# Extraction of $\alpha$ from $B\to\pi\pi$ decays using the available information on the hadronic amplitudes

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**UTfit** Collaboration

(paper in preparation)

4th International Workshop on
CKM
Unitarity Triangle
2006 Nagoya

# Motivations and results

1) A recent paper, J.Charles, A.Höcker, H.Lacker, F.R. Le Diberder, S.T'Jampens, hep-ph/0607246, discusses the extraction of $\alpha$ to strongly criticize the Bayesian approach.

We show that the differences between the frequentistic and Bayesian results are NOT due to the methods, but to the different physical assumptions on the hadronic amplitudes

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2) Useful information on the hadronic matrix element, which can be already extracted from the data, helps in reducing the eightfold ambiguity in the determination of $\alpha$ with the GL method

3) We show that, within the Bayesian approach, we obtain consistent results irrespectively on the parameterization used for the hadronic amplitudes

# The Gronau-London method:

$$A^{+-} \equiv A(B^0 \to \pi^+\pi^-) = e^{-i\alpha} T^{+-} + P$$

$$A^{00} \equiv A(B^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{2}}\left(e^{-i\alpha} T^{00} - P\right)$$

$$A^{+0} \equiv A(B^+ \to \pi^+\pi^0) = \frac{1}{\sqrt{2}} e^{-i\alpha}\left(T^{+-} + T^{00}\right)$$

CP: $\alpha \to -\alpha$

**6 parameters:**

$|T^{+-}|$, $|T^{00}|$, $|P|$,

$\delta^{00}$, $\delta^{P}$, $\alpha$

**6 observables:** $B^{+-}$, $B^{00}$, $B^{+0}$, $C^{+-}$, $S^{+-}$, $C^{00}$

$$B_{\pi\pi}^{+-,00} = \frac{1}{2}\left(\left|A^{+-,00}\right|^2 + \left|\bar{A}^{+-,00}\right|^2\right) \quad , \quad B_{\pi\pi}^{+0} = \frac{\tau_{B^+}}{\tau_{B^0}} \frac{1}{2}\left(\left|A^{+0}\right|^2 + \left|\bar{A}^{+0}\right|^2\right)$$

$$C_{\pi\pi}^{ij} = \frac{\left|A^{ij}\right|^2 - \left|\bar{A}^{ij}\right|^2}{\left|A^{ij}\right|^2 + \left|\bar{A}^{ij}\right|^2} \quad , \quad S_{\pi\pi}^{ij} = \frac{\mathrm{Im}\left[A^{ij}\bar{A}^{ij*}\right]}{\left|A^{ij}\right|^2 + \left|\bar{A}^{ij}\right|^2}$$

**8 or 0 solutions**

# The criticism of hep-ph/0607246

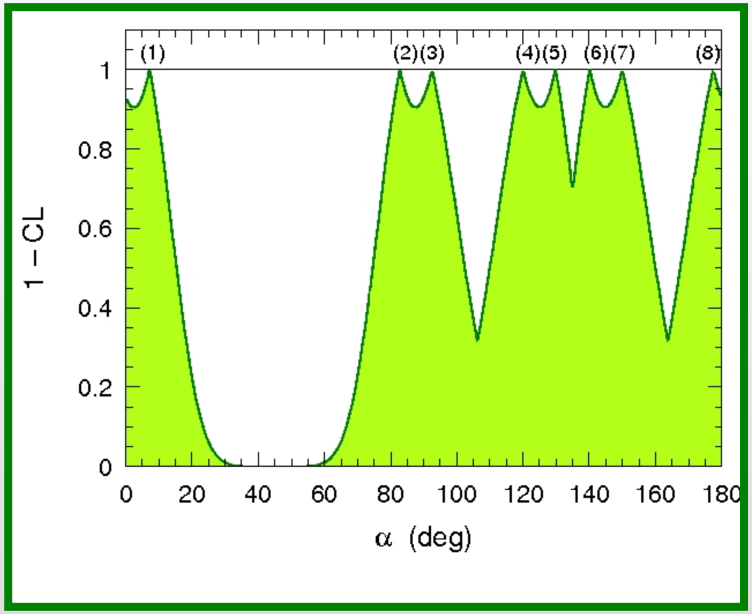
J.Charles, A.Höcker, H.Lacker, F.R. Le Diberder, S.T'Jampens

Comparison between the results of the frequentistic and Bayesian analysis in the determination of α with the Gronau and London method

## 1) The frequentistic result

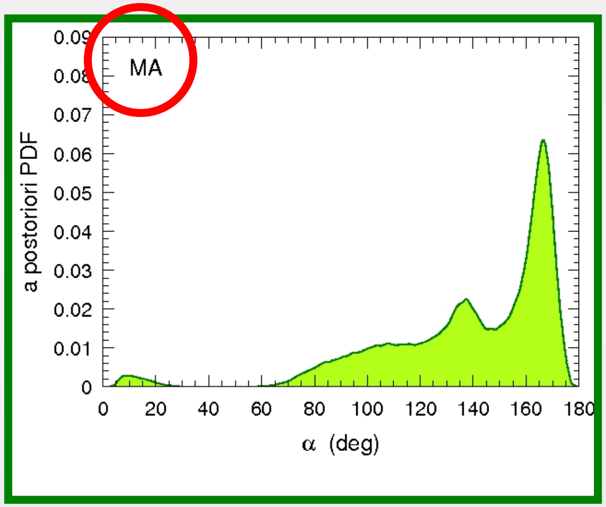
"The result is independent on the parameterization used"

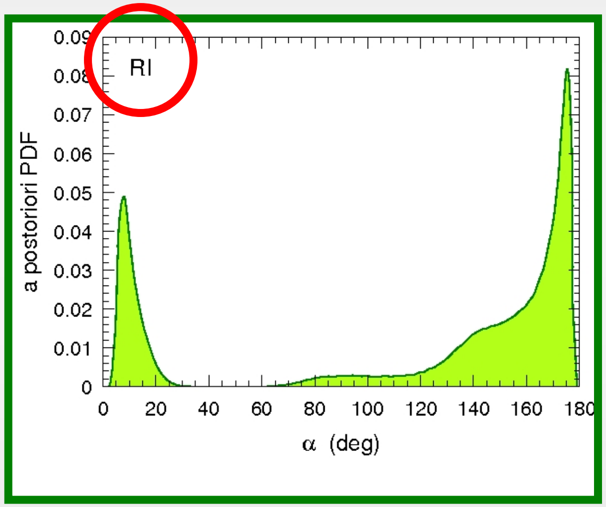
"The 8 solutions for α are clearly visible"

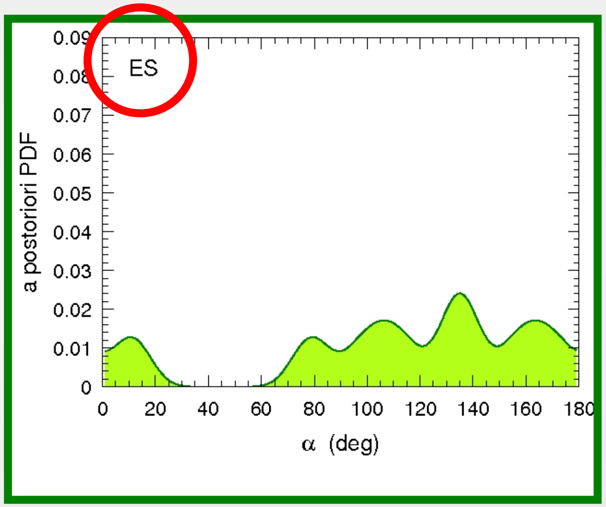


# 2) The Bayesian result

J.Charles et al: The posterior PDF for $\alpha$ shows a striking dependence on the choice of the amplitude parameterization (i.e. of the prior distribution)







- **MA**, **Modulus and Argument**: $|T^{ij}|$, $|P|$, $\delta_{ij}$, $\delta_P$ flat
- **RI**, **Real and Imaginary**: $Re[T^{ij}]$, $Im[T^{ij}|$, $Re[P]$… flat
- **ES**, **Explicit Solutions**: $B^{ij}$, $C^{ij}$, $S^{ij}$ flat

J.Charles et al: "Obviously, the choice of the priors cannot be irrelevant; hence, the Bayesian treatment is doomed to lead to results which depend on the decisions made, necessarily on unscientific basis, by the authors of a given analysis, for the choice of these PDFs"

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*A rather aggressive statement !!*

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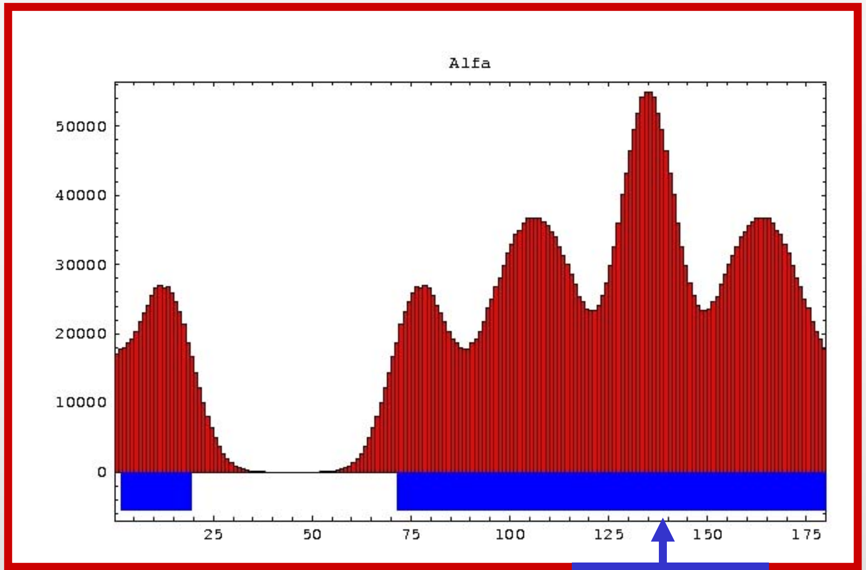
# Frequentistic vs. Bayesian analysis

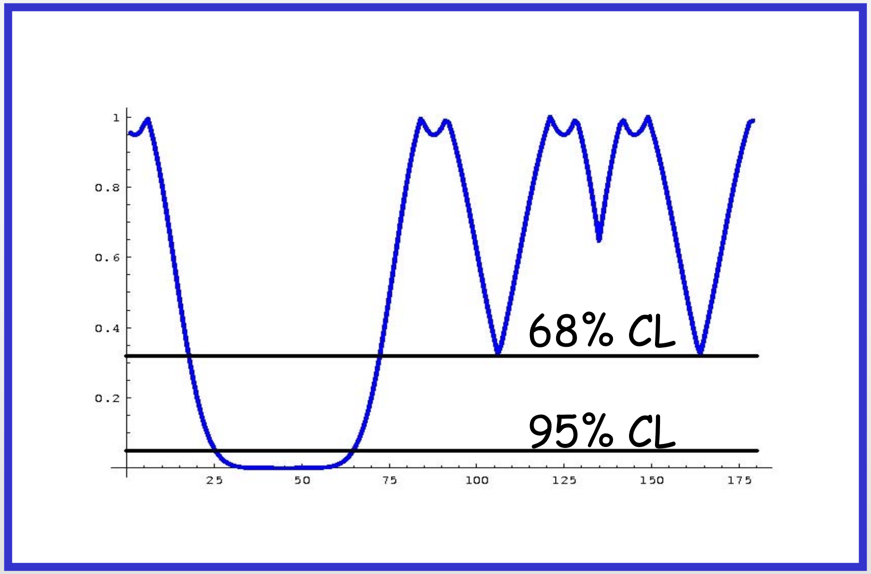
## Compare the 2 methods using the same assumptions

- In the frequentistic analysis: no additional information on the hadronic amplitudes is introduced (besides the GL method) →

- In the Bayesian approach: extract BR's and CP parameters with gaussian p.d.f. according with their experimental values and errors



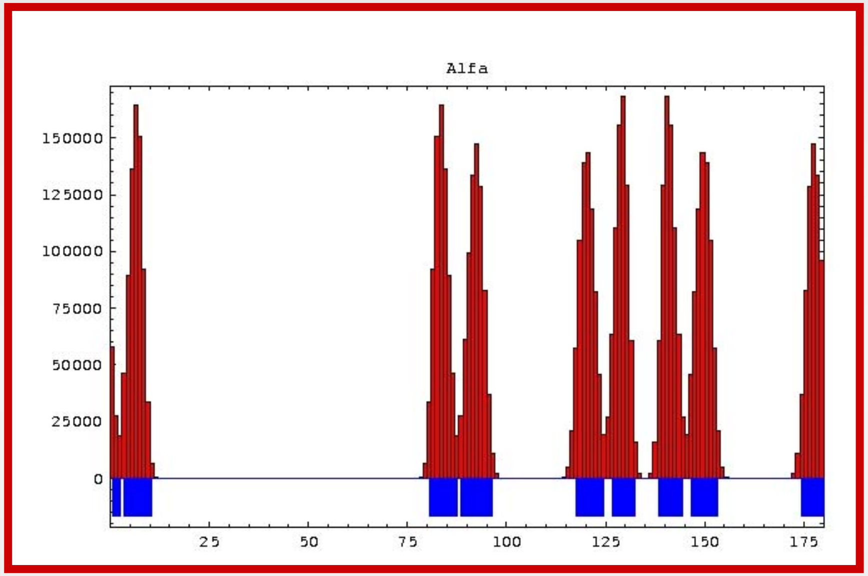
95% CL

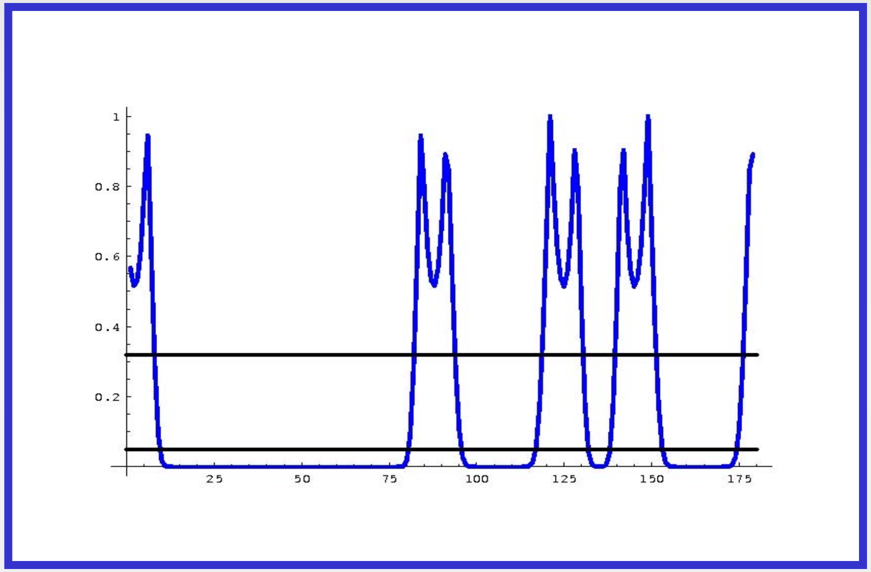


**THE TWO APPROACHES GIVE EQUIVALENT RESULTS**

# A second comparison:

Reduce the experimental errors by a factor of 10 at fixed central values





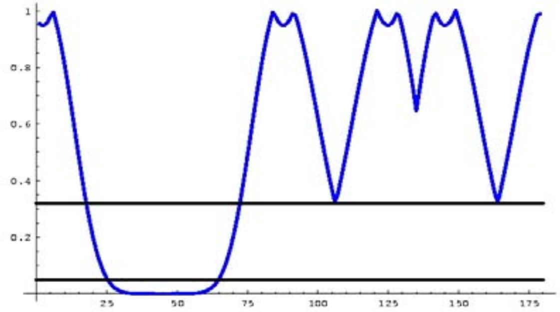
The eight solutions begin separating both in the Bayesian and frequentistic case

➡ Provided the same assumptions are done, the two approaches lead to similar results

# Are the 8 peaks a crucial requirement?

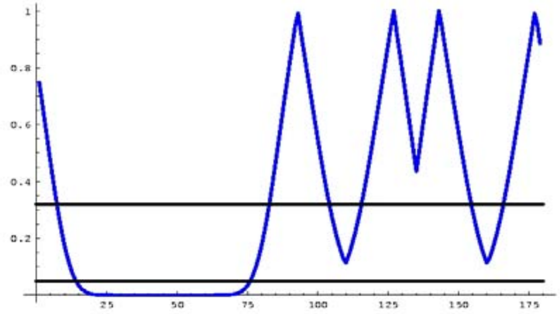
**J.Charles et al:**

| | | |
|---|---|---|
| $B_{\pi\pi}^{+0}\times 10^6$ | $B_{\pi\pi}^{+-}\times 10^6$ | $B_{\pi\pi}^{00}\times 10^6$ |
| $5.5\pm 0.6$ | $5.1\pm 0.4$ | $1.45\pm 0.29$ |
| $C_{\pi\pi}^{+-}$ | $S_{\pi\pi}^{+-}$ | $C_{\pi\pi}^{00}$ |
| $-0.37\pm 0.10$ | $-0.50\pm 0.12$ | $-0.28\pm 0.40$ |



**Present HFAG averages:**

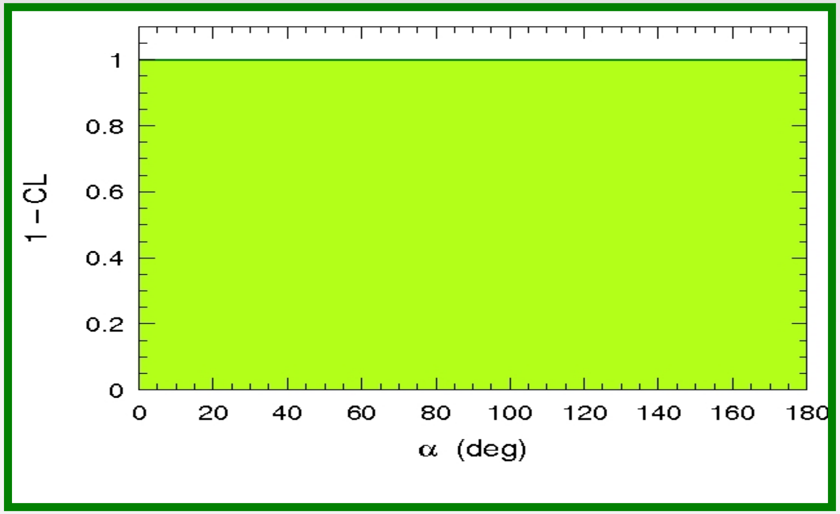
| | | |
|---|---|---|
| $B_{\pi\pi}^{+0}\times 10^6$ | $B_{\pi\pi}^{+-}\times 10^6$ | $B_{\pi\pi}^{00}\times 10^6$ |
| $5.7\pm 0.4$ | $5.2\pm 0.2$ | $1.31\pm 0.21$ |
| $C_{\pi\pi}^{+-}$ | $S_{\pi\pi}^{+-}$ | $C_{\pi\pi}^{00}$ |
| $-0.39\pm 0.07$ | $-0.59\pm 0.09$ | $-0.37\pm 0.32$ |

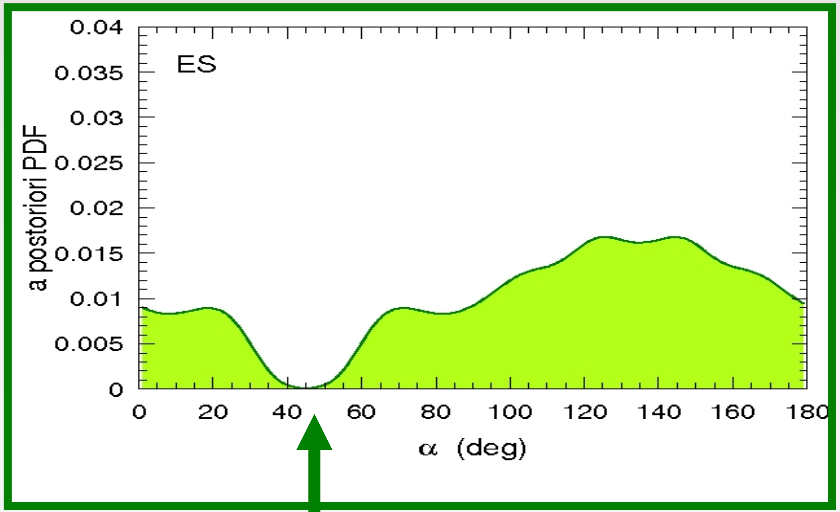


With present experimental values the 8 solutions are not visible anymore also in the frequentistic case

# Removing essential information: $B^0 \to \pi^0\pi^0$

[J.Charles et al: hep-ph/0607246]





**Why the region around $\alpha = 45^o$ is excluded ?**

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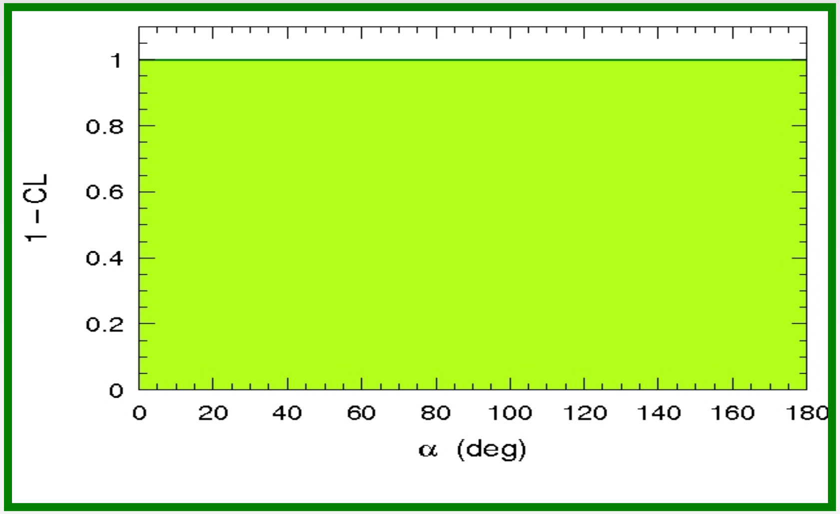
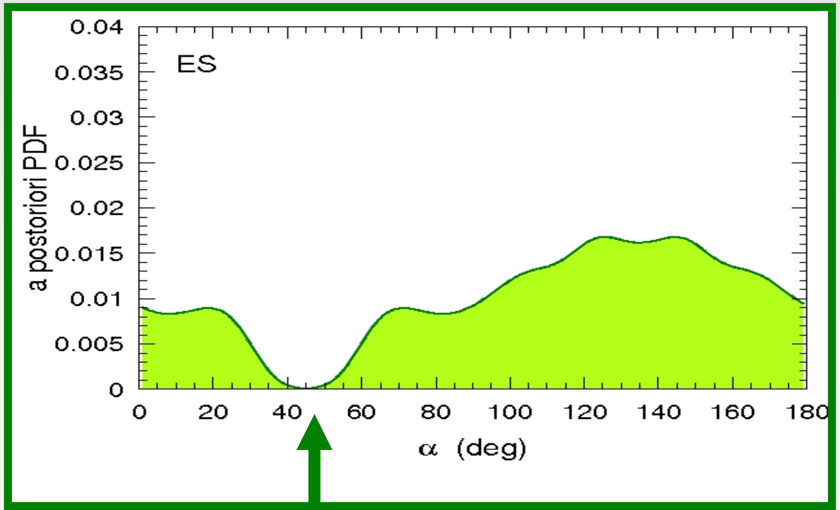
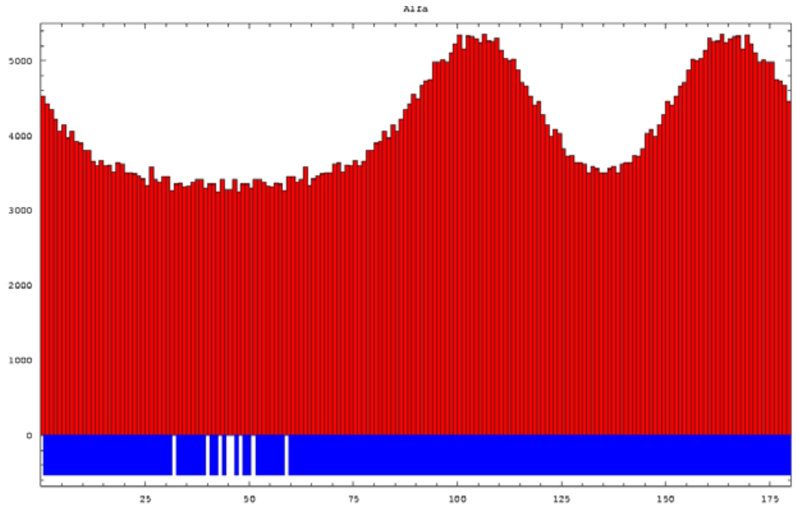




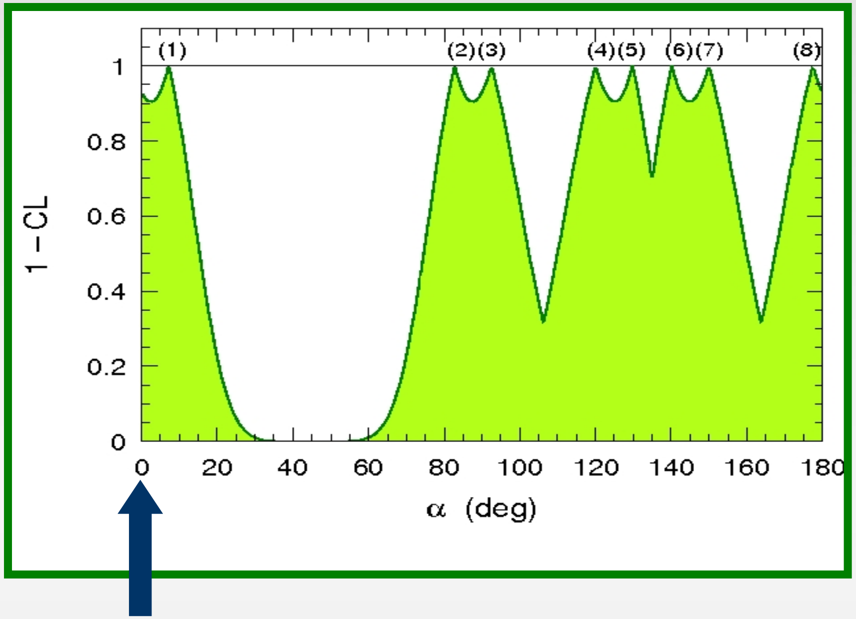
**Why the region around $\alpha$=45° is excluded ?**

**We find:**



**The result must be wrong…**

# Different physical assumptions



In the frequentistic analysis, the region around $\alpha=0$ is not excluded, despite the experimental observation of CP violation. It requires infinitely large hadronic matrix elements. But can we assume

$$\boxed{\langle \pi\pi | O | B \rangle \gg (M_{\mathrm{Planck}})^3 \text{ ?}}$$

# Adding information on the hadronic amplitudes

The GL method already requires some a priori "minimal assumptions" on strong interactions, namely:

- Flavour blind and CP conserving strong interactions

- Negligible isospin symmetry breaking effects, including e.m. corrections

Besides, we also believe that

- QCD is the theory of strong interactions

- QCD is a renormalizable theory with a dimensionless coupling constant and a natural scale $\Lambda_{QCD}$ ~ 1 GeV

Therefore we do not expect:

$$\langle \pi\pi | O | B \rangle \sim (1\ \mathrm{TeV})^3 \text{ or } (M_{\mathrm{Planck}})^3$$

- Two scales enter in the process, $M_B$ and $\Lambda_{QCD}$

$$\boxed{\langle \pi\pi | \hat{O} | B \rangle \sim f_\pi M_B^2 f^+(0) \sim f_\pi M_B^2 \left( \frac{\Lambda_{QCD}}{M_B} \right)^{3/2} \sim M_B^{1/2} \Lambda_{QCD}^{3/2}}$$

Note: the scaling law has a more general validity than factorization

This gives: $\boxed{T^{ij} \sim 1}$

[We use "natural units": the BR x $10^6$ are simply given by the squared amplitude]

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- Using strict factorization

$$BR(B_d^0 \to \pi^+\pi^-) \propto \left| C_1(M_B) \langle \pi^+\pi^- | O_1 | B_d^0 \rangle + C_2(M_B) \langle \pi^+\pi^- | O_2 | B_d^0 \rangle \right|^2 =$$

$$= \left| \frac{1}{3} C_1(M_B) + C_2(M_B) \right|^2 \left( M_B^2 f_\pi f^+(0) \right)^2$$

This gives $\boxed{|T^{+-}| = 1.0}$

Indeed, theoretical predictions for $B^{+-}$ exist since many years
Ciuchini et al. '98, BBNS '99, Keum et al. '02, …

- ## Scaling between B and D decays

The dependence on $M_H$ cancels in the decay rate. In the strict factorized limit (P=0):

$$R = \frac{\Gamma(B_d^0 \to \pi^+\pi^-)}{\Gamma(D^0 \to \pi^+\pi^-)} \overset{P=0}{=} \frac{\left|V_{ub}V_{ud}^*\right|^2}{\left|V_{cd}V_{ud}^*\right|^2}$$

$$\left|T^{+-}\right|^2 = BR(B_d^0 \to \pi^+\pi^-)\times 10^6 = BR(D^0 \to \pi^+\pi^-)\times 10^6 \left(\frac{\tau_{B_d^0}}{\tau_{D^0}}\right) R_{P=0}$$

This gives $\boxed{|T^{+-}| = 1.3}$

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- ## Extract $P_s$ from the $B_s \to K^+K^-$ decay

Up to Cabibbo suppressed terms $BR(B_s^0 \to K^+K^-) \sim \left|P_s\right|^2 \overset{SU(3)}{=} \left|P\right|^2$

$\boxed{|P_s| = 1.2}$ → $\boxed{|P| \leq 2.5}$ → $\boxed{|T^{+-}| \approx 1}$

assuming that SU(3) breaking
effects are not larger than 100%

# Using the available information (priors)

In previous UTfit analyses: $|T^{ij}| \leq 10$, $|P| \leq 10$, arbitrary phases

We were not enough Bayesian !

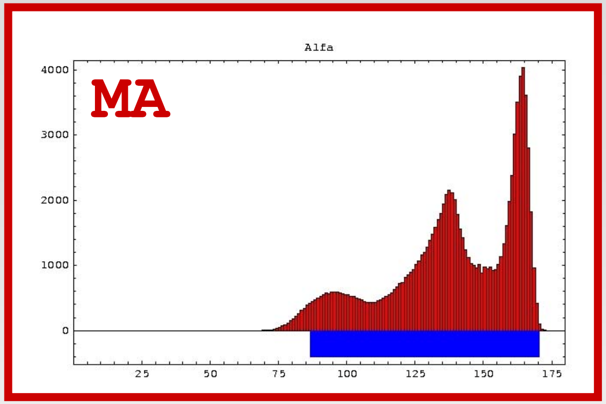
Now: $|T^{ij}| \leq 4$, $|P| \leq 2.5$, arbitrary phases

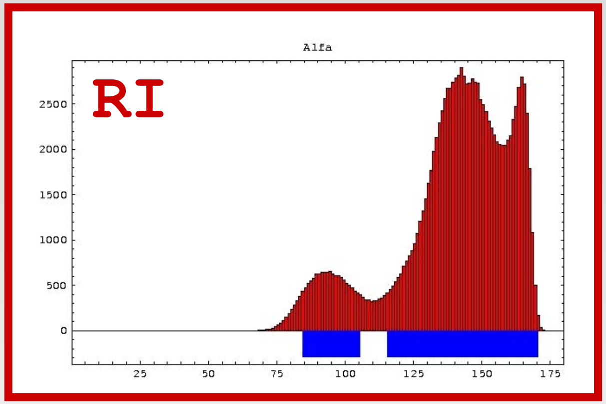
SU(3) breaking $\leq$ 100%

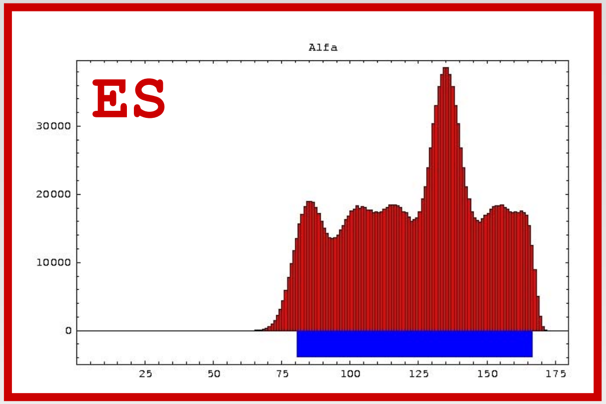
[ $|T^{ij}| \leq 10$ produces the same results]

Present HFAG averages:

| $B_{\pi\pi}^{+0} \times 10^6$ | $B_{\pi\pi}^{+-} \times 10^6$ | $B_{\pi\pi}^{00} \times 10^6$ | $C_{\pi\pi}^{+-}$ | $S_{\pi\pi}^{+-}$ | $C_{\pi\pi}^{00}$ |
|---|---|---|---|---|---|
| $5.7 \pm 0.4$ | $5.2 \pm 0.2$ | $1.31 \pm 0.21$ | $-0.39 \pm 0.07$ | $-0.59 \pm 0.09$ | $-0.37 \pm 0.32$ |







$$\alpha \in \left[ 80^0, 170^0 \right] \quad \text{at } 95\%$$

**1)** **The information on the matrix elements helps in eliminating some of the eight solutions, including the pathological solution at $\alpha \sim 0$**

**Note:**

**$\alpha < 2^o$ → $|T+-| > 30$ → $|P| \sim 30$ → SU(3) breaking~3000% !!**

**$m_s/m_d \sim 10$ → SU(2) breaking~300% (Gronau-London??)**

**2)** **There is substantially no difference among the results obtained with different parameterizations (same 95% probability region and similar 68% one)**

# Conclusions:

- Contrary to the claim of hep-ph/0607246, the differences between the frequentistic and Bayesian results are NOT due to the difference in the two approaches, but to the different physical assumptions on the hadronic amplitudes

- The present information on the hadronic matrix elements already allows a substantial reduction of the eightfold ambiguity in the determination of $\alpha$, in particular by eliminating the solution at $\alpha \sim 0$ that corresponds to unphysical values of the amplitudes

- The information on the hadronic matrix elements substantially eliminates in the Bayesian approach the dependence of the results on the a priori PDF the hadronic amplitudes which was noticed in hep-ph/0607246