Inclusive $\bar{B} \to X_s\gamma$: total width and spectrum

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- Computing the total $\bar{B} \to X_s\gamma$ BF in the on-shell scheme
- $\bar{B} \to X_s\gamma$ spectrum by Dressed Gluon Exponentiation
- Results and uncertainty estimates for BF and moments as a function of the cut $E_\gamma > E_0$

References:

Andersen & Gardi, “Radiative B decay spectrum: DGE at NNLO”, [hep-ph/0609250].
Andersen & Gardi, “Taming the $\bar{B} \to X_s\gamma$ spectrum by dressed gluon exponentiation”,[hep-ph/0502159]
Gardi, “Inclusive distributions near kinematic thresholds” [hep-ph/0606080].

Computing the total $\bar{B} \to X_s\gamma$ width in the on-shell scheme

Consider the well–known example of the (charmless) semileptonic width

$$\Gamma\left(\bar{B} \longrightarrow X_u l\bar{\nu}\right) = \frac{G_F^2 |V_{\rm ub}|^2 m_b^5}{192\pi^3} \underbrace{\left[1 + s_1 \frac{\alpha_s(\mu)}{\pi} + s_2(\mu)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + \cdots\right]}_{G_u}.$$

- The series for G_u is badly divergent... — a renormalon at $u = 1/2$
- Exact cancellation of the leading $\mathcal{O}(\Lambda)$ ambiguity between G_u and the pole mass m_b^5
  Beneke et al., Bigi et al. 1994
- Usually one resorts to some mass scheme where there is no such renormalon, but which one to choose?
- There is an alternative: compute in the on-shell scheme by resumming the series for $m_b/m_{\overline{\rm MS}}$ and the series for G_u, in the same prescription.
- Advantages:
  - We know the large–order asymptotic behavior, so the resummation introduces very little uncertainty — better than an arbitrary renormalon–free mass scheme!
  - Running–coupling effects are resummed: renormalization–group invariance!

Computing the total $\bar{B} \to X_s\gamma$ width in the on-shell scheme

Consider now the $\bar{B} \to X_s\gamma$ width

$$\Gamma(\bar{B} \longrightarrow X_s\gamma) = \frac{\alpha_{\rm em} G_F^2}{32\pi^4} |V_{\rm tb}V_{\rm ts}^*|^2 \left(m_b^{\overline{\rm MS}}(m_b)\right)^2 m_b^3 \sum_{i,j,\, i\leq j} C_i(\mu)C_j(\mu)\, G_{ij}(E_0,\mu),$$

$$= \frac{\alpha_{\rm em} G_F^2}{32\pi^4} |V_{\rm tb}V_{\rm ts}^*|^2 \left(m_b^{\overline{\rm MS}}(m_b)\right)^2 m_b^3 \underbrace{\left[f_0(\mu) + f_1(\mu)\frac{\alpha_s(\mu)}{\pi} + f_2(\mu)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + \cdots\right]}_{F}$$

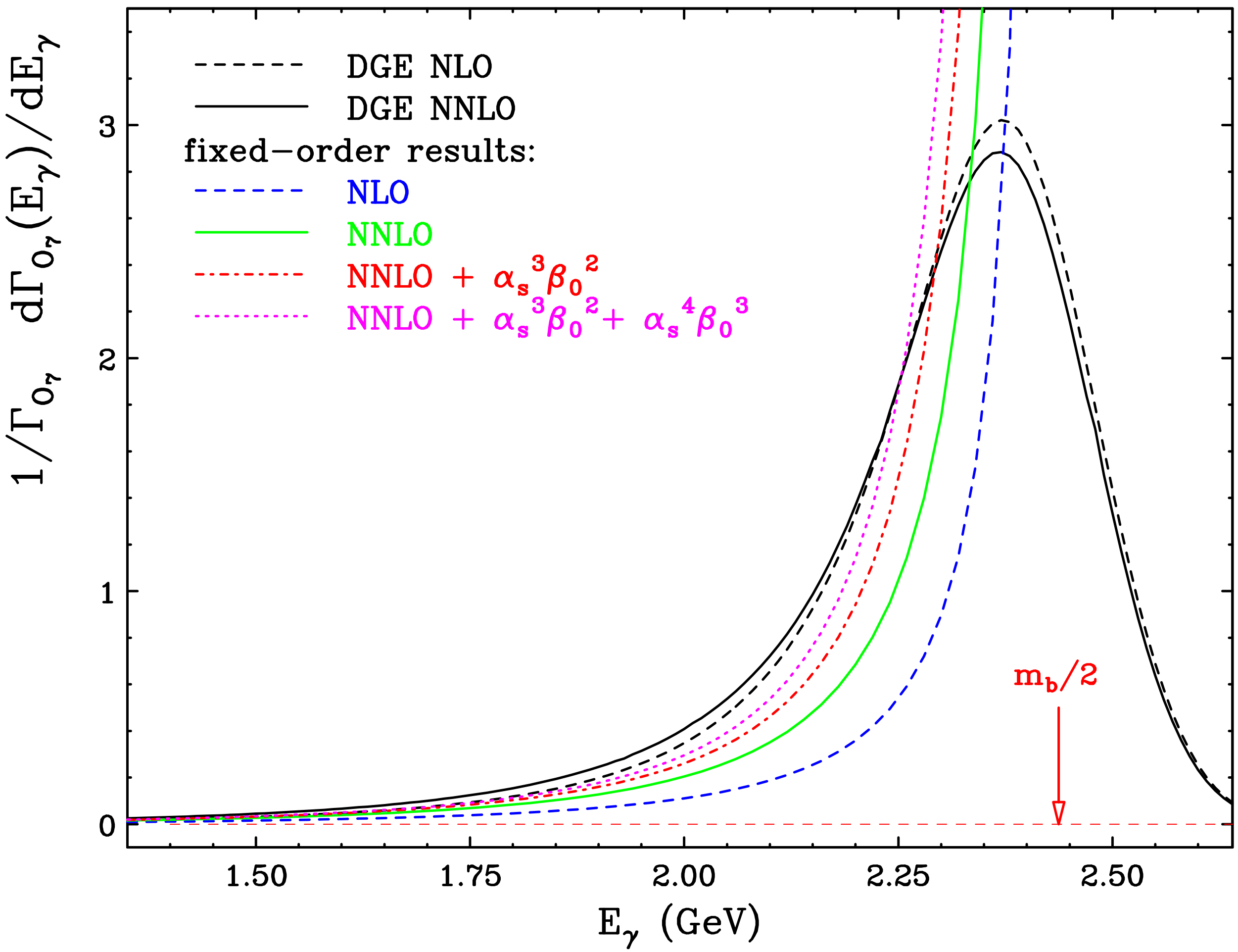
- Here scale dependence is associated also with renormalization of the operators; we wish to treat F at fixed order and use μ dependence as an uncertainty measure.
- Therefore, instead of summing the series for F, we use the semileptonic width:

$$\Gamma(\bar{B} \longrightarrow X_s\gamma) = \frac{\alpha_{\rm em} G_F^2}{32\pi^4} |V_{\rm tb}V_{\rm ts}^*|^2 \left(m_b^{\overline{\rm MS}}(m_b)\right)^2 \times \underbrace{m_b^3\, G_u^{3/5}}_{\text{Resummed: prescription independent}} \times \left[F/G_u^{3/5}\right]_{\text{Fixed Order}}$$

- At NLO we get: $\mathrm{BF}(E_\gamma > m_b/20) = 322 \cdot 10^{-6}$
  and with $\beta_0\alpha_s^2$ NNLO corrections to G_{ij}:
  $\mathrm{BF}(E_\gamma > m_b/20) = \left(357 \pm 40_{(\mu)} \pm 19_{(\mu_c)} \pm 21_{(\mathrm{param.})}\right) \cdot 10^{-6}$.

The $\bar{B} \to X_s\gamma$ spectrum: DGE *vs.* the fixed–order expansion

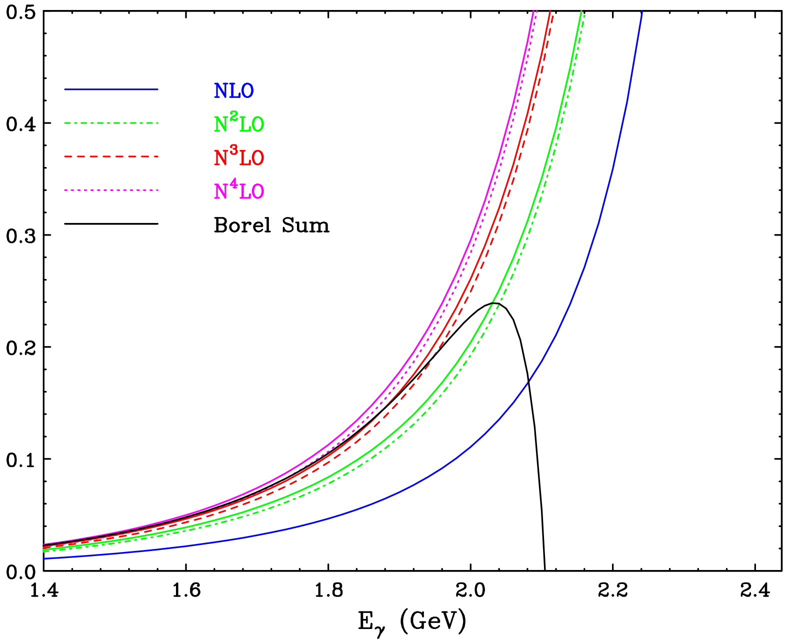
- Stability
- Support



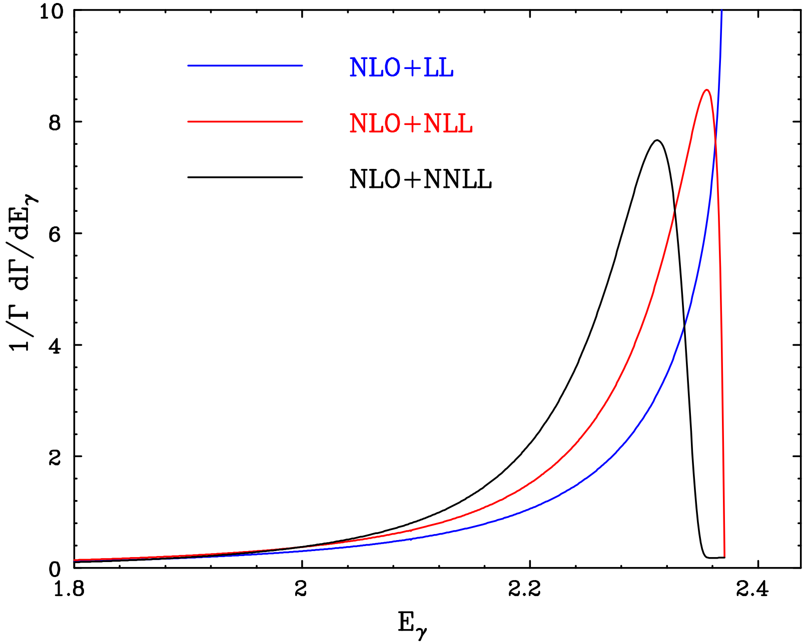
$\bar{B} \to X_s\gamma$ spectrum: Sudakov logs and running–coupling effects

- Sudakov logarithms are parametrically leading near the endpoint ($E_\gamma = m_b/2$)
- But over most of the range, running–coupling effects are the largest corrections!

Summation of running–coupling corrections in E_γ space (O_7):



Sudakov resummation (O_7) fixed logarithmic accuracy:



- When considered separately, neither Sudakov resummation nor renormalon resummation yield a viable description of the spectrum!

Dressed Gluon Exponentiation

Renormalon resummation:

summation of PT to power accuracy,

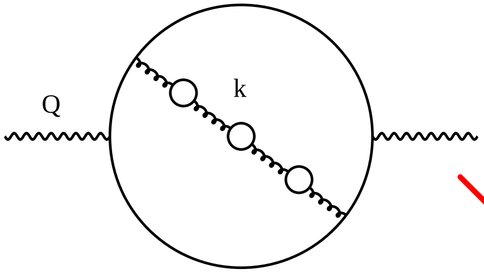
giving access to NP power corrections.

Renormalons $\equiv n \to \infty$ asymptotics,

running coupling $\Longrightarrow$ non-summable series

$\int_0 dk^2 \left[\alpha_s \beta_0 \ln(k^2/Q^2)\right]^n \Longrightarrow \left[\alpha_s \beta_0\right]^n n!$

dressing the gluon: $\alpha_s(\mu^2) \longrightarrow \alpha_s(k^2)$



Sudakov resummation:

corrections that become large near

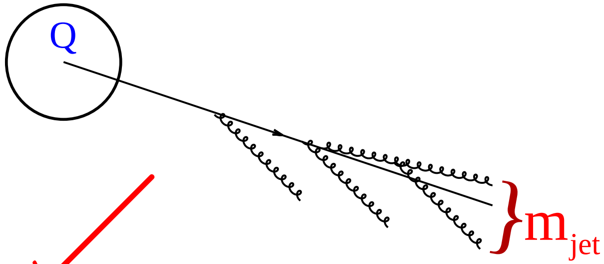
the exclusive phase–space boundary,

$m_{\rm jet} \to 0$

multiple soft and collinear emission

$\left[C_F \alpha_s \ln^2(m_{\rm jet}^2/Q^2)\right]^n$

exponentiation in moment space

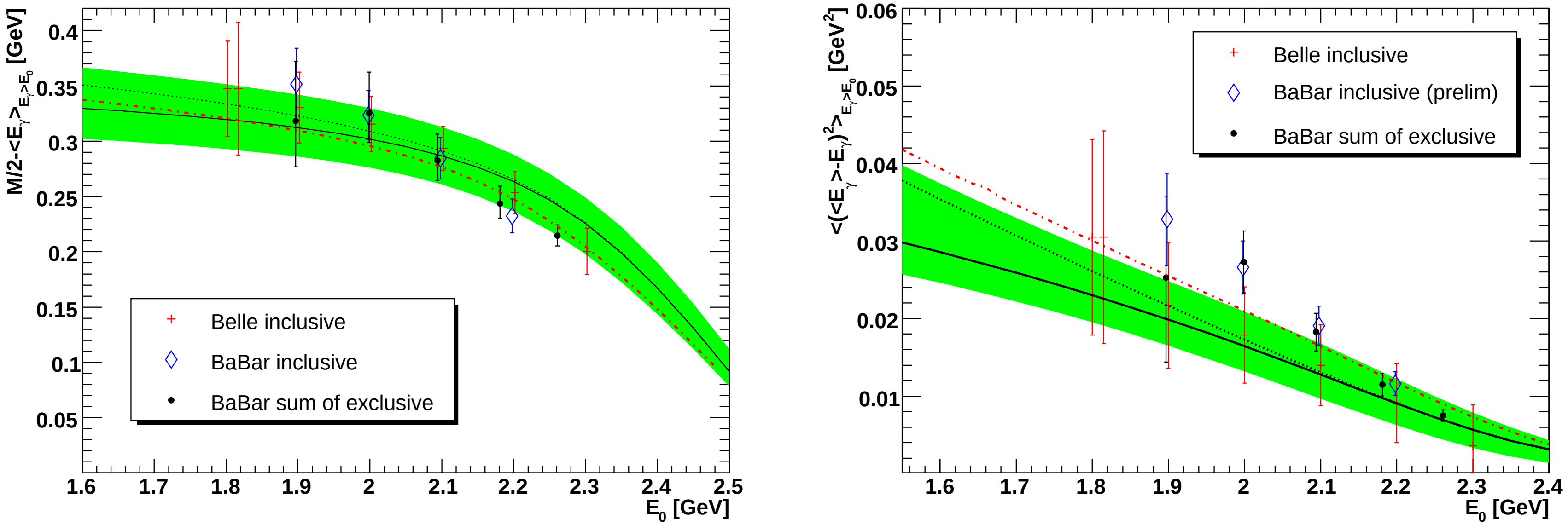


Dressed Gluon Exponentiation

E_γ moments as a function of the cut: DGE *vs.* data

$$\langle E_\gamma \rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \frac{d\Gamma(E_\gamma)}{dE_\gamma} E_\gamma$$

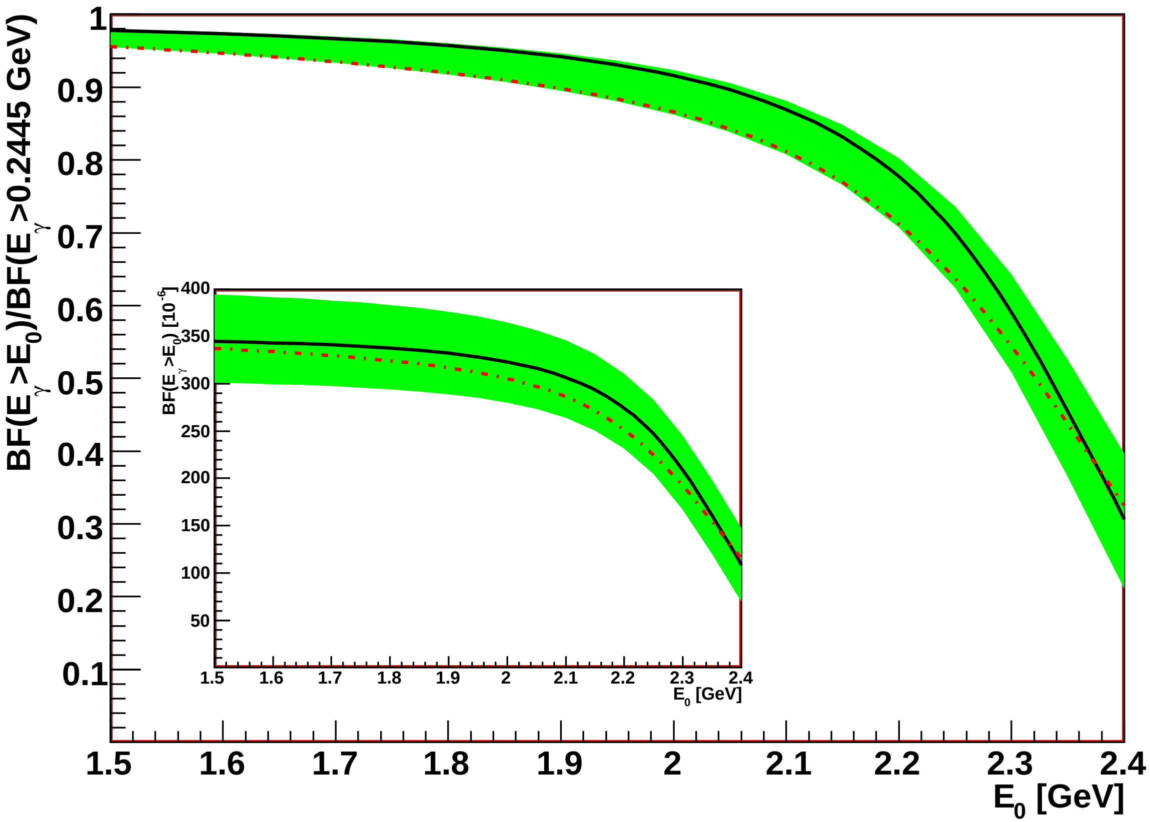
$$\left\langle \left(\langle E_\gamma \rangle_{E_\gamma > E_0} - E_\gamma \right)^n \right\rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \frac{d\Gamma(E_\gamma)}{dE_\gamma} \left(\langle E_\gamma \rangle_{E_\gamma > E_0} - E_\gamma \right)^n$$



- The uncertainty in $\langle E_\gamma \rangle$ is dominated by m_b for $E_\gamma < 2.2$ GeV and by power corrections $E_\gamma > 2.2$ GeV
- $E_\gamma > 1.8$ GeV: the uncertainty in the variance is dominated by power corrections

The $\bar{B} \to X_s\gamma$ BF and its cut–dependence

- The uncertainty in the BF cut–dependence is dominated by perturbative corrections!
  - Here only the G_{77} spectrum is matched to NNLO
  - Completing the NNLO calculation would help reducing the uncertainty.
- The cut–dependence uncertainty is smaller than the error on the total BF!



Andersen & Gardi

[hep-ph/0609250]

Conclusions

- New approach to evaluate the total BF using the on-shell scheme and the charmless semileptonic width
  - At NLO we get: $\mathrm{BF}(E_\gamma > m_b/20) = 322 \cdot 10^{-6}$
    and with $\beta_0 \alpha_s^2$ NNLO corrections:
    $\mathrm{BF}(E_\gamma > m_b/20) = \left(357 \ \pm 40_{(\mu)} \ \pm 19_{(\mu_c)} \ \pm 21_{(\mathrm{param.})}\right) \cdot 10^{-6}$.
- Significant progress in computing the spectrum.
- The BF cut dependence is still dominated by perturbative corrections:
  - For G_{77}, DGE (with NNLL accuracy) has now been matched to NNLO
  - But other matrix elements are still computed at NLO!
- The uncertainty related with the BF cut dependence is smaller than other uncertainties in the total BF.
- E_γ moments are more sensitive to m_b and to power corrections: possibility to measure these parameters!