Understanding $\eta^{(\prime)} K^{(*)}$ branching ratios

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Exp. results since PDG2006

- measurements of $B \to \eta' K^*$ from BaBar and strict upper bounds from Belle

  Babar : hep-ex/0607109

  $$Br(\eta' K^{*+}) = (4.9^{+1.9}_{-1.7} \pm 0.8) \cdot 10^{-6}$$
  $$Br(\eta' K^{*0}) = (3.8 \pm 1.1 \pm 0.5) \cdot 10^{-6}$$

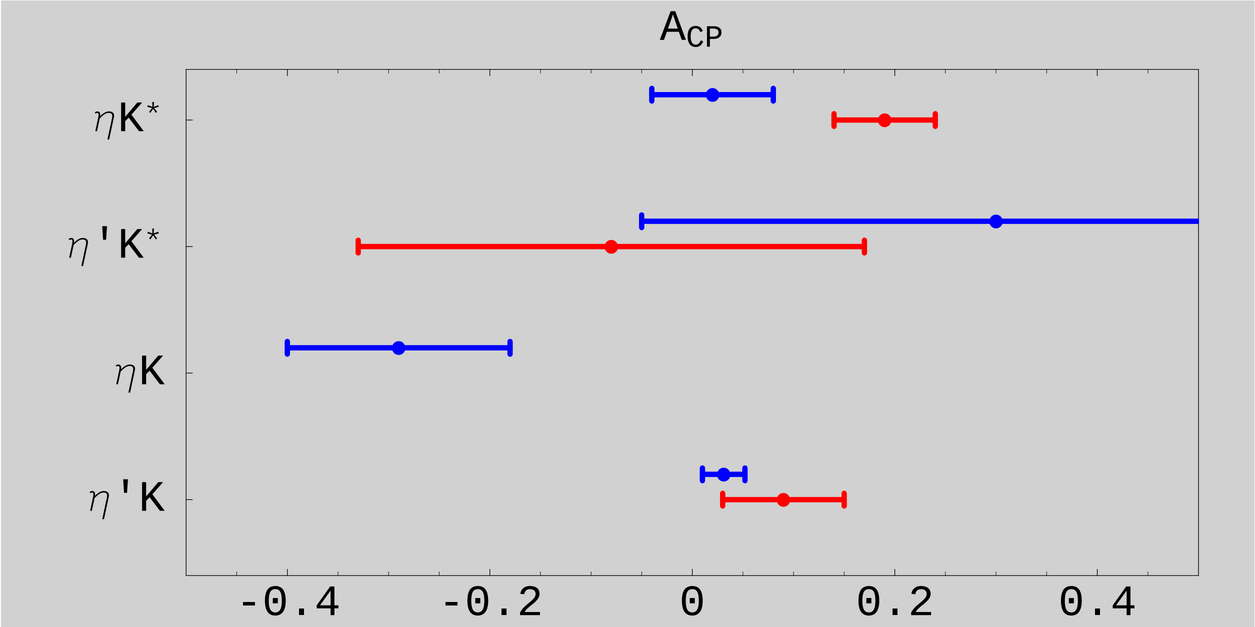
  Belle ($90\% CL$): J. Schuemann, talk at DPF 2006

  $$Br(\eta' K^{*+}) < 2.8 \cdot 10^{-6}$$
  $$Br(\eta' K^{*0}) < 2.6 \cdot 10^{-6}$$

- updates of many other modes
  - Belle: $\eta' K^+$, $\eta K^+, \eta' K^0$ hep-ex/0603001, hep-ex/0608033
  - BaBar and Belle: $\eta K^{*0/+}, \eta K^0$ hep-ex/0608005, hep-ex/0608034, hep-ex/0607063, hep-ex/0608033

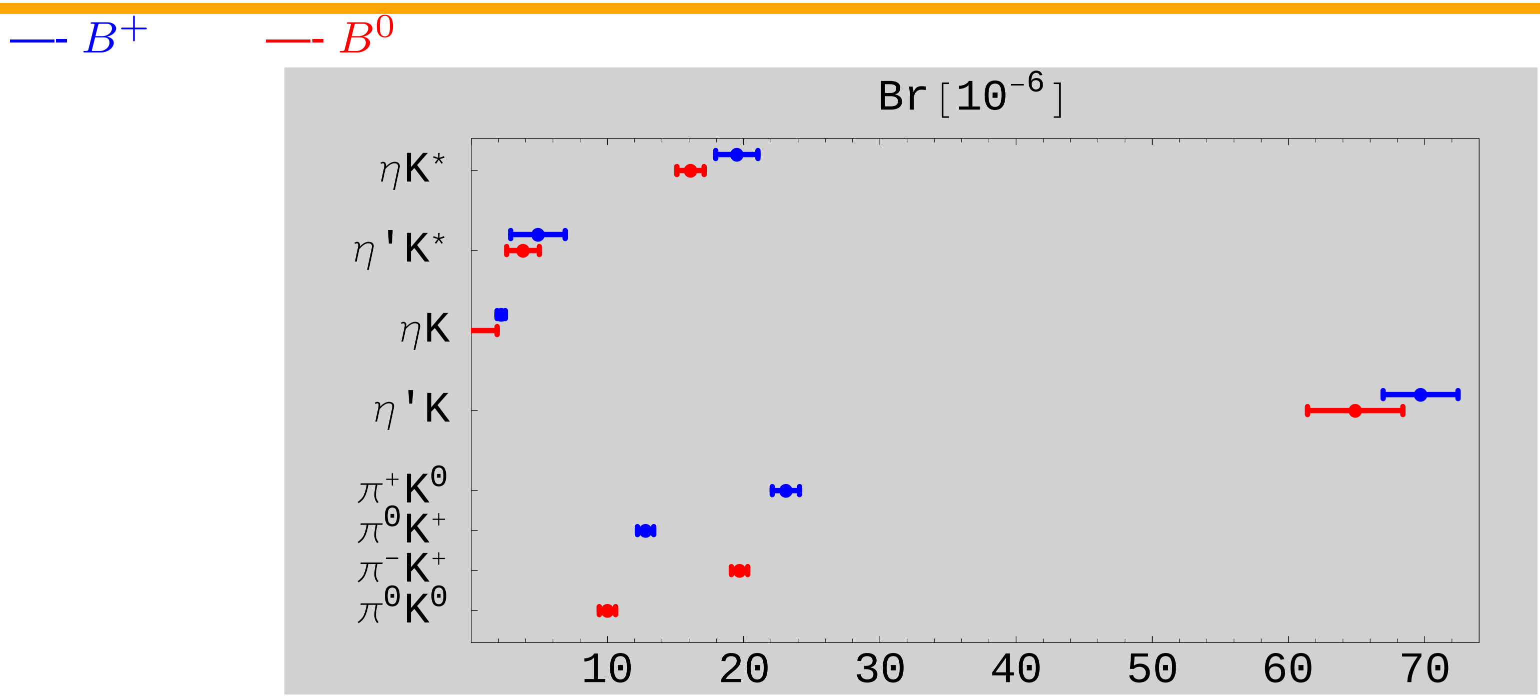
A_{CP}

—- B^+ —- B^0



- $A_{CP}(\eta K^+) = -0.29 \pm 0.11 \leftarrow 2.6\sigma$ away from zero
- $A_{CP}(\eta K^{*0}) = 0.19 \pm 0.05 \leftarrow 3.8\sigma$ away from zero (with both Belle and BaBar having new measurements)
- other consistent with zero

Questions



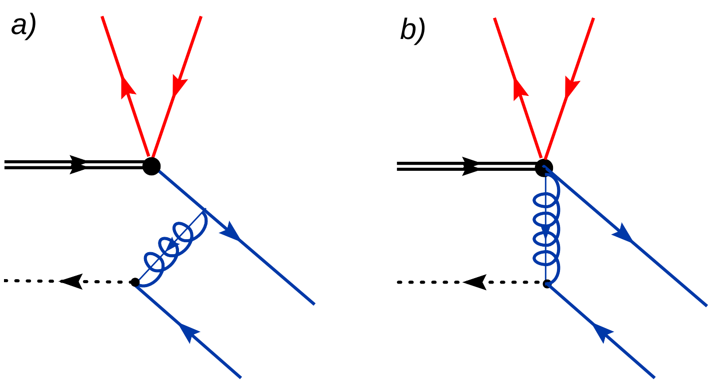
- why $\mathrm{Br}(\mathrm{B} \to \eta' \mathrm{K}) \gg \mathrm{Br}(\mathrm{B} \to \eta \mathrm{K})$?
- why in $B \to \eta^{(\prime)} K^*$ inverted hierarchy?
- why $\mathrm{Br}(\mathrm{B} \to \eta' \mathrm{K}) \gg \mathrm{Br}(\mathrm{B} \to \mathrm{K}\pi)$?

Factorization formula

- $1/m_b$ expansion $\Rightarrow$ (soft-collinear) effective field theory
- for start only nonisosinglet final states (such as $B \to \pi\pi$)
- at LO in $1/m_B$

Bauer, Pirjol, Rothstein, Stewart 2004
Beneke, Buchalla, Neubert, Sachrajda 1999

$$A(B \to M_1 M_2) \propto f_{M_1}\phi_{M_1}(u) \otimes T_{1J}(u,z) \otimes \zeta_J^{BM_2}(z)$$
$$+ f_{M_1}\phi_{M_1}(u) \otimes T_{1\zeta}(u)\zeta^{BM_2} + 1 \leftrightarrow 2 + \lambda_c^{(f)} A_{cc}^{M_1 M_2}$$



- a) introd. matrix el. ζ^{BM}
- b) function $\zeta_J^{BM}(z)$, calc. in $\alpha_S(\sqrt{\Lambda m_B})$ expansion

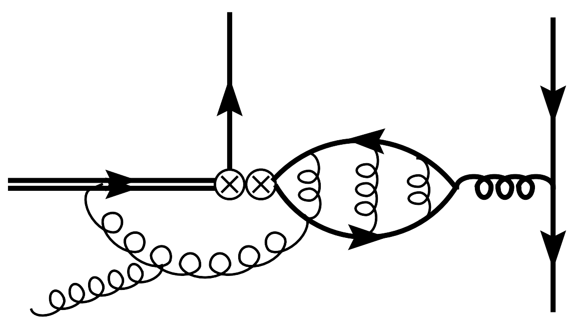
- LO in $\alpha_S(m_b)$: $f_+^{BP}(0) = \zeta^{BP} + \zeta_J^{BP}$, $\quad f_+^{BP}(0) + f_-^{BP}(0) = 2\zeta_J^{BP}$

Different theor. approaches

- at LO in $\alpha_S(m_b)$: $T_{1,2}$ constant, $T_{1J,2J}$ only funcs. of u
- this simplifies the factorization formula

$$A_{B\to M_1 M_2} \propto f_{M_1}\phi_{M_1}(u)\otimes T_{1J}(u)\zeta_J^{BM_2} + f_{M_1}T_{1\zeta}\zeta^{BM_2} + 1\leftrightarrow 2 + \lambda_c^{(f)}A_{cc}^{M_1M_2}$$

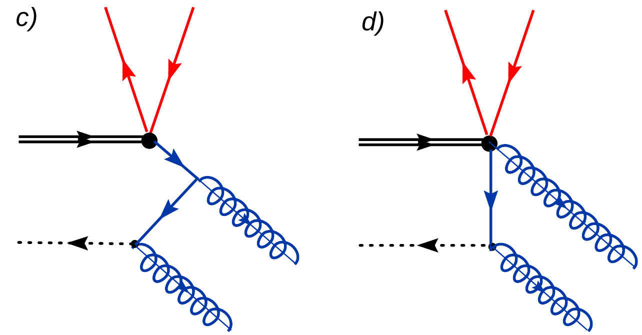
  - QCDFact.: ζ_J^{BM} calculated, ζ^{BM} from form factors
  - BPRS: ζ^{BM}, ζ_J^{BM} fit from $B\to PP$ data
  - pQCD: everything calculated
- charming penguins $A_{cc}^{M_1M_2}$
  - BBNS: perturbative
  - BPRS: nonperturbative



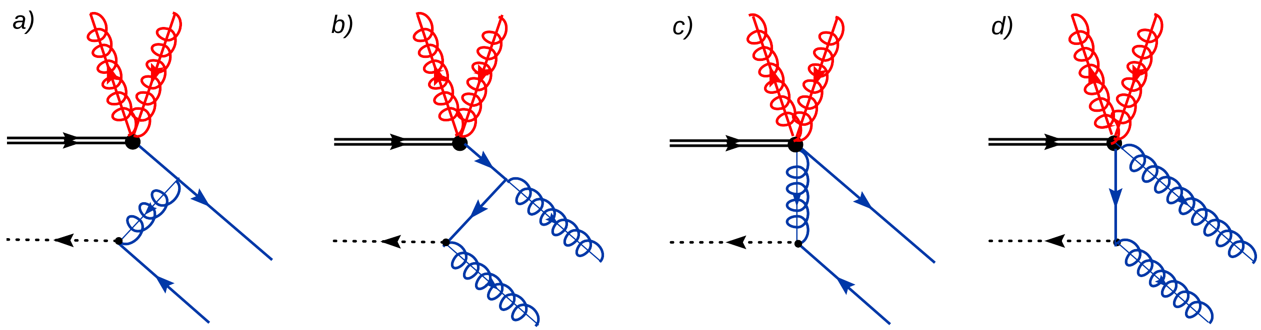
Isosinglet final states

Beneke, Neubert, 2002
Williamson, JZ 2006

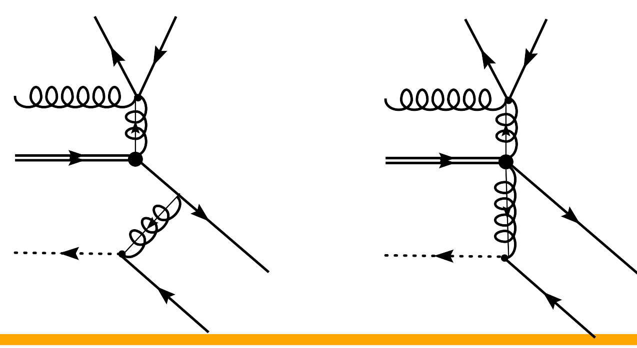
- additional operators in $\text{SCET}_{\text{I}} \to \text{SCET}_{\text{II}}$ matching, that contribute only for η, η'



- at $\alpha_S(m_b)$ also operators (at this order only from O_{8g})



- heavy quark–soft quark–soft gluon terms

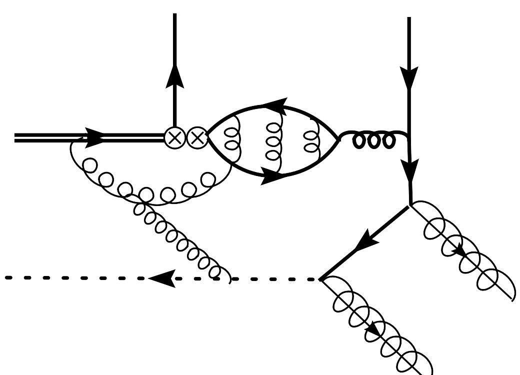


Factorization including isosinglets

- extended factorization formula

$$A(B \to M_1 M_2) \propto f_{M_1}\phi_{M_1}(u) \otimes T_{1J}(u,z) \otimes \zeta_J^{BM_2}(z)$$
$$+ f_{M_1}\phi_{M_1}(u) \otimes T_{1\zeta}(u)\zeta^{BM_2} + 1 \leftrightarrow 2 + \lambda_c^{(f)} A_{cc}^{M_1 M_2}$$
$$+ \delta_{M_1,\eta^{(\prime)}}\big[f^1_{M_1}\bar{\Phi}^g_{M_1}(u) \otimes [T^g_{1J}(u,z) \otimes \zeta_J^{BM_2}(z) + T^g_{1\zeta}(u)\zeta^{BM_2}]$$
$$+ (f_{M_1}\phi_{M_1}(u) \otimes \zeta_3^{BM_2}(u) + f^1_{M_1}\bar{\Phi}^g_{M_1}(u) \otimes \zeta_{3g}^{BM_2}(u))\big]$$

- $\zeta^{B\eta^{(\prime)}}$, $\zeta_J^{B\eta^{(\prime)}}$ receive contribs. from gluonic operators
- for isosinglets also "gluonic charming penguins" $\Rightarrow$ in SU(3) limit one additional parameter A_{ccg}



Gluonic contributions

- $\zeta_{(J)g}$ connected to $B \to \eta^{(\prime)}$ semileptonic form factors, not known
- $\zeta_{3(g)}, A_{ccg}$ not known from anywhere else
- in QCD factorization (Beneke, Neubert):
  - at LO in $\alpha_S(\sqrt{\Lambda m_b})$: $\zeta_3^{BM}(u) = \zeta_3/(u\bar{u})$, ζ_{3g}^{BM} const.
  - ζ_g, ζ_3 varied in some (reasonable) range, ζ_{Jg}, ζ_{3g} set to zero
- in SCET analysis (Williamson, JZ):
  - SU(3) used, $\zeta_{(J)g}, A_{ccg}$ determined from data (fit)
  - ζ_3 omitted, even so $\zeta_{(J)g}$ poorly determ., 2 solutions
- measuring Br, A_{CP} for more modes with η, η' in the final state can help a lot!

Comment about diagrammatic approach

- in "diagrammatic" SU(3) fits dynamical assump.: "annihilation-like" ampl. neglected for η, η' final states
  - 4 additional reduced matrix elements for isosinglets, only one (s) taken nonzero
  - in SCET counting all 4 LO in $1/m_b$: keeping only s corresponds to $\zeta_{(J)g} \ll \zeta_{(J)}$ limit not to $m_b \to \infty$ limit

$B \to K\eta$ vs $B \to K\eta'$

- Lipkin '91: constructive and destructive interf.

$$A_{\bar{B}\to\bar{K}\eta'} = \cos\phi A_{\bar{B}\to\bar{K}\eta_s} + \sin\phi A_{\bar{B}\to\bar{K}\eta_q}$$
$$A_{\bar{B}\to\bar{K}\eta} = -\sin\phi A_{\bar{B}\to\bar{K}\eta_s} + \cos\phi A_{\bar{B}\to\bar{K}\eta_q}$$

  with $\phi = (39.3 \pm 1.0)^\circ$, so that $\cos\phi \simeq \sin\phi$

- If $A_{\bar{B}\to\bar{K}\eta_q} \simeq A_{\bar{B}\to\bar{K}\eta_s}$
  - $\Rightarrow$ a constructive interference in $A_{\bar{B}\to\bar{K}\eta'}$
  - $\Rightarrow$ a destructive interference in $A_{\bar{B}\to\bar{K}\eta}$

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- If $A_{\bar{B}\to\bar{K}\eta_q} \simeq A_{\bar{B}\to\bar{K}\eta_s}$
  - $\Rightarrow$ a constructive interference in $A_{\bar{B}\to\bar{K}\eta'}$
  - $\Rightarrow$ a destructive interference in $A_{\bar{B}\to\bar{K}\eta}$
- exactly what happens in QCDFact, SCET
  - penguin dominated mode

$$A_{\bar{B}\to\bar{K}\eta_q} \simeq \frac{1}{\sqrt{2}}(P^{K\eta_q} + 2P_g^{K\eta_q}) + \dots$$
$$A_{\bar{B}\to\bar{K}\eta_s} \simeq P^{K\eta_s} + 2P_g^{K\eta_s} + \dots$$

$B \to K\eta^{(\prime)}$ hierarchy

- SU(3) break. $1 + \Delta r_{(g)} = P_{(g)}^{K\eta_s} / P_{(g)}$, $\quad P_{(g)} \equiv P_{(g)}^{K\eta_q}$

$$A_{B^- \to \eta K^-} \propto (\sqrt{2} - r_g \tan\phi) P_g + \left(\frac{1}{\sqrt{2}} - r \tan\phi\right) P + \cdots$$
$$= (0.59 + 0.8\Delta r_g) P_g - (0.11 + 0.8\Delta r) P + \cdots$$
$$A_{B^- \to \eta' K^-} \propto (r_g + \sqrt{2}\tan\phi) P_g + \left(r + \frac{\tan\phi}{\sqrt{2}}\right) P + \cdots$$
$$= (2.16 + \Delta r_g) P_g + (1.59 + \Delta r) P + \cdots$$

- $Br(B \to \eta K) \ll Br(B \to \eta' K)$ for most $\arg(P_g/P)$, no cancelation between P and P_g needed
- the suppression is larger for P than for P_g
- $P_g, \Delta r_{(g)} = 0 \Rightarrow Br(\eta K) \sim O(10^{-7})$ and not $\sim O(10^{-6})$

Ehancement of $B \to \eta' K$

- $Br(B \to \eta' K)$ enhanced over $Br(B \to \pi K)$

$$\frac{A_{B^- \to \eta' K^-}}{A_{\bar{B}^0 \to \pi^+ K^-}} \simeq \left(r \cos\phi + \frac{\sin\phi}{\sqrt{2}} \right) \frac{P}{P} + \left(r_g \cos\phi + \sqrt{2} \sin\phi \right) \frac{P_g}{P} + \cdot$$

$$\simeq 1.22 + 0.8\Delta r + (1.67 + 0.8\Delta r_g) \frac{P_g}{P}$$

- for $\Delta r = 0.3$, $P_g = 0 \quad \Rightarrow \frac{Br(\eta' K^-)}{Br(\pi^+ K^-)} = 2.1$
- for $\Delta r_{(g)} = 0, P_g = 0.5P \quad \Rightarrow \frac{Br(\eta' K^-)}{Br(\pi^+ K^-)} = 4.2$
- while experimentally $\frac{Br(\eta' K^-)}{Br(\pi^+ K^-)} = 3.54 \pm 0.18$
- SU(3) breaking and/or gluonic content of η' can naturally explain the enhancement see also Gerard, Kou, 2006

More precisely

- fit of SCET param. to all available data assuming SU(3) (errors dominated by $\delta_{SU(3)} \sim O(0.3)$, not shown)

Williamson, JZ, 2006

$$P = 4.9\, e^{i157^\circ} \mathrm{MeV}$$

$$P_g^{\mathrm{Sol.I}} = 3.5\, e^{-i104^\circ} \mathrm{MeV}, \qquad P_g^{\mathrm{Sol.II}} = 3.6\, e^{+i64^\circ} \mathrm{MeV}$$

- in QCD factorization (default scenario, central values)

Beneke, Neubert, 2003

$$P = 4.0\, e^{-i173^\circ} \mathrm{MeV}, \qquad P_g = 0.6 \left(\frac{F_2}{0.1}\right) e^{-i173^\circ} \mathrm{MeV}$$

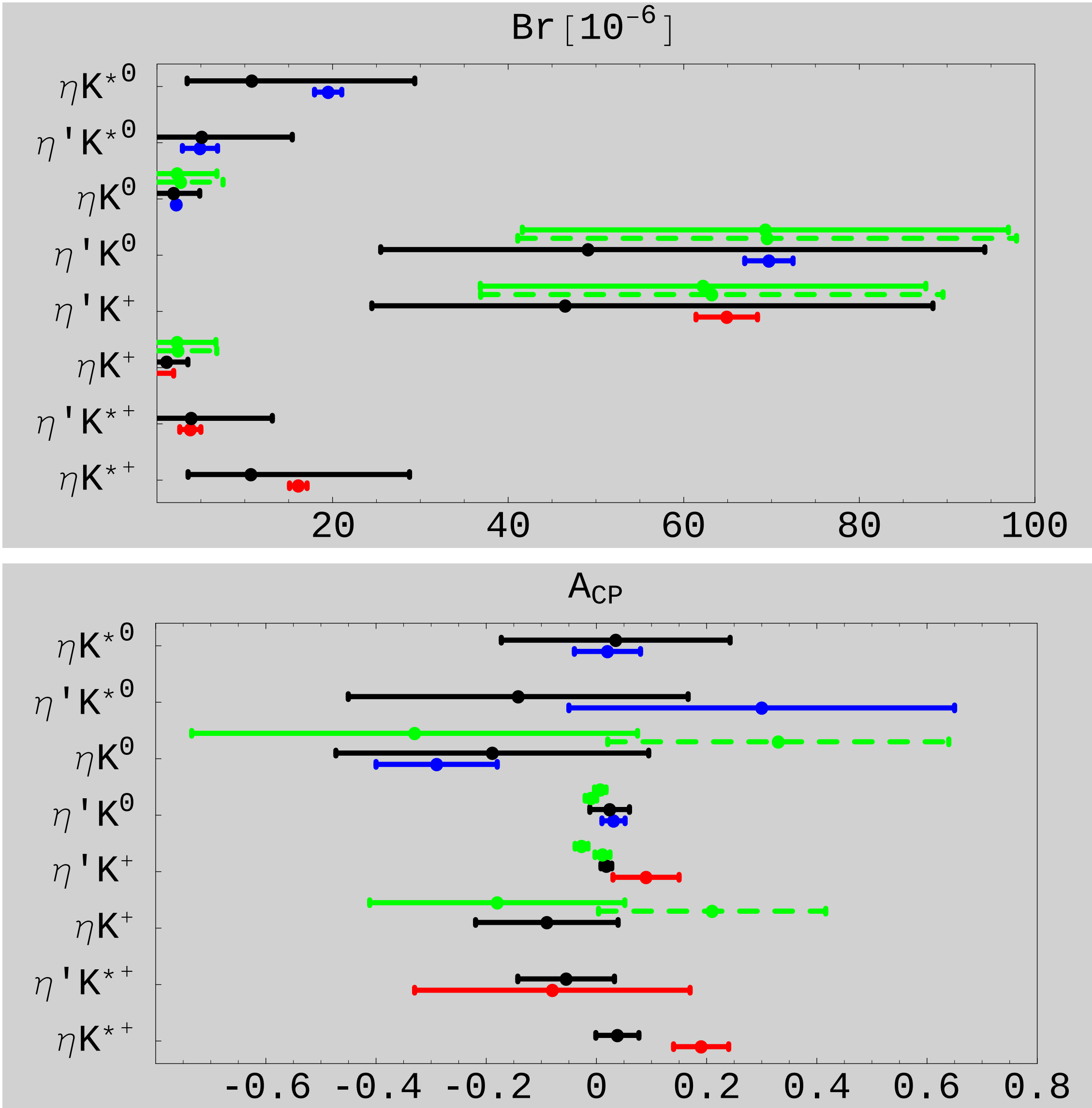
while for SU(3) breaking

$$\Delta r = 0.33 \pm 0.31, \qquad \Delta r_g = 0.25 \pm 0.06$$

- need more experimental data to have a better control over unknown parameters
- measurement of direct CP asymmetries crucial

Results

—- $(B^+)_{\rm exp}$ —- $(B^0)_{\rm exp}$ —- BN —- SCET Sol I - - - SCET Sol II



$B \to K^* \eta^{(\prime)}$

- two topologies (spectator goes in V or P)
- Lipkin's symmetry argument not sufficient for this case, since symmetries do not relate the two topologies
- further dynamical information required
- no numerical analysis in SCET yet
- in QCDFact the inverted hierarchy comes from a flip of sign (compared to KP) in chirally enhanced $1/m_b$ term

$$\hat{\alpha}_4(PK^*) = \alpha_4(PK^*) + \overbrace{r_\chi^{K^*}}^{\text{small}} \alpha_6(PK^*) \leftarrow \text{becomes smaller}$$

$$\hat{\alpha}_4(K^*P) = \alpha_4(K^*P) - \underbrace{r_\chi^{P}}_{\text{large}} \alpha_6(K^*P) \leftarrow \text{flips sign}$$

$F_2 = 0 \Rightarrow \hat{\alpha}_4(K^*\eta_s) = \hat{\alpha}_4(\eta_q K^*) = 0$, remaning nonzero

Conclusions

- Lipkin argument +SU(3) breaking+ contributions due to gluonic content of η' naturally explains the hierarchy of $Br(K\eta') \gg Br(K\eta)$ and $Br(K\eta') \gg Br(K\pi)$
- situation is more complicated in $B \to K^*\eta^{(\prime)}$ decays, two topologies, in QCDFactorization the inverted hierarchy comes from formally $1/m_b$ suppressed terms

Backup slides

Counting of parameters

- assuming isospin,LO in $1/m_b, \alpha_S(m_b)$, neglecting ζ_3
  - $B \to \pi\pi$: 4 real parameters $\zeta_{(J)}^{B\pi}$, $A_{cc}^{\pi\pi}$ vs. 8 observables (6 measured)
  - $B \to \pi\eta^{(\prime)}$: 8 new parameters $\zeta_{(J)}^{B\eta_{q,s}}$, $A_{cc}^{\pi\eta_{q,s}}$ beyond $B \to \pi\pi$ vs. 19 observables (4 measured)
  - similarly $B \to \pi K$ vs. $B \to K\eta^{(\prime)}$
- at present in the analysis of isosinglets SU(3) needs to be used (this can be relaxed with more data)
- in the SU(3) limit 8 real parameters: ζ, ζ_J, A_{cc} and the "gluonic" ζ_g, ζ_{Jg}, A_{ccg} (these only for isosinglets)
- compare with 18 complex reduced matrix elements in most general SU(3) decomposition

Relation to diagrammatic approach

- in SU(3) limit, for $\Delta S = 0$ decay (at LO in $\alpha_S(m_b)$)

$$t \propto V_{ub}V_{ud}^* \Big[\Big(C_1 + \frac{C_2}{N}\big(1 + \langle x^{-1}\rangle\big)\Big)\zeta_J + \Big(C_1 + \frac{C_2}{N}\Big)\zeta \Big] + \ldots$$

$$c \propto V_{ub}V_{ud}^* \Big[\Big(C_2 + \frac{C_1}{N}\big(1 + \langle x^{-1}\rangle\big)\Big)\zeta_J + \Big(C_2 + \frac{C_1}{N}\Big)\zeta \Big] + \ldots$$

$$p \propto V_{cb}V_{cd}^* A_{cc} + V_{ub}V_{ud}^* F(\zeta, \zeta_J) + \ldots$$

- at LO in $1/m_b$ no annihilation and exchange amplitudes
- at LO in $1/m_b$ and $\alpha_S(m_b)$ strong phase only in p (A_{cc})
- no color suppression (in SCET counting $\zeta \sim \zeta_J$)

$$t \propto V_{ub}V_{ud}^* \Big[0.77\zeta_J + 1.03\zeta \Big] + \ldots, \qquad c \propto V_{ub}V_{ud}^* \Big[1.23\zeta_J + 0.12\zeta \Big]$$