Penguin polution estimates relevant for ϕ_2/α extraction

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Precision determination of $\alpha(\gamma)$

- precision β determination in $b \to c\bar{c}s$ $(B \to \psi K_S)$:

$$\beta = (21.7^{+1.3}_{-1.2})^\circ$$

with ultimate theory error below percent level

Ciuchini, Pierini, Silvestrini, 2005
Boos, Mannel, Reuter, 2004
H.-n. Li, S. Mishima, 2006

- Sets high standards for α, γ determinations
- what theory errors on determination of α ?
  - isospin
  - SU(3)
  - $1/m_b$

cf. also talk by B. Melic

α in $B \to \pi^+\pi^-$

- two amplitudes with different weak phases

$$A(B^0 \to \pi^+\pi^-) = Te^{i\gamma} + Pe^{i\delta}, \qquad r = P/T$$

- time dependent decay width

$$\Gamma(B^0(t) \to \pi^+\pi^-) \propto \Gamma_{\pi^+\pi^-}\left[1 + C_{\pi^+\pi^-}\cos\Delta m t - S_{\pi^+\pi^-}\sin\Delta m t\right]$$

- expanding in r

$$S_{\pi^+\pi^-} = \sin 2\alpha + 2r\cos\delta\sin(\beta+\alpha)\cos 2\alpha + O(r^2)$$

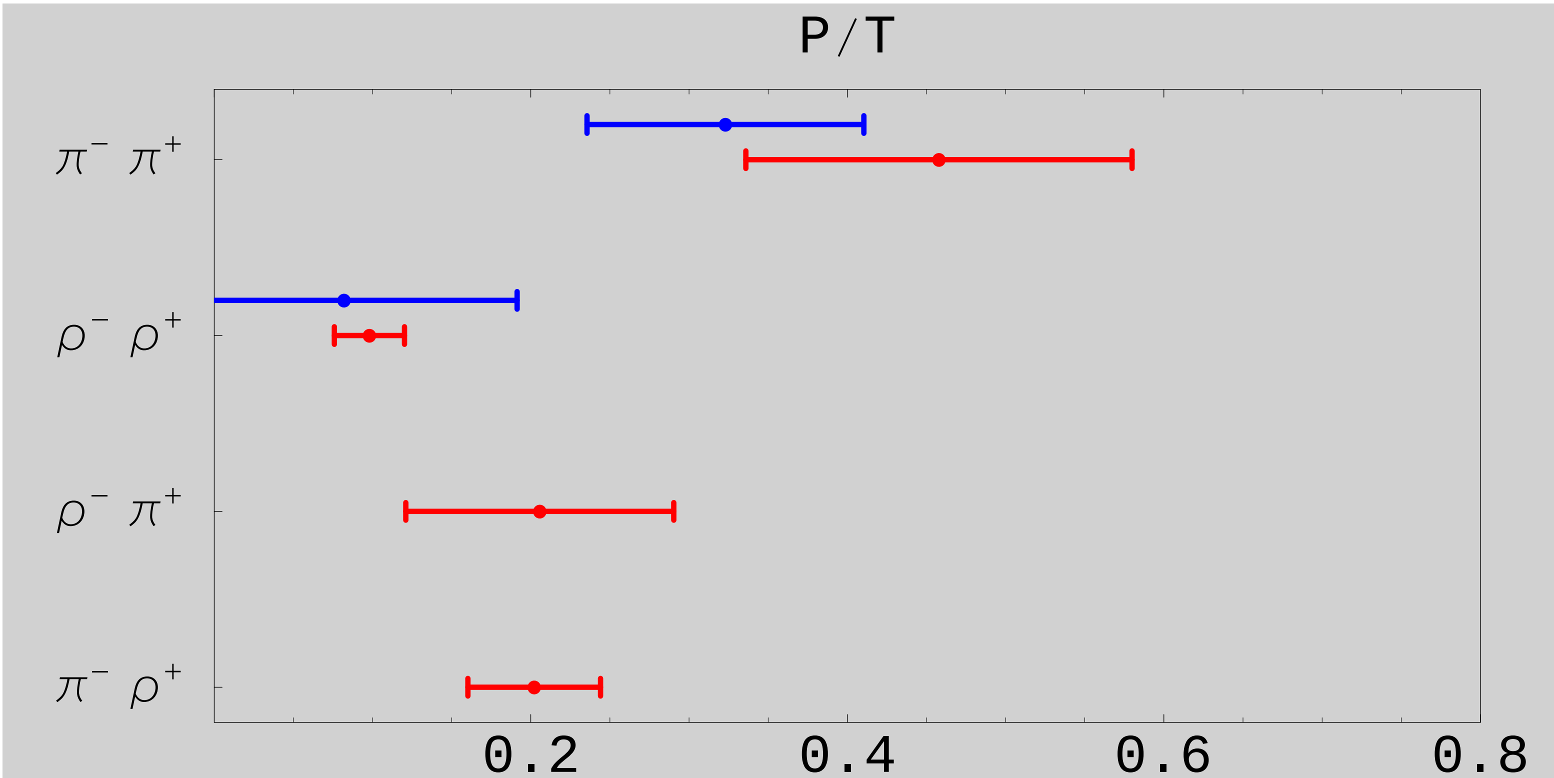
- 3 measurables vs. 4 unknowns: T, r, δ, γ

to fix r additional input required

- larger $r \Rightarrow$ larger difference $\sin 2\alpha - \sin 2\alpha_{\text{eff}}$

Size of P/T

—- isospin —- SU(3)



- hierarchy: $r(\pi^+\pi^-) > r(\rho^+\pi^-) \sim r(\pi^+\rho^-) > r(\rho^+\rho^-)$
- in SU(3) (and $1/m_b$) approaches: expect the same hierarchy in the theory errors

Isospin

Isospin breaking

- the standard methods for obtaining α from $B \to \pi\pi, \rho\rho, \rho\pi$ use isospin

  Gronau, London (1990)
  Snyder, Quinn (1993)

- theory error on α due to isospin breaking
  - d and u charges different
  - $m_u \neq m_d$
- extends the basis of operators to EWP $Q_{7,\dots,10}$
- mass eigenstates do not coincide with isospin eigenstates: $\pi - \eta - \eta'$ and $\rho - \omega$ mixing
- reduced matrix elements only appr. related by Clebsch Gordan coeff.
- may induce $\Delta I = 5/2$ operators not present in H_W

Sizes of isospin breaking

- not all isospin breaking effects can be calculated/constrained at present
- the ones that can be are of expected size
$$\sim (m_u - m_d)/\Lambda_{QCD} \sim \alpha_0 \sim 1\%$$
- EWP effect known model indep. (negl. $Q_{7,8}$)
Neubert, Rosner; Gronau, Pirjol, Yan;Buras, Fleischer (1999)
$$\Delta\alpha_{\rm EWP} = (1.5 \pm 0.3 \pm 0.3)^\circ$$
the same for $\pi\pi, \rho\rho, \rho\pi$
- for $\pi^0 - \eta - \eta'$ mixing M. Gronau, J.Z. (2005), S. Gardner (2005)
$$|\Delta\alpha_{\pi\pi}^{\pi-\eta-\eta'}| < 1.6^\circ$$

Theory error for $B(t) \to \rho\rho$

- since $\Gamma_\rho \neq 0 \Rightarrow I = 1$ contributions possible
  Falk, Ligeti, Nir, Quinn (2003)
  - $O(\Gamma_\rho^2/m_\rho^2)$ effect
  - possible to constrain experimentally
- isospin breaking
  M. Gronau, JZ (2005)
  - EWP same as for $B \to \pi\pi$
  - $\rho - \omega$ mixing, integrated effect $< 2\%$
  - other: $g_I \equiv g(\rho_I \to \pi^+\pi^-) \neq g_c \equiv g(\rho^+ \to \pi^+\pi_3)$
    [PDG: $g_c/g_I - 1 = (0.5 \pm 1.0)\%$]

Effect of isospin breaking $B \to \rho\pi$

- Snyder-Quinn: isospin breaking affects only the relation between penguins!
  - not true for the pentagon analysis
- largest contribution from EWP because they are related to tree (and easily taken into account)
- other isospin breaking effects are $P/T \sim 0.2$ suppressed
  - for example

$$|\Delta\alpha_{\pi-\eta-\eta'}| \leq 0.1^\circ$$

  - this does not include all isospin breaking but shows the suppression exists

$$SU(3)$$

Basic idea

- in $\Delta S = 1$ decays:

$$T' \sim V_{ub}V_{us}^* \qquad\qquad P' \sim V_{cb}V_{cs}^*$$

- in $\Delta S = 0$ decays:

$$T \sim V_{ub}V_{ud}^* \qquad\qquad P \sim V_{cb}V_{cd}^*$$

- thus P'/T' is $1/\lambda^2$ enhanced over $P/T \Rightarrow$ can use it to bound P/T

$B \to \rho\rho$ and SU(3)

- $P/T \sim 0.1 \Rightarrow$ can SU(3) analysis compete with isospin?
- Note: in $\rho\rho$ isospin analysis the theory errors not necessarily P/T suppressed

Stronger bound on P/T from SU(3)

Beneke, Gronau, Rohrer, Spranger (2006)

- use SU(3) related $\mathcal{B}_L(K^{*0}\rho^+)$ which mostly P

$$\mathcal{B}(K^{*0}\rho^+) = (9.3 \pm 1.7)\cdot 10^{-6}, f_L = 0.48^{+0.09}_{-0.08},$$

- Long. CP-av. rate: $|A_L(K^{*0}\rho^+)|^2_{\rm CP-av} = F\left(\frac{|V_{cs}|f_{K^*}}{|V_{cd}|f_\rho}\right)P^2$
  - SU(3) breaking factorized: $F=1$, expect corrections
  - $F \neq 1$: nonfact. SU(3)-breaking, P^c_{EW}, PA
  - conservative range: $0.3 \leq F \leq 1.5$
- $F = 1 \Rightarrow P/T = 0.10 \pm 0.02$
- include pre-ICHEP06 exp. errors on S_L, C_L and vary F:

$$\alpha = [91.2^{+9.1}_{-6.6}\,(\rm exp)^{+1.2}_{-3.9}\,(\rm th)]^\circ$$

- th. error from F is smaller than $|\alpha - \alpha_{\rm eff}|_{\bf isospin} < 22.4^\circ$

Lesson

- once we know $(P/T)_{\rho^+\rho^-}$ is small

  $$\Downarrow$$

  SU(3) breaking (or other corrections) introduce a small error in α
- could be comparable to theory errors in isospin analysis of $B \to \rho\rho$

SU(3) approach to α from $B \to \rho\pi$

- in Snyder-Quinn crucial interf. regions in Dalitz plot
- yet ρ bands do not overlap head-on
- alternative: use just $\rho^{\pm}\pi^{\mp}$ final state without interference info + SU(3) Gronau, JZ (2004)
- to constrain $p_{\pm}/t_{\pm}$ in $A(B^0 \to \rho^{\pm}\pi^{\mp}) = e^{i\gamma}t_{\pm} + p_{\pm}$ use SU(3) related modes $B^0 \to K^{*+}\pi^-, K^+\rho^-$ and $B^+ \to K^{*0}\pi^+, K^0\rho^+$
- @90% CL: $0.16 \le |p_+/t_+| \le 0.24$ $0.12 \le |p_-/t_-| \le 0.29$
- SU(3) breaking on extracted α are small, of order $p_{\pm}^2/t_{\pm}^2$ $\Rightarrow$ MC study with up to 30% SU(3) breaking on penguins gives $\sqrt{\langle(\alpha^{\text{out}} - \alpha^{\text{in}})^2\rangle} \sim 2^\circ$, but some could be larger, needs further study

α from $a_1^{\pm}\pi^{\mp}$

cf. talk by F. Palombo

- isospin analysis practically impossible (either 4-body final states or pentagon relations)
- SU(3) very similar to $\rho\pi$ — M. Gronau, JZ, 2005
  - complication: K_{1A} from the SU(3) multiplet of a_1 an admixture of two mass eigenstates
- method works much better if smaller p/t
  - if $p/t \ll 1$ can use bounds
  - otherwise general SU(3) fit (what the size of errors?)
- to have bounds at least 3 SU(3) related modes needed

$$\Big\{\big(\overline{\Gamma}(B^0 \to K_1^+(1400)\pi^-)\ \&\ \overline{\Gamma}(B^0 \to K_1^+(1200)\pi^-)\big)\ \text{or}$$
$$\big(\overline{\Gamma}(B^+ \to K_1^0(1400)\pi^+)\ \&\ \overline{\Gamma}(B^+ \to K_1^0(1200)\pi^+)\big)\Big\}$$

and

$$\Big\{\overline{\Gamma}(B^0 \to a_1^- K^+)\ \text{or}\ \overline{\Gamma}(B^+ \to a_1^+ K^0)\Big\}$$

$$1/m_b$$

"α" from $B \to \pi\pi$ using $1/m_b$

Buchalla, Safir (2003, 2004)

- make CKM factors in penguin-to-tree explicit

$$\frac{P}{T} = \frac{2 r_p e^{i\delta}}{\sqrt{\bar\rho^2 + \bar\eta^2}}$$

- another relation between $\bar\rho, \bar\eta$ from $\tau \equiv \tan\beta$
- then to corrections of $O(r_p^2)$

$$\bar\eta = \frac{1 + \tau S_{\pi\pi} - \sqrt{1 - S_{\pi\pi}^2}}{(1+\tau^2) S_{\pi\pi}} (1 + \mathrm{Cos}[\phi] r_p)$$

- in QCDFact $r_p = 0.107 \pm 0.031$
- 30% error on r_p translates to $\sim 3\%$ error on $\bar\eta$, but have to be sure about the phase

Conclusions

- the isospin breaking effects that have been analysed are of expected size
- alternative and competitive ways of analysing the same data using info from SU(3) or $1/m_b$ exist
- in $B \to \rho\rho$ SU(3) approach should be used at present instead of isospin analysis

Backup slides

$B \to \pi\pi$ and isospin

Gronau, London (1990)

- completely general isospin decomposition

$$
\begin{aligned}
A_{+-} = \langle \pi^+\pi^- | H | B^0 \rangle &= -A_{1/2} + \tfrac{1}{\sqrt{2}} A_{3/2} - \tfrac{1}{\sqrt{2}} A_{5/2} \\
A_{00} = \langle \pi^0\pi^0 | H | B^0 \rangle &= \tfrac{1}{\sqrt{2}} A_{1/2} + A_{3/2} - A_{5/2} \\
A_{+0} = \langle \pi^+\pi^0 | H | B^+ \rangle &= \tfrac{3}{2} A_{3/2} + A_{5/2}
\end{aligned}
$$

- neglecting $A_{5/2} \sim \alpha A_{1/2}$ (i.e. $\sim 1\%$ correction)

$$
\begin{aligned}
A_{+-} + \sqrt{2} A_{00} &= \sqrt{2} A_{+0} \\
\bar{A}_{+-} + \sqrt{2} \bar{A}_{00} &= \sqrt{2} \bar{A}_{+0}
\end{aligned}
$$

- neglecting EWP $\Rightarrow$ A_{+0} only tree contribs.

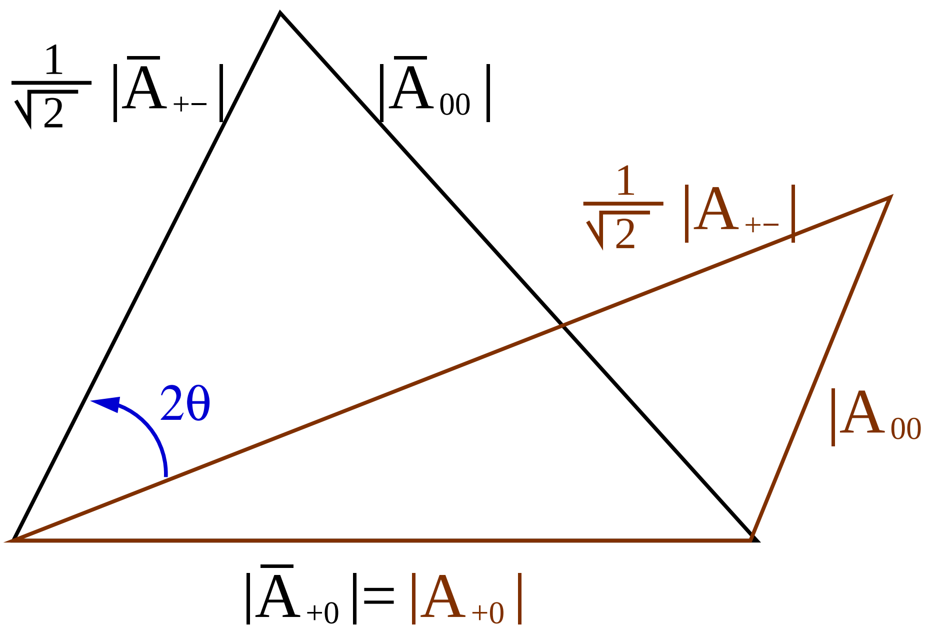
$$
e^{i\gamma} A_{+0} = e^{-i\gamma} \bar{A}_{+0} \quad \Rightarrow \quad |A_{+0}| = |\bar{A}_{+0}|
$$

Gronau-London triangle

- $\sin 2\alpha$ from
  $\Gamma(B^0(t) \to \pi^+\pi^-) \propto [1 + C_{\pi\pi} \cos \Delta m t - S_{\pi\pi} \sin \Delta m t]$

$$\boxed{\sin(2\alpha_{\rm eff}) = S_{\pi\pi}/\sqrt{1 - C_{\pi\pi}^2}}$$

$$\boxed{2\alpha = 2\alpha_{\rm eff} - 2\theta}$$



$\pi^0 - \eta - \eta'$ mixing

- π^0 w.f. has η, η' admixtures

$$|\pi^0\rangle = |\pi_3\rangle + \epsilon|\eta\rangle + \epsilon'|\eta'\rangle$$

where $\epsilon = 0.017 \pm 0.003,\ \epsilon' = 0.004 \pm 0.001$ Kroll (2004)

- GL triangle relations no longer hold

$$A_{+-} + \sqrt{2}A_{00} - \sqrt{2}A_{+0} \neq 0$$
$$\bar{A}_{+-} + \sqrt{2}\bar{A}_{00} - \sqrt{2}\bar{A}_{+0} \neq 0$$

- varying the phases of $A_{0\eta^{(\prime)}}$, $\bar{A}_{0\eta^{(\prime)}}$ gives bound

$$|\Delta\alpha_{\pi-\eta-\eta}| \leq \sqrt{2\frac{\tau_+}{\tau_0}}\left(\epsilon\sqrt{\frac{\mathcal{B}_{0\eta}}{\mathcal{B}_{+0}}} + \epsilon'\sqrt{\frac{\mathcal{B}_{0\eta'}}{\mathcal{B}_{+0}}}\right)$$

leading to M. Gronau, J.Z. (2005), S. Gardner (2005)

$$\boxed{|\Delta\alpha_{\pi-\eta-\eta'}| < 1.6^\circ}$$

Electroweak penguins

- separate triangle relations still hold
- neglecting $Q_{7,8}$

Neubert, Rosner; Gronau, Pirjol, Yan; Buras, Fleischer (1999)

$$H_{\rm eff,EWP}^{\Delta I=3/2} = -\frac{3}{2}\frac{C_9+C_{10}}{C_1+C_2}\frac{V_{tb}^*V_{td}}{V_{ub}^*V_{ud}}H_{\rm eff,c-c}^{\Delta I=3/2}$$

$\Rightarrow e^{i\gamma}A_{+0} = e^{-i(\gamma+2\delta)}\bar{A}_{+0}$, but still $|A_{+0}| = |\bar{A}_{+0}|$

$\Rightarrow$ $\alpha = \alpha_{\rm eff} - \theta - \delta$ with $\delta = (1.5 \pm 0.3 \pm 0.3)^\circ$

conservatively $\sim 2(|c_7|+|c_8|)/(|c_9|) < 0.2$

- the same relation

$$e^{i\gamma}T = e^{-i(\gamma+2\delta)}\bar{T}$$

holds for $\Delta I = 3/2$ (tree) amplitudes in $\rho\rho$ and $\rho\pi$ system

t conv. vs. c conv.

- c-convention:

$$A = Te^{i\gamma} + Pe^{i\delta}, \qquad r = P/T$$

$$S = \sin 2\alpha + 2r\cos\delta\sin(\beta+\alpha)\cos 2\alpha + O(r^2)$$

- t-convention

$$A = -Te^{i\gamma} + Pe^{i(-\beta+\delta)}, \qquad r = P/T$$

$$S = \sin 2\alpha + r\cos\delta\big(\sin(\alpha) - \sin(3\alpha)\big) + O(r^2)$$

$$\frac{2\sin(\beta+\alpha)\cos 2\alpha}{\sin(\alpha)-\sin(3\alpha)} \sim -0.9$$