12.8	39.4	1.4	$3.1_{-2.0}^{+2.4} \pm 1.2$	
$a_0^0(\eta_{3\pi})\pi^+$	1_{-6}^{+8}	9.5	22.6	0.3	$1.2_{-3.2}^{+3.9} \pm 1.7$	
$a_0^0 \pi^+$				1.4	$2.6_{-1.7}^{+2.0} \pm 1.0$	< 5.8
$a_0^0(\eta_{\gamma\gamma})K^+$	0_{-3}^{+5}	12.4	39.4	0.3	$0.3_{-0.6}^{+1.1} \pm 0.4$	
$a_0^0(\eta_{3\pi})K^+$	6_{-5}^{+7}	9.1	22.6	0.5	$1.9_{-2.9}^{+3.8} \pm 2.5$	
$a_0^0 K^+$				0.4	$0.4_{-0.7}^{+1.1} \pm 0.3$	< 2.5
$a_0^0(\eta_{\gamma\gamma})K^0$	0_{-5}^{+6}	15.0	13.3	0.5	$1.4_{-2.4}^{+3.5} \pm 1.2$	
$a_0^0(\eta_{3\pi})K^0$	4_{-4}^{+5}	9.7	7.8	1.2	$6.6_{-5.4}^{+7.8} \pm 2.8$	
$a_0^0 K^0$				1.0	$2.8_{-2.4}^{+3.1} \pm 1.1$	< 7.8

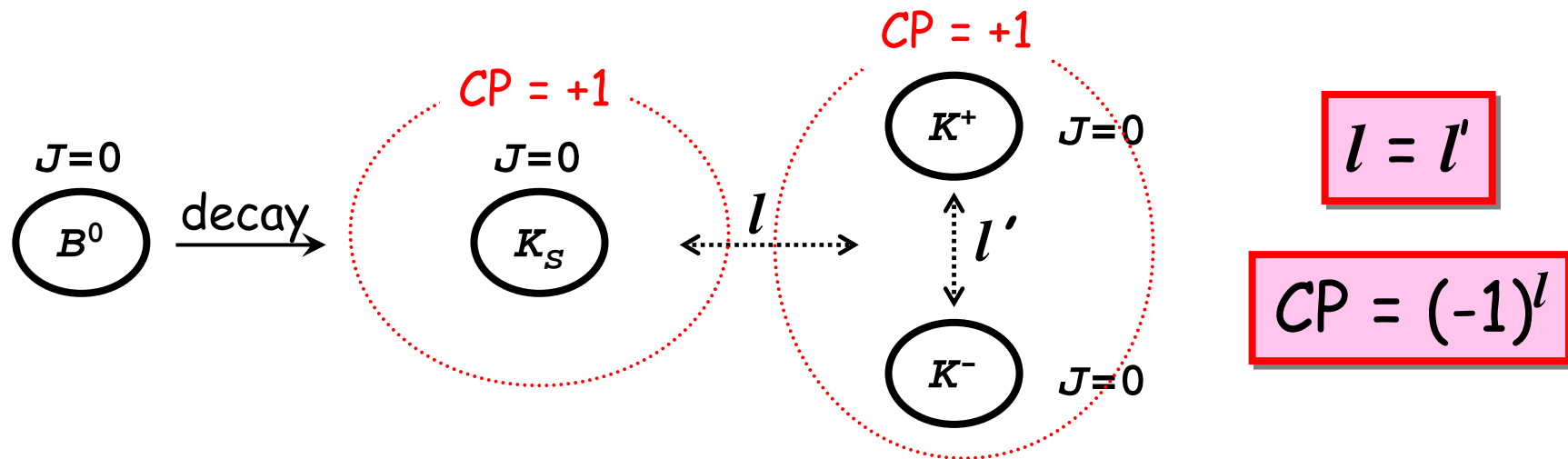
See also: Grossman, Ligeti, Nir, Quinn Phys. Rev. D68

$B^0 \rightarrow K^+ K^- K_S$: CP Mixture

Since $B^0 \rightarrow K^+ K^- K_S$ is 3-body decay,
the final state is a mixture of $CP = \pm 1$



$CP = \pm 1$ fraction is equal to that of $\lambda = \text{even/odd}$



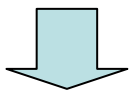


$B^0 \rightarrow K^+ K^- K_S$: CP Mixture - cont'd

Assuming isospin symmetry

($b \rightarrow s$ conserves isospin; $b \rightarrow u$ is small)

$$\mathcal{B}(B^+ \rightarrow K^+ K^0 \bar{K}^0) = \mathcal{B}(B^0 \rightarrow K^0 K^+ K^-) \times \frac{\tau_{B^+}}{\tau_{B^0}}$$



l -even fraction in $|K^+ K^- \rangle$ is equal to that in $|K^0 \bar{K}^0 \rangle$ system

$$|K^0 \bar{K}^0 \rangle = \frac{\alpha}{\sqrt{2}} (|K_S K_S \rangle + |K_L K_L \rangle) + \beta |K_S K_L \rangle$$

CP = +1 $l = \text{even}$ $l = \text{odd}$

$$|K^+ K^0 \bar{K}^0 \rangle = \frac{\alpha}{\sqrt{2}} (|K^+ K_S K_S \rangle + |K^+ K_L K_L \rangle) + \beta |K^+ K_S K_L \rangle$$

First measurement with 78 fb⁻¹:

$$\begin{aligned} \alpha^2 &= 2 \frac{\mathcal{B}(B^+ \rightarrow K^+ K_S K_S)}{\mathcal{B}(B^0 \rightarrow K^0 K^+ K^-)} \times \frac{\tau_{B^0}}{\tau_{B^+}} \\ &= \frac{\mathcal{B}(B^+ \rightarrow K^+ K_S K_S)}{\mathcal{B}(B^0 \rightarrow K_S K^+ K^-)} \times \frac{\tau_{B^0}}{\tau_{B^+}} \\ &= \underline{1.03 \pm 0.15(\text{stat}) \pm 0.05(\text{syst})} \end{aligned}$$

The most recent update with 353 fb⁻¹:

$$f_{\text{CP-even}} = 0.93 \pm 0.09 \pm 0.05$$

$B^0 \rightarrow \varphi K_S$

$$\sin 2\beta_{\text{eff}} = 0.44 \pm 0.27 \pm 0.05$$

$B^0 \rightarrow K^+ K^- K_S$ (non φ)

$$\sin 2\beta_{\text{eff}} = 0.60 \pm 0.18 \pm 0.04^{+0.19}_{-0.12}$$

See also: M. Gronau, J. Rosner, Phys. Lett. B564