CKM 2006

Extracting CKM γ phase from $B^{\pm} \to K^{\pm} \pi^{+} \pi^{-}$ and $B^{0}, \overline{B}^{0} \to K_{S} \pi^{+} \pi^{-}$

hep-ph/0608268

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1) Feasibility of $|A|^2 + |A'|^2$ fit

2) Extracting γ

2 vs 3 body amplitudes

$B^0 \rightarrow K^+ \pi^-$ $\quad a\, e^{i\,\delta}$

$B^0 \rightarrow K^*(890)^+ \pi^-$

$B^0 \rightarrow K^*(1430)^+ \pi^-$

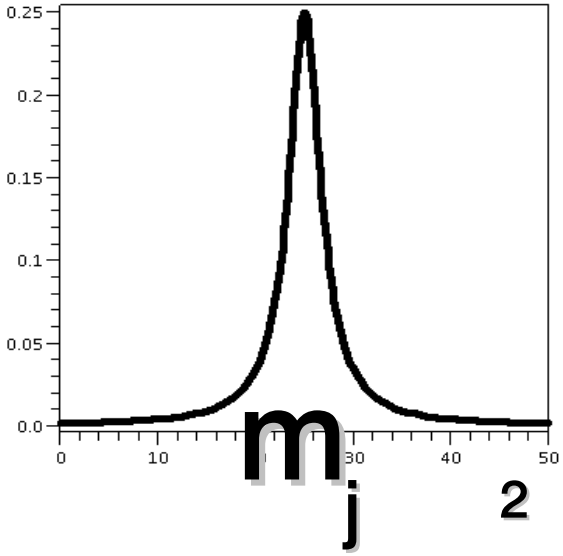
$B^0 \rightarrow K_s\, \rho(770)^0$

$B^0 \rightarrow K_s\, f_0(980)$

$B^0 \rightarrow K_s\, \chi_{c0}$

$K_s \pi^+ \pi^-$

$a_j\, e^{i\,\delta_j} \times BW_j(E)$



Analysis Formalism*

$$A(x) = \sum_{\text{all interm states } i} a_i \, e^{\,i\,\delta_i} \; S_i^J(x)$$

a_i → magnitude , δ_i → strong + weak phases

$x = \{s_{l\,m}, s_{m\,n}\}$ → Dalitz variables

$$s_{l\,m} = (p_l^{\mu} + p_m{}^{\mu})^2 = m_l^2 + m_m{}^2 + 2\,E_l E_m - 2\,p_l\,p_m \cos\theta_{l\,m}$$

S_i^J →form factors x BW x angular term (spin J)

*AKA Isobaric Model

Toy Model

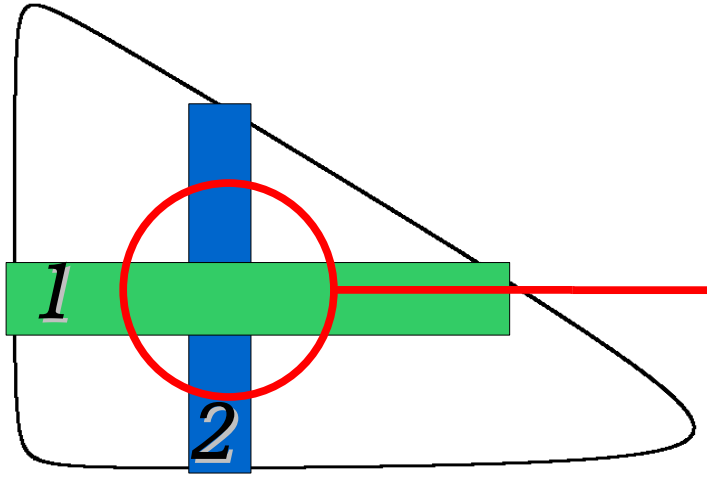
$$A = BW_1 + r\, e^{i\phi}\, BW_2$$

- Phase space (Dalitz Plot) points distribution $\propto$

$$|A|^2 = |BW_1|^2 + r^2 |BW_2|^2 +$$

$$+ 2r \cos\phi \, \mathrm{Re}\, BW_1 BW_2^* + 2r \sin\phi \, \mathrm{Im}\, BW_1 BW_2^*$$

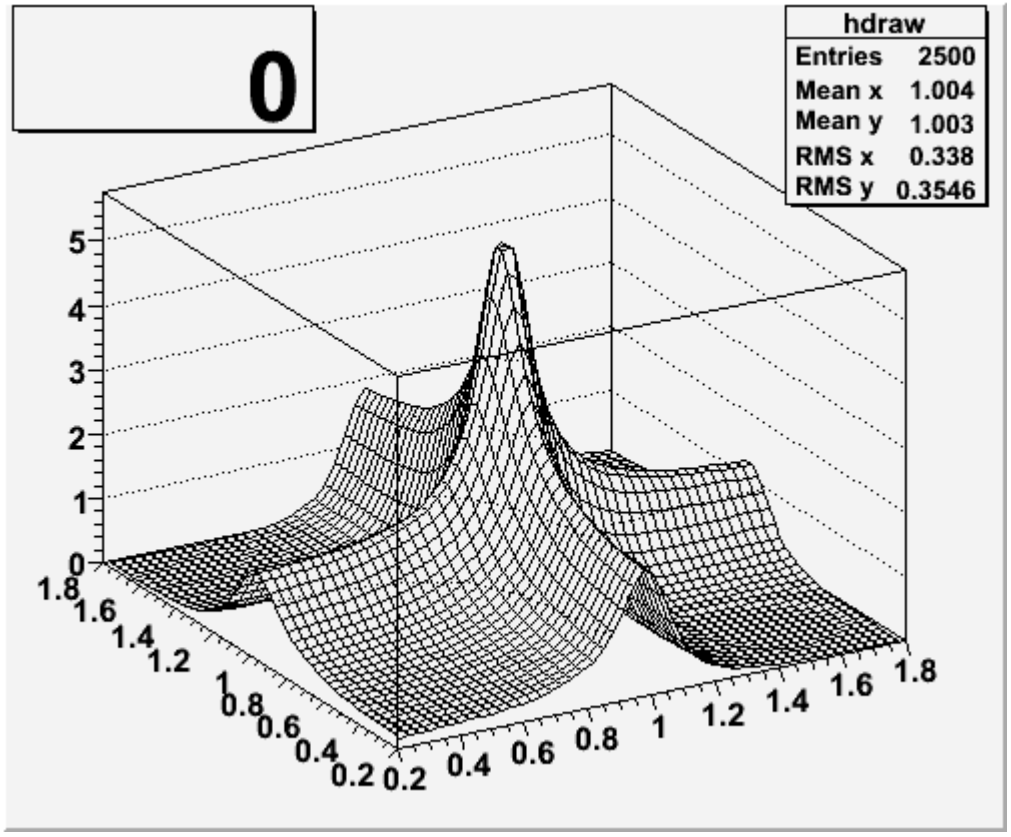
DP

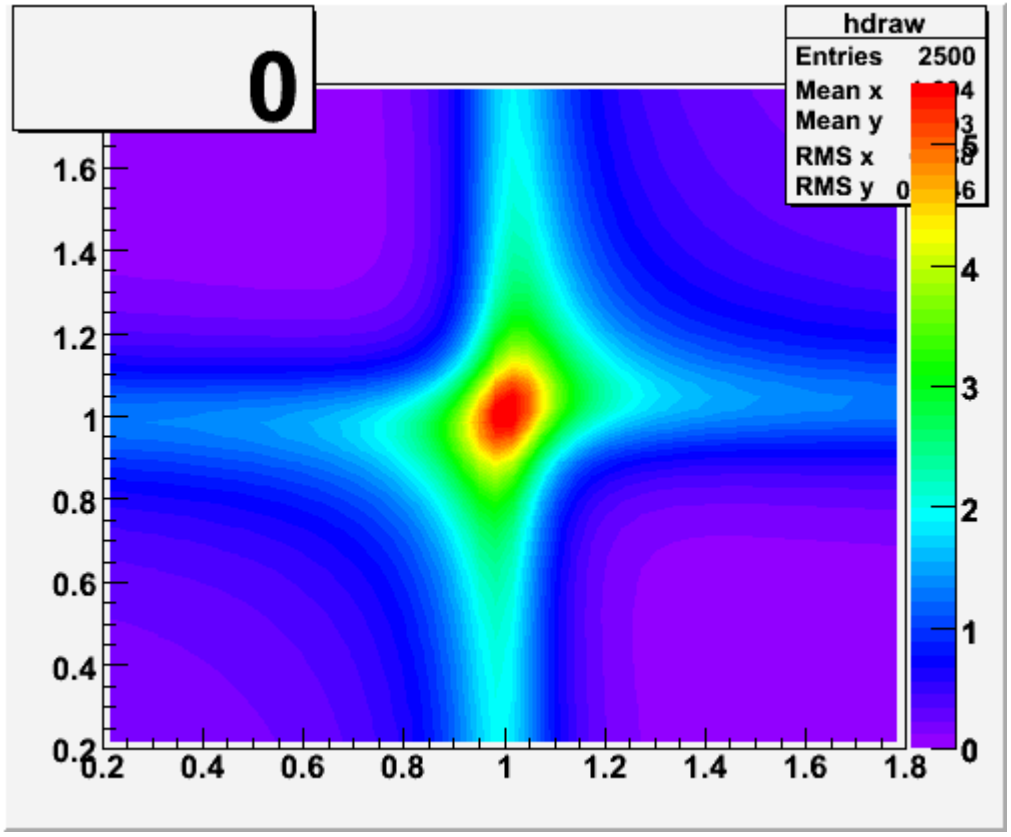


interference

Toy Model

$r=1,\ \phi \rightarrow 0 \,..\, 2\pi$

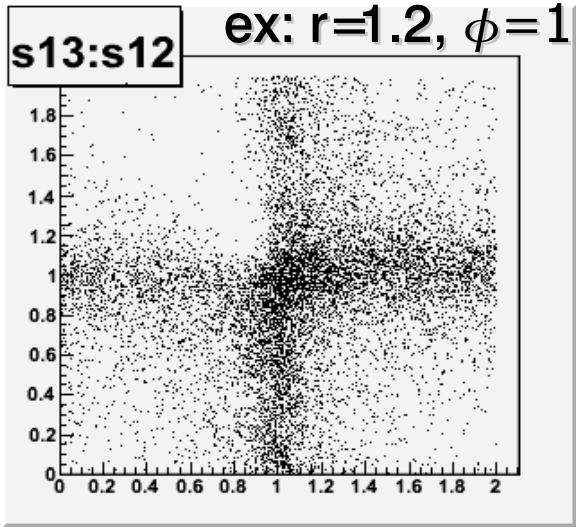




- surface univocally defined by the set of parameters {r, ϕ }

Fitting $|A|^2$

- given a set of points modelled by A :



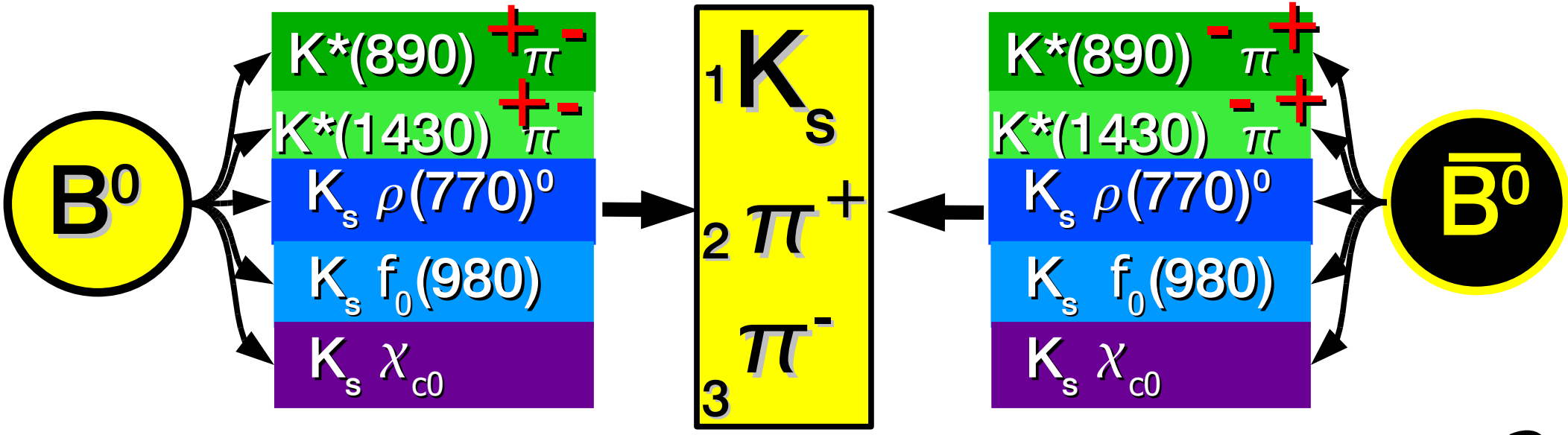
$$PDF = \frac{1}{M} |BW_1 + r\, e^{i\phi} BW_2|^2$$

$$M = \int ds_{13} ds_{23} |A|^2$$

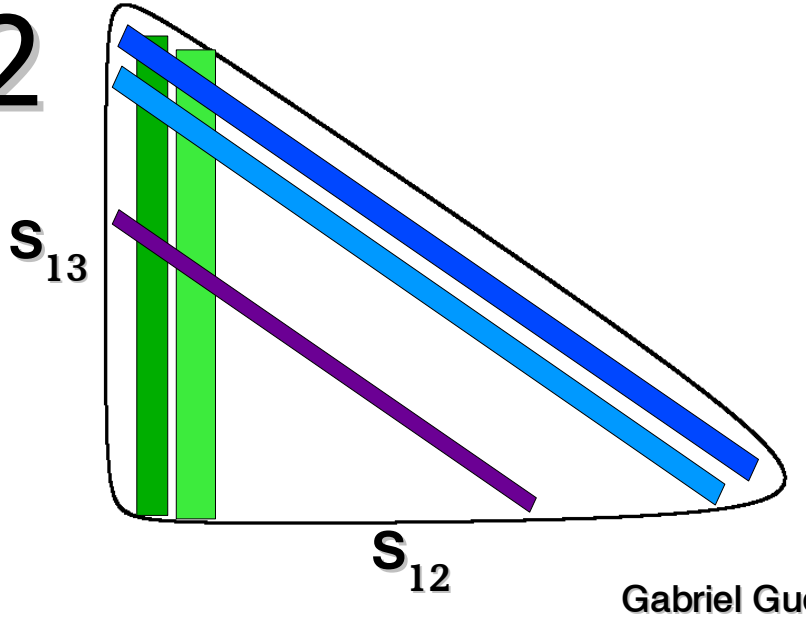
Likelihood Fit : $r, \phi \rightarrow$ extracted

- Fit of Isobaric amplitudes in 3 body decays has been successfully used in many analysis from BaBar, Belle, Focus, etc
- ex: σ(500) discovery, Fermilab E791, Phys Rev Lett 86, 770

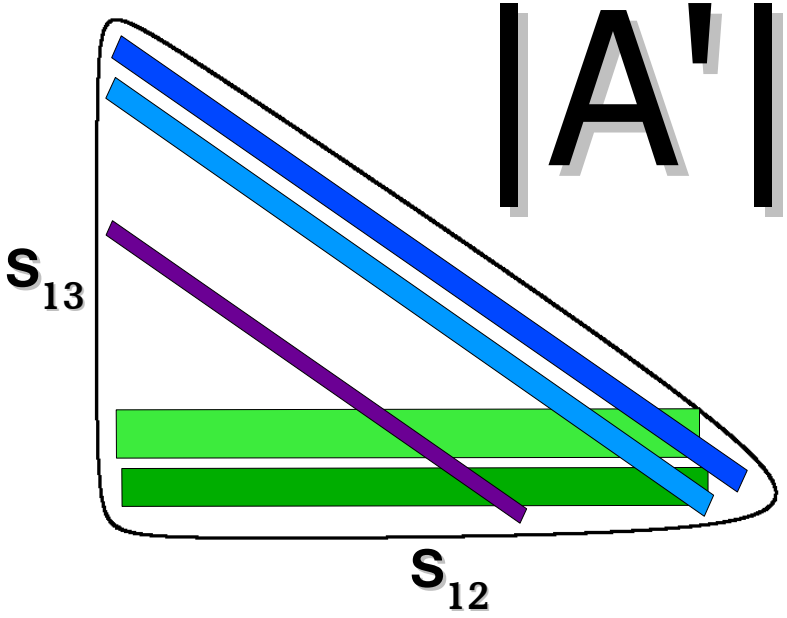
$B^0 \to K_s\ \pi^+\ \pi^-$ resonances

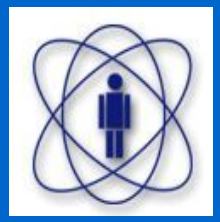


$|A|^2$



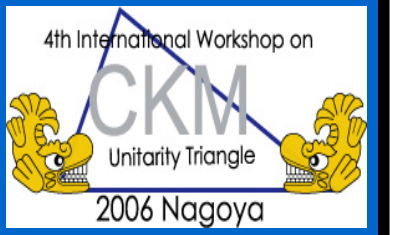
$|A'|^2$





1) Feasibility of $|A|^2 + |A'|^2$ fit

$B^0 \to K_s\ \pi^+\ \pi^-$ amplitudes



time independent

$$B^0 \to K_s\ \pi^+\ \pi^- : A, \quad \bar{B}^0 \to K_s\ \pi^-\ \pi^+ : A'$$

time dependence $\to$ B^0/ $\bar{B}^0$ mixing

Prob. (t=0) B^0 $\to$ (t=t) $K_s\ \pi^+\ \pi^-$

$$M(t) = e^{-(\Gamma/2 - iM)t}[A \cos(\Delta m\, t/2) - i\, q/p\, A' \operatorname{sen}(\Delta m\, t/2)]$$

Prob. (t=0) $\bar{B}^0$ $\to$ (t=t) $K_s\ \pi^+\ \pi^-$

$$M'(t) = e^{-(\Gamma/2 - iM)t}[A' \cos(\Delta m\, t/2) - i\, p/q\, A \operatorname{sen}(\Delta m\, t/2)]$$

Good news

if $|p/q|=1$

G. Burdman *et al*, Phys Rev 45, 187

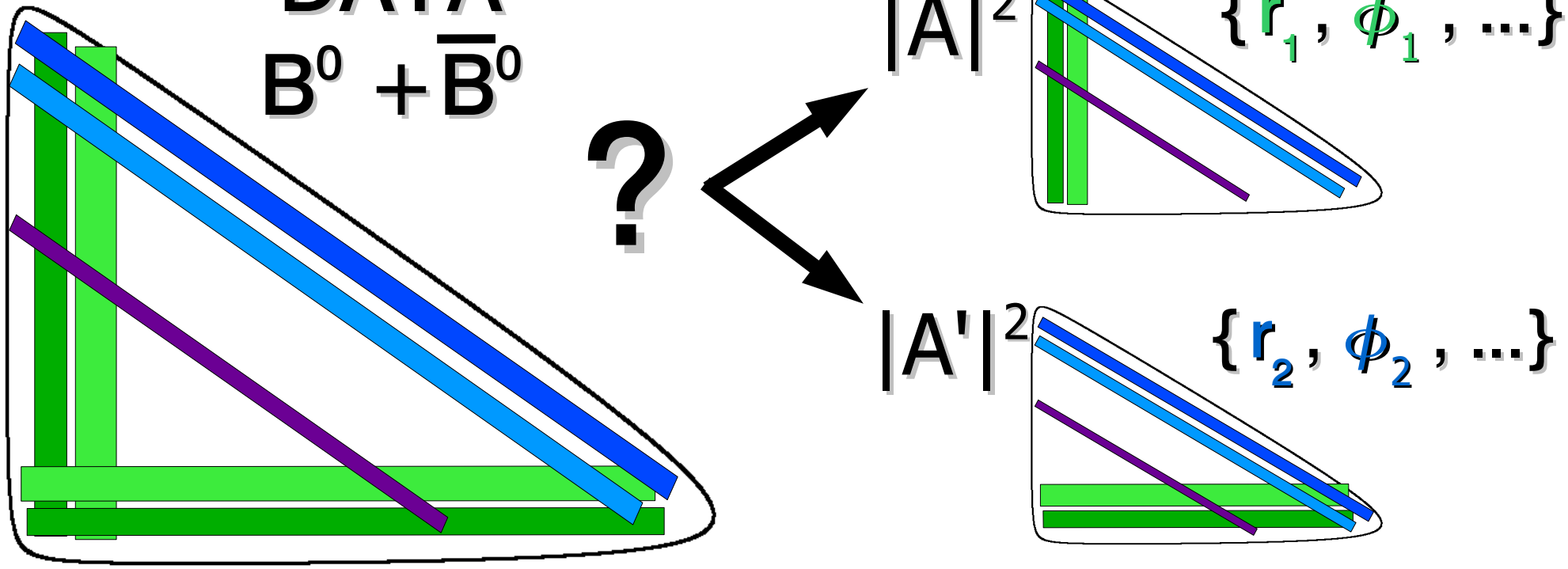
$$\text{Prob.}\ (\ B^0 \text{ or } \bar{B}^0 \to K_s\, \pi^+ \pi^-\ )\ \propto$$

$$|M(t)|^2+|M'(t)|^2 = e^{-\Gamma t}\ (\ |A|^2+|A'|^2\ )$$

mixing effects cancels out!

time independent amp → NO TAGGING

In practice



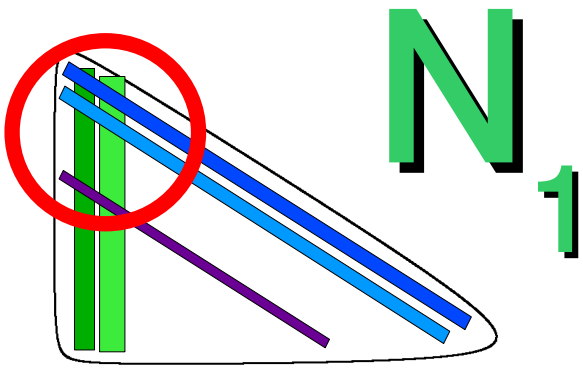
Is possible to extract parameters separately from A and A' in a joint sample?

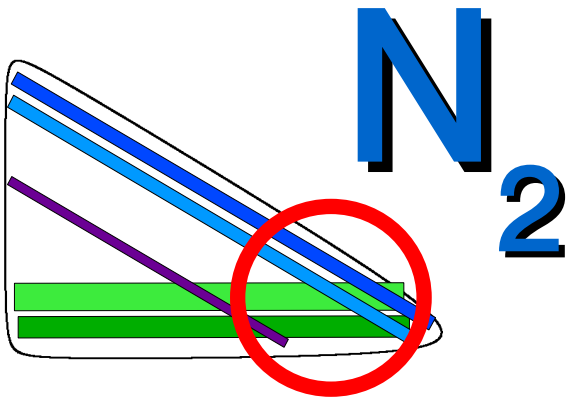
In practice

Yes!

reason: different, non-overlap interferences

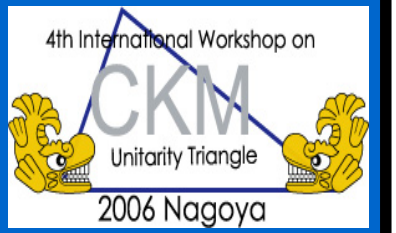
→ different surfaces





also possible to extract the number of events N_1, N_2 of each set, measuring direct CPV

Fit $|A|^2 + |A'|^2$ how to



PDF =

$$\frac{1}{M_1 + M_2}\{ |S_1 + r_1 e^{i\phi 1} S_2 + \ldots|^2 + |r S_1 + r_2 e^{i\phi 2} S_3 + \ldots|^2 \}$$

$$M_1 = \int ds_{13} ds_{23} \, |A|^2, \qquad M_2 = \int ds_{13} ds_{23} \, |A'|^2$$

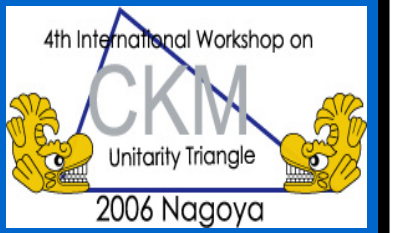
Likelihood Fit

r_1, ϕ_1, r_2, ϕ_2, r, ... → *univocally extracted*

r → scale, fundamental for measuring N_1 and N_2

N_1, N_2 → $N_1 / N_2 = M_1 / M_2$

Feasibility test



- We generated and fitted 100 samples of **100K** $B^0 + \bar{B}^0$ events each, using the method discussed previously

- The input resonances magnitudes and phases were inspired in BaBar results **hep-ex:0507004**

- The errors that will be shown are **RMS** from the 100 sample distributions

Feasibility test

decay	*B0 / $\overline{B0}$*	*input*	*fitted*
χ_{c0} Ks	a0 / b0	0.3 / 0.3	Fixed / (0.3 ± 0.03)
	δ0 / ϕ0	3.78 / 3.78	fixed / fixed
K*(890) π	a1 / b1	1.17 / 1.3	(1.17 ± 0.06) / (1.30 ± 0.01)
	δ1 / ϕ1	0.4 / 5.98	(0.41 ± 0.08) / (5.99 ± 0.07)
K*(1430) π	a2 / b2	2.45 / 2.72	(2.45 ± 0.11) / (2.72 ± 0.13)
	δ2 / ϕ2	0.375 / 6.0	(0.39 ± 0.08) / (6.00 ± 0.06)
ρ^0 Ks	a3 / b3	0.6 / 0.6	(0.60 ± 0.04) / (0.60 ± 0.04)
	δ3 / ϕ3	1.2 / 1.2	(1.22 ± 0.09) / (1.20 ± 0.07)
f_0 Ks	a4 / b4	1.03 / 1.03	(1.02 ± 0.06) / (1.04 ± 0.05)
	δ4 / ϕ4	2.3 / 2.3	(2.30 ± 0.07) / (2.30 ± 0.08)

- $N_1 / N_2 = 0.84 \pm 0.12$
- small errors

Belle results

- Recently, Belle released the final result for the untagged amplitude analysis of $B^0 \to K_s \pi^+ \pi^-$:

  hep-ex: 0610081

- Against the theoretical previsions, they assume no CP violation, i.e. $A(x,y) = A'(y,x)$
- They comment about measuring an average. This technical difficulty would be avoided if they made the joint fit as proposed here

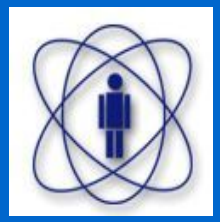
Extracting γ

Two step

- Analyse $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ to extract *penguin* amplitude and phase from K* resonances
- Analyse joint (not-tagged) B^0, $\overline{B}^0 \to K_s\pi^{+}\pi^{-}$, using the $B^{\pm}$ penguin to extract *tree* parameters and γ

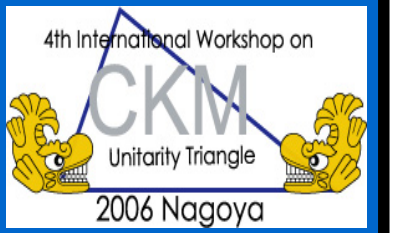
Depends on

- Possibility to measure independently B^0 and $\overline{B}^0$ parameters in the joint sample (we can do it as shown before)
- 3 widely accepted hypothesis (we can test them)



2) Extracting γ

$B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ contributions



$B^{\pm} \to K^{*}(890)^{0}\,\pi^{\pm}$: $a_p\, e^{i\,\delta p}$

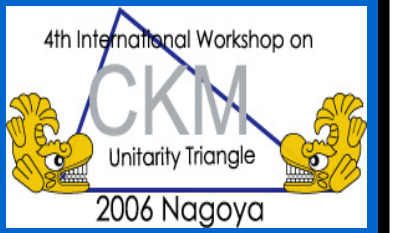
$B^{\pm} \to K^{*}(1430)^{0}\,\pi^{\pm}$: $a'_p\, e^{i\,\delta p'}$

$B^{\pm} \to K^{\pm}\,\rho,\ f_0$: A, B

$B^{\pm} \to K^{\pm}\,\chi_{c0}$: $a_t\, e^{i\,\delta t}$

1st hypothesis : K* from $B^{\pm}$ has only penguin contribution

$B^0 \to K_s \pi^+ \pi^-$ contributions



$B^0 \to K^*(890)^+ \pi^- : a_t \, e^{i(\delta t + \gamma)} + a_p \, e^{i \delta p}$

$B^0 \to K^*(1430)^+ \pi^- : a'_t \, e^{i(\delta t' + \gamma)} + a'_p \, e^{i \delta p'}$

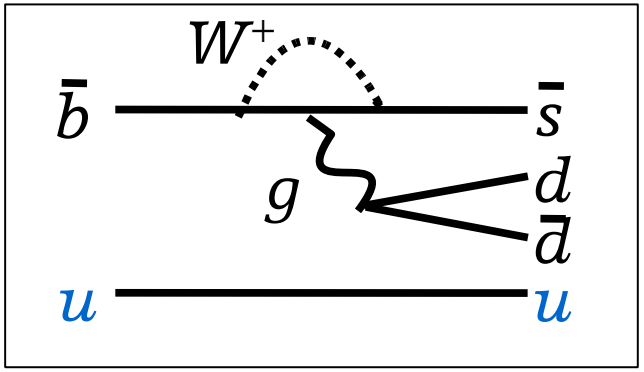
$B^0 \to K_s \, \rho, \, f_0 : A', B'$

$B^0 \to K_s \, \chi_{c0} : a_t \, e^{i \delta t}$

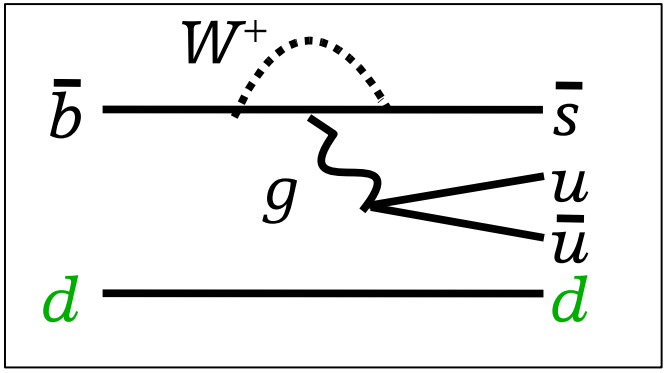
2nd hypothesis : penguin from B^0 K^* is the same of $B^{\pm}$

2nd hypothesis

- penguin from $B^{\pm}$ K^* is the same of B^0



$B^{\pm}$

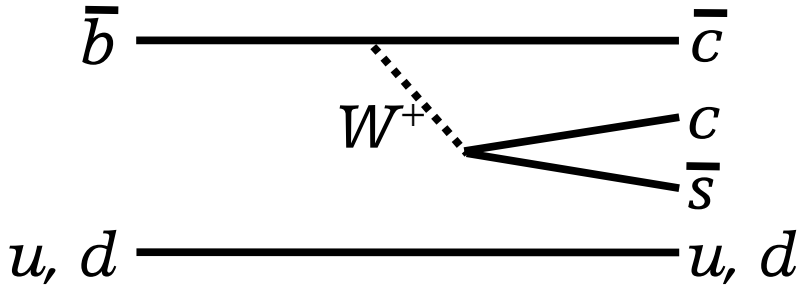


B^0

- Flavour SU(3) symmetry

χ_{c0}

- amplitudes and phases are always measured relative to a fixed one
- to compare amplitudes and phases from B^0 and $B^{\pm}$, we must measure them relative to the same anchor
- we choose the χ_{c0}, because we expect the same contribution to $B^{\pm}$ and B^0, based on its graphic



3rd hypothesis : χ_{c0} is the same for $B^{\pm}$ and B^0

Idea

$B^{\pm} \to K^*(890)^0 \pi^{\pm}$: $a_p e^{i\delta p}$

$B^0 \to K^*(890)^+ \pi^-$: $a_p e^{i\delta p} + a_t e^{i\varphi}$

$\overline{B}^0 \to K^*(890)^- \pi^+$: $a_p e^{i\delta p} + a_t e^{i\theta}$

$\varphi = \delta_t + \gamma$

$\theta = \delta_t - \gamma$

knowing penguin → can extract a_t, φ , a_t and θ

for model consistency, we must measure $a_t = a_t$

γ

The ability of measuring γ is related to the value of γ and the ratio* $r = a_t / a_p$ → We can measure it

In our Monte Carlo 100x100k test samples we input:

$$\gamma = 69^\circ \text{ , } r = 0.45$$

and measure:

$$\gamma = (\varphi - \theta) / 2 = 69^\circ \pm 5^\circ$$

* divergent theoretical previsions: Beneke,Neubert Nucl. Phys **B675**, 333(2003) and Buras *et al*, Phys. Rev.Lett **92** 101804 (2004)

Hypothesis

1st hypothesis : K* from $B^{\pm}$ has only penguin contribution

test in the charged fit, check if magnitude $(K^* \pi^+) =$ magnitude $(K^* \pi^-)$

BaBar confirmed this hypothesis : hep-ex/0507004

2nd hypothesis : penguin from $B^{\pm}$ K* is the same of B^0

test $a_t(B^0) = a_t(\overline{B^0})$

3rd hypothesis: χ_{c0} is the same *for* $B^{\pm}$ and B^0

test $\Gamma_{B^0 \to K_s \chi_{c0}} = \Gamma_{\overline{B^0} \to K_s \chi_{c0}} = \Gamma_{B^{\pm} \to K^{\pm} \chi_{c0}}$

Based on theoretical papers

- C. W. Chiang *et al,* Phys. Rev. D69, 034001 (2004)
- M. Beneke and M. Neubert, Nucl. Phys, B675, 333 (2003)

If one fails → **interesting anyway, unexpected physical effects !**

Conclusion

- Joint fits to non tagged B^0 $\bar{B}^0$ data samples can be made and this is a general result that be applied to other channels: $B^0 \rightarrow \pi^+\pi^-\pi^0$, $B_s \rightarrow D^0 K^+K^-$, $D^0 \rightarrow K_s \pi^+ \pi^-$ ...
- With 100K evts and all the hypothesis mentioned we measure γ within 5°
- We can measure direct CP asymmetry in $B^0, \bar{B}^0 \rightarrow K_s \pi^+ \pi^-$
- We can measure $r = a_t / a_p$ and $\delta_t - \delta_p$
- χ_{c0} is used as reference. If f_0 is dominated by ss component , it could be used as anchor and this method applied at current B factories statistics