Introduction to ϕ_2 from charmless decays

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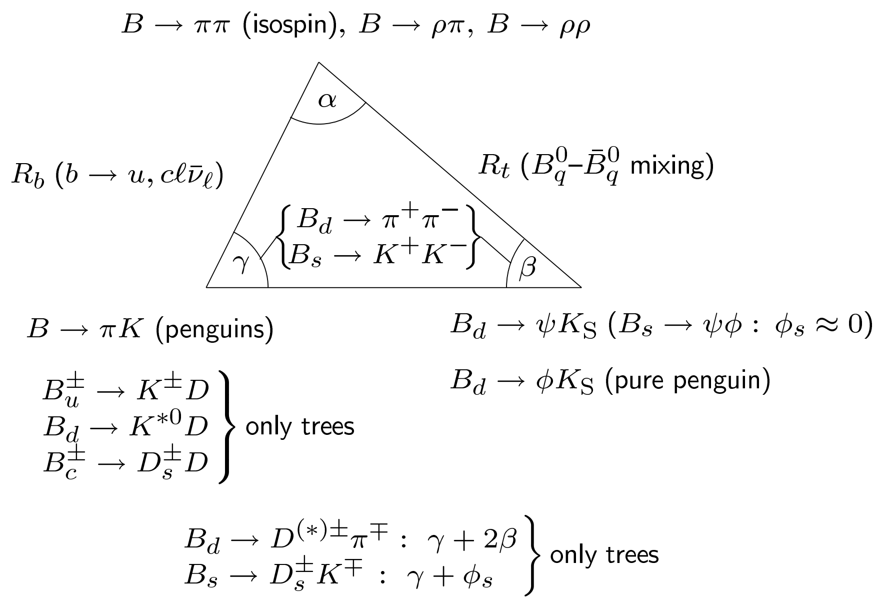
Contents

Contents

R. Fleischer, hep-ph/0608010



$$\phi_2 \equiv \alpha \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*|$$

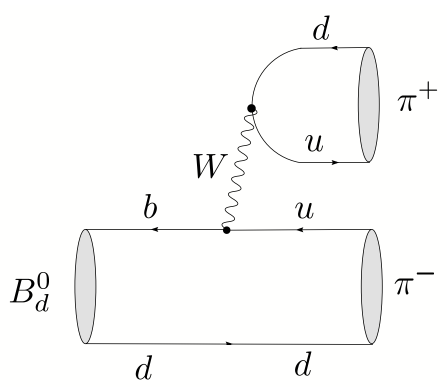
SM prediction (indirect measurements of α):

-combining measurements of $|V_{us}|, |V_{ud}|, |V_{ub}|, |V_{cb}|$, CPV from $K^0 - \overline{K}^0$, $B_{d,s} - \overline{B}_{d,s}$ mixing and $\sin 2\beta$:

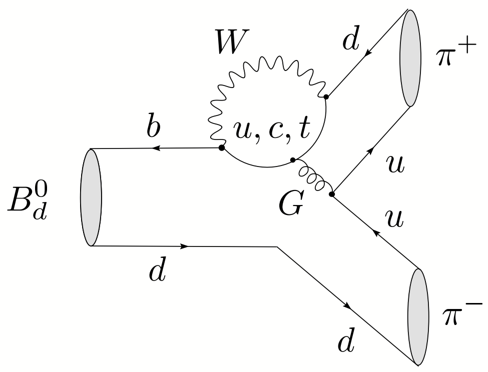
$$\alpha = (98.2 \pm 7.7)^\circ \qquad \text{(UTfit)} \qquad\qquad \alpha = (100.0^{+4.5}_{-7.3})^\circ \qquad \text{(CKMfitter)}$$

M.Gronau, D.London, Phys.Rev.Lett. 65, 3381 (1990)

in general there are two amplitudes with different WEAK phases:



(a) tree diagram



(b) penguin diagrams

CP-ASYMMETRY for tagged B^0 ($\overline{B}^0$):

$$a_{CP}(t) = \frac{\Gamma[\overline{B}(t) \to \pi^+\pi^-] - \Gamma[B^0(t) \to \pi^+\pi^-]}{\Gamma[\overline{B}(t) \to \pi^+\pi^-] + \Gamma[B^0(t) \to \pi^+\pi^-]} = S_{+-} \sin(\Delta m\, t) - C_{+-} \cos(\Delta m\, t)$$

$$= a^{\pm}_{\text{mixed}} \sin(\Delta m\, t) - a^{\pm}_{\text{dir}} \cos(\Delta m\, t)$$

$$S_{+-} \equiv a^{\pm}_{\text{mixed}} = \frac{2\text{Im}\lambda}{1+|\lambda|^2} \qquad C_{+-} \equiv a^{\pm}_{\text{dir}} = \frac{1-|\lambda|^2}{1+|\lambda|^2}$$

λ is related to the B^0-$\overline{B}^0$ mixing through p/q and to the ratio of the amplitudes:

$$\lambda = \frac{q}{p}\frac{\overline{A}}{A}$$

NO PENGUIN AMPLITUDES: $S_{+-} = \sin 2\alpha \qquad C_{+-} = 0$

EXPERIMENT: $S_{+-} = -0.59 \pm 0.09 \qquad C_{+-} = -0.39 \pm 0.07$

this can be parametrized as

$$\begin{aligned} \sin(2\alpha_{\text{eff}}) &= S_{+-}/\sqrt{1-C^2_{+-}} \\ \alpha_{\text{eff}} &= \alpha - \theta = \alpha - \arg(A_{+-}/\overline{A}_{+-}) \end{aligned}$$

α from $a_{CP}(t) \to$ requires the knowledge P/T which includes several nonperturbative amplitudes:

$$\begin{aligned} \overline{A}(\overline{B} \to \pi^+\pi^-) &= -\lambda_u(T+P_u) - \lambda_c P_c - \lambda_t P_t = e^{-i\gamma}T_{\pi\pi} + e^{i\phi}P_{\pi\pi} \\ \sqrt{2}\overline{A}(\overline{B} \to \pi^0\pi^0) &= \lambda_u(-C+P_u) + \lambda_c P_c + \lambda_t P_t = e^{-i\gamma}C_{\pi\pi} - e^{i\phi}P_{\pi\pi} = e^{-i\gamma}(T_{-0} - T_{\pi\pi}) - e^{i\phi}P_{\pi\pi} \\ \sqrt{2}\overline{A}(B^- \to \pi^-\pi^0) &= -\lambda_u(T+C) = e^{-i\gamma}T_{-0} \end{aligned}$$

HOW TO DETERMINE α ? $\to$ BY USING FLAVOUR (ISOSPIN) SYMMETRIES !

see talk by J. Zupan at this workshop

HOW TO DETERMINE α ? $\to$ BY USING FLAVOUR (ISOSPIN) SYMMETRIES !

see talk by J. Zupan at this workshop

ISOSPIN FOR $\pi\pi$ SYSTEM:

- u-d symmetry - broken by the quark masses - neglected
- EW penguins are neglected
- $\pi\pi$ system - $I = 0, 2$ due to the Bose statistics

weak Hamiltonian contains tree-level operators $\mathcal{O}_{1,2}$ ($\Delta I = 1/2, 3/2$), gluonic penguins $\mathcal{O}_{3,..,6}$ ($\Delta I = 1/2$) and electroweak penguins $\mathcal{O}_{7,..,10}$ ($\Delta I = 1/2, 3/2$ - neglected):

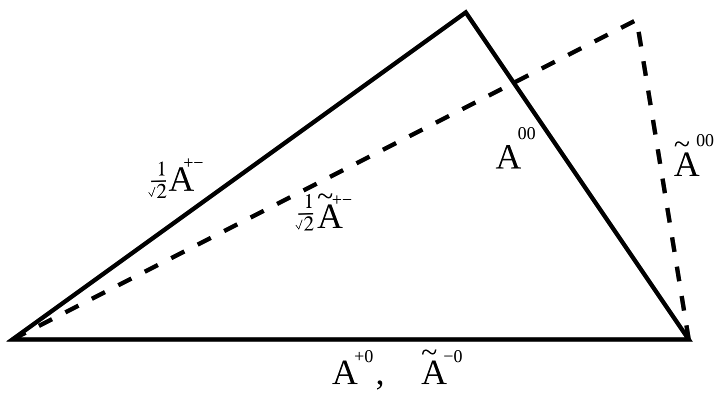
$$\mathcal{H}_{weak} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pd}^* \left\{ C_1(\mu)\mathcal{O}_1^p + C_2(\mu)\mathcal{O}_2^p + \sum_{i=3,..,10} C_i(\mu)\mathcal{O}_i \right\}$$

$$\begin{aligned}
\frac{1}{\sqrt{2}}\overline{A}(\overline{B} \to \pi^+\pi^-) &= A_{I=2} - A_{I=0} \\
\overline{A}(\overline{B} \to \pi^0\pi^0) &= 2A_{I=2} + A_{I=0} \\
\overline{A}(B^- \to \pi^-\pi^0) &= 3A_{I=2}
\end{aligned}$$

if there are no EW penguins, $A_{I=2}$ receives contributions only from tree operators

ISOSPIN RELATION BETWEEN $\overline{A}$ AMPLITUDES (and their CP-conjugate amplitudes):

$$\frac{1}{\sqrt{2}}\overline{A}(\overline{B} \to \pi^+\pi^-) + \overline{A}(\overline{B} \to \pi^0\pi^0) = \overline{A}(B^- \to \pi^-\pi^0)$$



$$\tilde{A} = e^{2i\gamma}\overline{A}$$

mixed CP asymmetries:

$$S_{+-} \sim \mathrm{Im}\lambda_{+-} = \mathrm{Im}\left[e^{2i\alpha}\left[\frac{1-\overline{A}_0/\overline{A}_2}{1-A_0/A_2}\right]\right] == \mathrm{Im}\left[e^{2i\alpha}\left[\frac{1-|\overline{A}_0/\overline{A}_2|e^{\pm i\overline{\theta}}}{1-|A_0/A_2|e^{\pm i\theta}}\right]\right]$$

$$S_{00} \sim \mathrm{Im}\lambda_{00} = \mathrm{Im}\left[e^{2i\alpha}\left[\frac{1+1/2|\overline{A}_0/\overline{A}_2|e^{\pm i\overline{\theta}}}{1+1/2|A_0/A_2|e^{\pm i\theta}}\right]\right]$$

▷ we have eightfold discrete ambiguity in the determination of α

CONCLUSIONS:

- the theoretical uncertainty due to the penguin diagrams (gluonic) are eliminated - angle α can be determined by tagged measurement of all $\pi\pi$ modes
- in the absence of the full information (in particular present measurement of a_{dir}^{00} does not provide significant bound on α), one can set several bounds on θ angle ($\alpha_{\rm eff} = \alpha - \theta$) by predicting amplitudes T, P, C by using models and some assumptions:

  M.Gronau, D. London, N. Sinha, R. Sinha, Phys.Lett.B 514, 315 (2001)
  D. Pirjol, Phys.Rev.D 60, 054020 (1999); R. Fleischer, Phys.Lett.B 459, 306 (1999)
  G. Buchalla, A.S. Safir, Phys. Rev. Lett. 93, 021801 (2004)

  Y. Grossman, A. Höcker, Z. Ligeti, D. Pirjol, Phys.Rev.D 72, 094033 (2005)

see talk by A. Bevan at this workshop

A.E.Snayder, H.R.Quinn, Phys.Rev.D 48, 2139(1993); H.R.Quinn, J.P.Silva, Phys.Rev.D 62, 054002 (2000)

ISOSPIN FOR $\rho\pi$ SYSTEM:

- much more complicated since $\rho^{\pm}\pi^{\mp}$ final states are not CP eigenstates
- $\rho\pi$ system has $I = 0, 1, 2$ - four independent isospin amplitudes
- pentagon isospin analysis or a Dalitz plot analysis of $\pi^+\pi^-\pi^0$ final decay state

$$
\begin{aligned}
A(B^+ \to \rho^+\pi^0) &= \frac{1}{2}\sqrt{\frac{3}{2}}A_{2,3/2} - \frac{1}{2}\sqrt{\frac{1}{2}}A_{1,3/2} + \sqrt{\frac{1}{2}}A_{1,1/2}\left[-\sqrt{\frac{1}{6}}A_{2,5/2}\right] \\
A(B^+ \to \rho^0\pi^+) &= \frac{1}{2}\sqrt{\frac{3}{2}}A_{2,3/2} + \frac{1}{2}\sqrt{\frac{1}{2}}A_{1,3/2} - \sqrt{\frac{1}{2}}A_{1,1/2}\left[-\sqrt{\frac{1}{6}}A_{2,5/2}\right] \\
A(B^0 \to \rho^+\pi^-) &= \frac{1}{2}\sqrt{\frac{1}{3}}A_{2,3/2} + \frac{1}{2}A_{1,3/2} + \frac{1}{2}A_{1,1/2} + \sqrt{\frac{1}{6}}A_{0,1/2}\left[+\frac{1}{2}\sqrt{\frac{1}{3}}A_{2,5/2}\right] \\
A(B^0 \to \rho^-\pi^+) &= \frac{1}{2}\sqrt{\frac{1}{3}}A_{2,3/2} - \frac{1}{2}A_{1,3/2} - \frac{1}{2}A_{1,1/2} + \sqrt{\frac{1}{6}}A_{0,1/2}\left[+\frac{1}{2}\sqrt{\frac{1}{3}}A_{2,5/2}\right] \\
A(B^0 \to \rho^0\pi^0) &= \sqrt{\frac{1}{3}}A_{2,3/2} - \sqrt{\frac{1}{6}}A_{0,1/2}\left[+\sqrt{\frac{1}{3}}A_{2,5/2}\right].
\end{aligned}
$$

- tree level contributions: $A_{2,3/2}$ and $A_{1,3/2}$
- other pieces: tree level + penguins (gluonic); EW penguins - neglected
- $\Delta I = 5/2$ from electromagnetic rescattering effects: $A_{2,5/2}$ - neglected

instead of triangular isospin relations as in $B \to \pi\pi$ here we have PENTAGON in the complex plane (and similarly for CP-conjugate amplitudes):

$$\sqrt{2}\left(A(B^+ \to \rho^+\pi^0) + A(B^+ \to \rho^0\pi^+)\right) = A(B^0 \to \rho^+\pi^-) + A(B^0 \to \rho^-\pi^+) + 2A(B^0 \to \rho^0\pi^0)$$

- time dependent Dalitz analysis of $B^0 \to \rho\pi \to \pi^+\pi^-\pi^0$ removes penguin contributions and extract α (no discrete ambiguities for 2α)
- there are two different ways how to do time dependent Dalitz analysis:

- Quasi-2-body approach - uses quasi-two body representations of $B^0 \to (\pi^\pm\pi^0)\pi^\mp$ decays corresponding to distinct bounds in the 3-pion Dalitz plot in the vicinity of the ρ resonances
- full time-dependent Dalitz analysis of $B^0 \to \pi^\pm\pi^\mp\pi^0$, accounting for the interference between intersecting ρ resonance bands and other resonances

BaBar obtained (hep-ex/0408099): $\alpha = (113^{+27}_{-17} \pm 6)^\circ$ see talk by G.Cavoto at this workshop

Belle obtained (hep-ex/0609003): $\alpha = (83^{+12}_{-23})^\circ$ see talk by C-C. Wang at this workshop

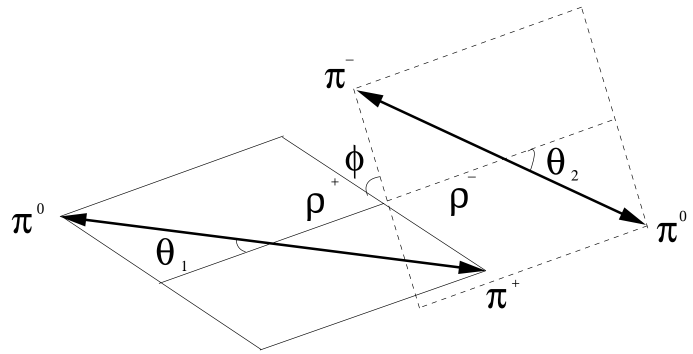
using BaBar analysis a model dependent constraint can be set (M.Gronau, J.Zupan, Phys.Rev.D 70, 074031 (2004)):

$$\alpha = (93 \pm 4_{\rm exp} \pm 6_{\rm th})^\circ$$

G.Kramer, W.Palmer, Phys.Rev.D 45, 193 (1992)

ISOSPIN FOR $\rho\rho$ SYSTEM:

- $\rho\pi$ system has $I = 0, 2$ states only $\to$ one can do similar isospin analysis as for $\pi\pi$ system
- S=0 particle decays into two S=1 particles $\to$ three helicity states $H = 0$ (longitudinal L), ± 1 (transversal T)
- $H = 0$ (L state) is CP-even state and dominates in the decay



$$\frac{d^2\Gamma}{\Gamma\, d\cos\theta_1\, d\cos\theta_2} = \frac{9}{4}\left[f_L \cos^2\theta_1 \cos^2\theta_2 + \frac{1}{4}(1 - f_L)\sin^2\theta_1 \sin^2\theta_2\right]$$

f_L = fraction of the $H = 0$ state in the decay

- by measuring BR's and a_{CP}'s of $B \to \rho^0\rho^0$ and $B \to \rho^\pm\rho^\mp$ one can determine α
- there are some corrections:

- due to the final width of the ρ meson ($I = 1$ contributions), but there are expected to be of order of 4%

- isospin violation

see talk by A. Somov at this workshop

if we include EW penguins ($\Delta I = 1/2, 3/2$):

$$\begin{aligned} \overline{A}(\overline{B} \to \pi^+\pi^-) &= e^{-i\gamma} T_{\pi\pi} + e^{i\phi} P_{\pi\pi}\,, \\ \sqrt{2}\overline{A}(\overline{B} \to \pi^0\pi^0) &= e^{-i\gamma}(T_{-0} - T_{\pi\pi}) - e^{i\phi} P_{\pi\pi}\,, \\ \sqrt{2}\overline{A}(B^- \to \pi^-\pi^0) &= e^{-i\gamma} T_{-0} + e^{i\beta} P_{EW}. \end{aligned}$$

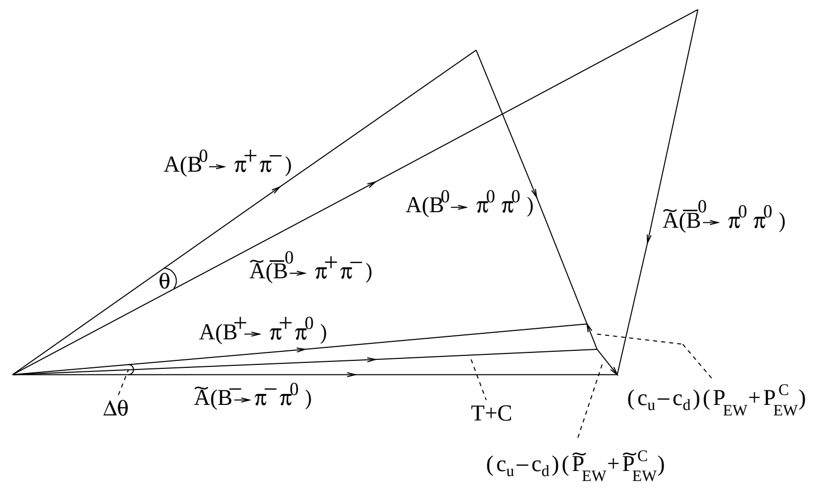
SU(2) triangular relations still hold true:

$$\begin{aligned} \frac{1}{\sqrt{2}}\overline{A}(\overline{B} \to \pi^+\pi^-) + \overline{A}(\overline{B} \to \pi^0\pi^0) = \overline{A}(B^- \to \pi^-\pi^0) \\ \frac{1}{\sqrt{2}}A(B \to \pi^+\pi^-) + A(B \to \pi^0\pi^0) = A(B^+ \to \pi^+\pi^0) \end{aligned}$$

but now $A(B^+ \to \pi^+\pi^0) \neq \tilde{A}(B^- \to \pi^-\pi^0)$

• there are model-dependent estimates of P_{EW}/T_{-0} giving SU(2) breaking corrections to α determination:

Deshpande, He, hep-ph/9408404; Gronau et al, hep-ph/9504327; Fleischer, hep-ph/9509204; Charles, hep-ph/9806468; Neubert, Rosner,hep-ph/9808493; Buras, Fleischer,Eur.Phys.J C 11, 93 (1999); Gronau, Pirjol, Yan, hep-ph/9810482; Beneke et al, hep-ph/0104110; etc....



• the correction due to the EW penguins is given by the ratio P_{EW}/T_{-0}, i.e. by the angle $\Delta\theta$; bounds can be set:

$$\cos\Delta\theta \geqslant 1 - 2\left|\frac{P_{EW}}{T_{-0}}\right|^2 \qquad \text{or} \qquad |\Delta\theta| \leqslant 2\left|\frac{P_{EW}}{T_{-0}}\right|$$

Gronau et al., Phys.Rev.D 52, 6374 (1995)

▷ effect of EW penguins on α is $<$ few degrees

S. Gardner, U. Meissner, Phys.Rev.D 65, 094004 (2002); J.Charles et al., hep-ph/0406184

it comes

i) from the insertion of the $d-u$ mass difference $\Delta I = 1$ operator
ii) by electromagnetic corrections ($O(\alpha)$ corrections)

without any assumptions on the operators one can write:

$$\frac{1}{\sqrt{2}}\overline{A}(\overline{B} \to \pi^+\pi^-) = \left(A_{I=2} - A_{I=2}^{5/2}\right) - A_{I=0}$$
$$\overline{A}(\overline{B} \to \pi^0\pi^0) = 2\left(A_{I=2} - A_{I=2}^{5/2}\right) + A_{I=0}$$
$$\overline{A}(B^- \to \pi^-\pi^0) = 3A_{I=2} + \sqrt{6}A_{I=2}^{5/2}$$

the isospin triangular relation is broken:

$$\frac{1}{\sqrt{2}}\overline{A}(\overline{B} \to \pi^+\pi^-) + \overline{A}(\overline{B} \to \pi^0\pi^0) - \overline{A}(B^- \to \pi^-\pi^0) \propto A_{I=2}^{5/2}$$

▷ effect of $\Delta I = 5/2$ corrections to α is $<$ then the current experimental precision

$\pi^0 - \eta - \eta'$ MIXING IN $B \to \pi\pi$; $\rho^0 - \omega$ MIXING IN $B \to \rho\pi$ AND $B \to \rho\rho$

S. Gardner, Phys.Rev.D 59, 077502 (1999); M. Gronau, J. Zupan, hep-ph/0502139

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S. Gardner, Phys.Rev.D 59, 077502 (1999); M. Gronau, J. Zupan, hep-ph/0502139

• $\pi^0 - \eta - \eta'$: if the $d - u$ mass difference is introduced, π^0 becomes contaminated by the isospin singlet components:

$$|\pi^0\rangle = |\pi_3\rangle + \epsilon|\eta\rangle + \epsilon'|\eta'\rangle$$

with $\epsilon = 0.017 \pm 0.003$, $\epsilon' = 0.004 \pm 0.001$

▷ effect of $\pi^0 - \eta\eta'$ mixing in $B \to \pi\pi$ decays to α determination is $< 1.4^\circ$

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▷ effect of $\pi^0 - \eta\eta'$ mixing in $B \to \pi\pi$ decays to α determination is $< 1.4^\circ$

• $\rho^0 - \omega$: ρ^0 and ω are mixtures of an isovector ρ_I and an isoscalar ω_I:

$$|\rho^0\rangle = |\rho_I\rangle - \epsilon_1|\omega_I\rangle \ , \qquad |\omega\rangle = |\omega_I\rangle + \epsilon_2|\rho_I\rangle$$

with $\epsilon_1, 2 < 0.01$.
▷ effect of $\rho^0 - \omega$ mixing in $B \to \rho\pi$ and $B \to \rho\rho$ decays to α determination is expected to be small

▷ ISOSPIN BREAKING EFFECTS IN $B \to \rho\gamma$, $B \to \omega\gamma$

A.Ali, A. Parkhomenko, hep-ph/0610149

$$\Delta \equiv \frac{1}{2} \frac{\Gamma(B^+ \to \rho^+\gamma)}{\Gamma(B^0 \to \rho^0\gamma)} - 1 = -0.36 \pm 0.27 \quad (\text{BaBar, hep-ex/0607099})$$

• calculated at the leading order:

$$\Delta_{LO} = -2\epsilon_A |\lambda_u| \cos\alpha + \epsilon_A |\lambda_u|^2$$

where ϵ_A parametrizes interference between penguin and annihilation contributions which can be calculated in some model

• since: $\lambda_u = \frac{V_{ub}V^*_{ud}}{V_{tb}V^*_{td}} = \left| -\frac{V_{ub}V^*_{ud}}{V_{tb}V^*_{td}} \right| e^{i\alpha}$ one can in principle get an independent determination of α

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▷ $B^0 \to a_1^\pm(1260)\pi^\mp$ DECAYS AND $SU(3)_f$ SYMMETRY

R. Aleksan et al, Nucl.Phys.B 361, 141 (1991); M. Gronau, J. Zupan, hep-ph/0512148

• recently $B^0 \to a_1^\pm\pi^\mp$ was observed

• the situation is similar to $B \to \rho\pi$, so one can apply isospin analysis and use SU(3) relations corrections from penguin amplitudes are bounded by relating $B^0 \to a_1^\pm\pi^\mp$ with corresponding $\Delta S = 1$ decays, $B \to a_1 K$ and $B \to a_1 K_{1A}$ (K_{1A} is admixture of the $K_1(1270)$ and $K_1(1400)$ resonances)

▷ $B^0 \to \rho\rho$ AND $B^+ \to K^{*0}\rho^+$ DECAYS WITH $SU(3)_f$ SYMMETRY

M. Beneke, M. Gronau, J. Rohrer, M. Spranger, hep-ph/0604005

• using of $SU(3)_f$ relations to extract penguin amplitudes and QCD factorization as a theoretical model they obtain:

$$\alpha = \left[91.2^{+9.1}_{-6.6}(\mathrm{exp})^{+1.2}_{-3.9}(\mathrm{th})\right]^\circ$$

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▷ $B^0 \to K^0\overline{K}^0$

A. Datta, M. Imbeault, D. London, J. Matias, hep-ph/0611280

• they set a relation for extracting α from $B^0 \to K^0\overline{K}^0$ decay

$$2\sin^2\alpha = \frac{\kappa\mathcal{B}}{|V_{ub}^* V_{ud}|^2 |T-P|^2}$$

• with present data no bound on α can be set

▷ $B^0 \to \rho\rho$ AND $B^+ \to K^{*0}\rho^+$ DECAYS WITH $SU(3)_f$ SYMMETRY

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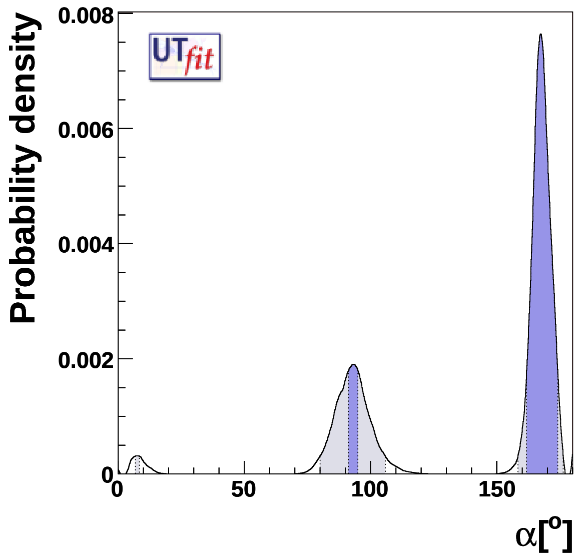
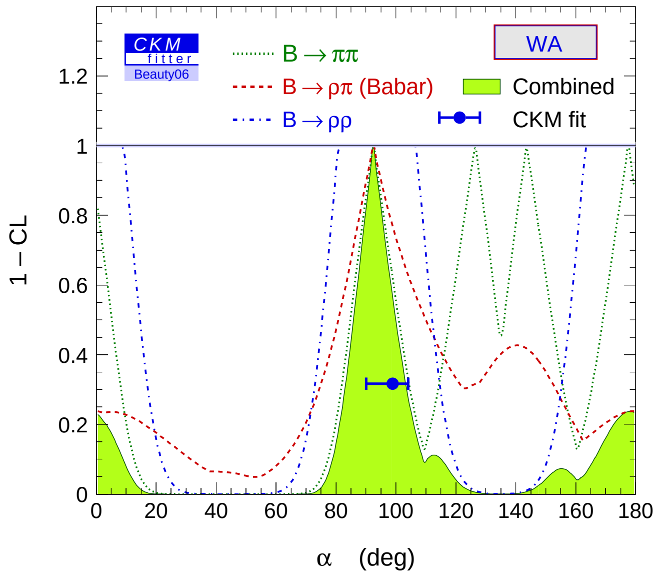
▷ AND FROM OTHER MODES ...

see talk by F. Palombo at this workshop

see talk by V. Lubicz at this workshop



see talk by S. T'Jampens at this workshop



indirect measurements of α:

$\alpha = (98.2 \pm 7.7)^\circ$ (UTfit) $\alpha = (100.0^{+4.5}_{-7.3})^\circ$ (CKMfitter)

direct measurements of α:

$\alpha = (92 \pm 7)^\circ$ (UTfit) $\alpha = (92.6^{+10.7}_{-9.3})^\circ$ (CKMfitter)

theoretical model dependent precisions:

precision from $B \to \pi\pi = 13^\circ$

precision from $B \to \rho\pi = 15.5^\circ$

precision from $B \to \rho\rho = 9^\circ$