New lattice results for B_K
from the RBC/UKQCD Collaborations

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RBC and UKQCD Collaborations

(Columbia University)

Calculations done using the QCDOC supercomputers

at RIKEN-BNL Research Center and the University of Edinburgh

Outline

The Kaon Bag Parameter

The bag parameter is needed to compare K and B physics in the unitarity triangle. The lattice provides the only first-principles method for determining B_K.

The Challenges

- Realistic sea-quark content with large volume
- Reliable renormalization factors for operators
- Control over operator mixing
- Multiple lattice spacings to find continuum limit

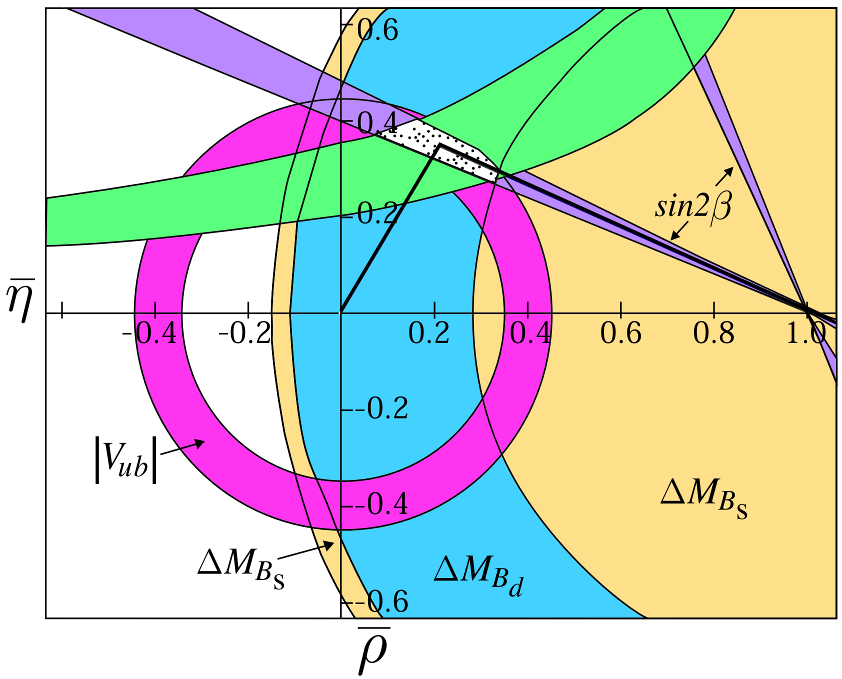
New Results

We present a lattice B_K measurement for 2+1 flavors, using domain-wall fermions, providing good chiral symmetry and nonperturbative renormalization.

The Kaon Bag Parameter

Important to CP Violation

Errors in the determination of ϵ are now dominated by uncertainty in the value of B_K.



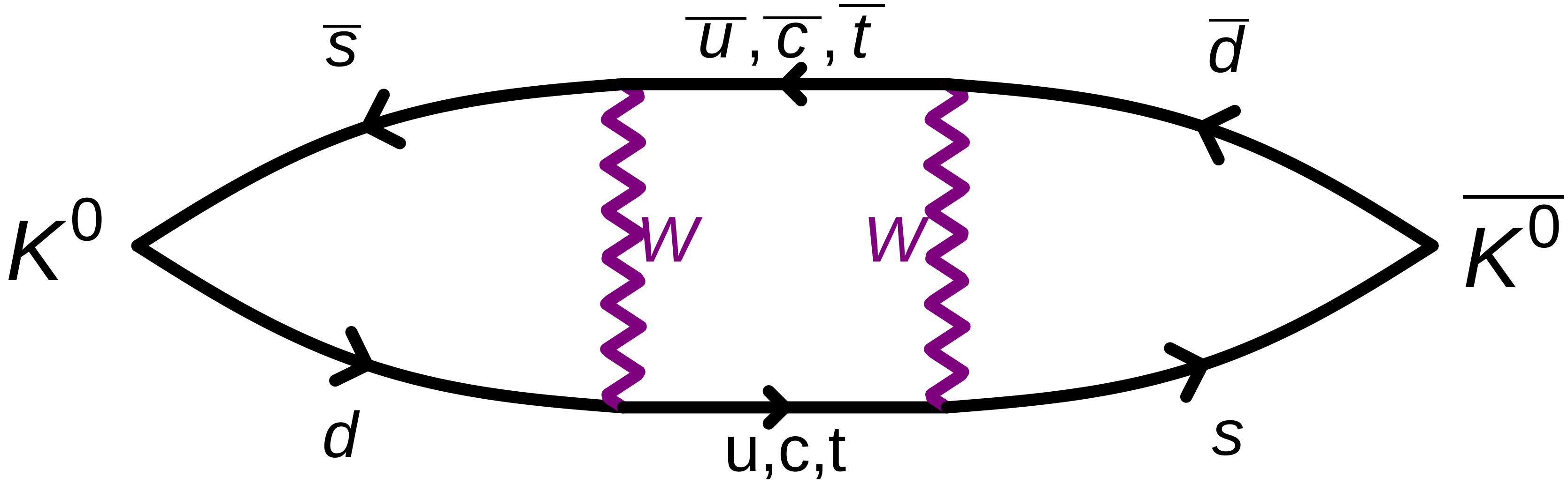
$$\epsilon = \hat{B}_K \, \mathrm{Im}\lambda_t \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} \times \{\mathrm{Re}\lambda_c \, [\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \mathrm{Re}\lambda_t \, \eta_2 S_0(x_t)\} \, e^{i\pi/4}$$

Constraint by ϵ denoted by green hyperbolic bands.

The Kaon Bag Parameter

Fundamental Diagram

B_K parametrizes the amount of mixing between neutral kaons due to weak interactions. At leading order in the Standard Model, we understand this mixing is due to the exchange of two W bosons:



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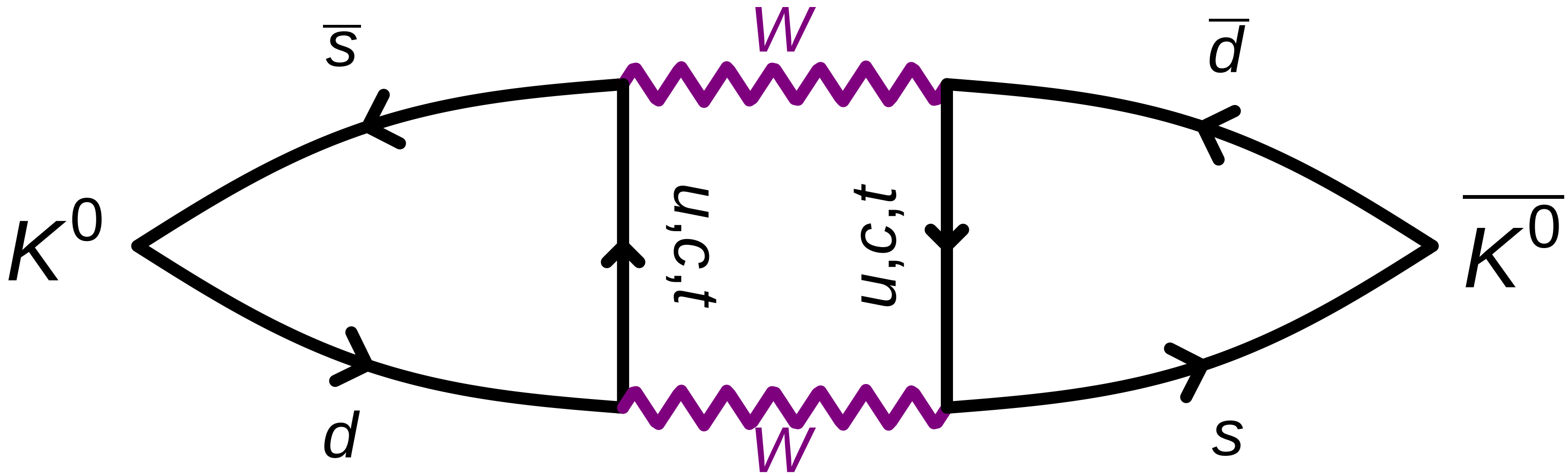
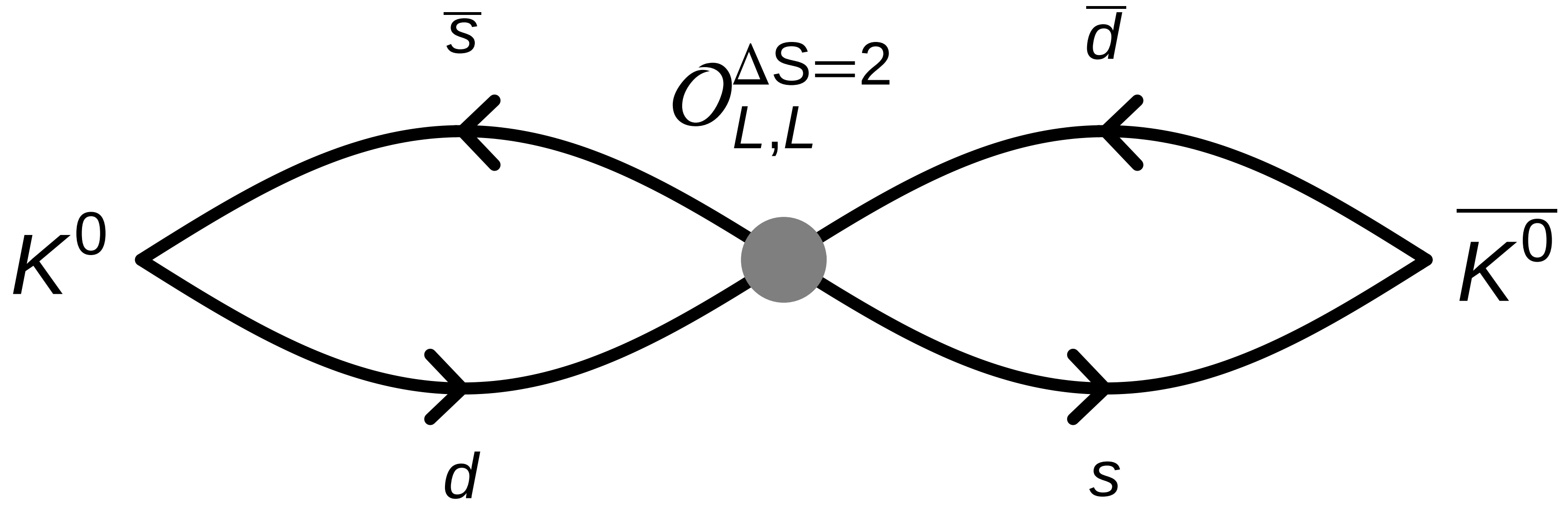


The Kaon Bag Parameter

Figure-Eight Diagram

After operator product expansion, we evaluate this QCD diagram. Operators with quantum numbers of the kaon are inserted at the initial and final times, creating a kaon and annihilating an anti-kaon respectively.



The Kaon Bag Parameter

Defi nition

The operator inserted at the center is the four-quark left-left operator that changes strangeness by two:

$$B_K = \frac{\langle \overline{K^0} | \mathcal{O}_{LL}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} f_K^2 M_K^2}$$

$$\mathcal{O}_{LL}^{\Delta S=2} = (\bar{s}d)_L (\bar{s}d)_L$$

$$\mathcal{O}(\mu) = \sum_i Z_i(\mu) \mathcal{O}_i^{\Delta S=2}$$

The Challenge

Operator Mixing

In the chiral limit, B_K contains only the operator of the form $VV + AA$:

$$\mathcal{O}^{\Delta S=2}_{VV+AA} = (\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d)$$

but if chiral symmetry is broken, operators with other chiral structures may enter. These operators are large and problematic.

$$\begin{aligned}
\mathcal{O}^{\Delta S=2}_{VV-AA} &= (\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) - (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d) \\
\mathcal{O}^{\Delta S=2}_{SS\pm PP} &= (\bar{s}d)(\bar{s}d) \pm (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) \\
\mathcal{O}^{\Delta S=2}_{TT} &= (\bar{s}\sigma_{\mu\nu} d)(\bar{s}\sigma_{\mu\nu} d)
\end{aligned}$$

The Solutions

Improved Staggered Fermions

Staggered fermions have large mixings, and the chiral perturbation theory to correct for taste-breaking is very challenging. This leads to very large systematic uncertainty.

Twisted-Mass QCD

Twisted mass protects certain operators from mixing with wrong chirality components. It suffers from lattice-artifact phases that are not yet well understood.

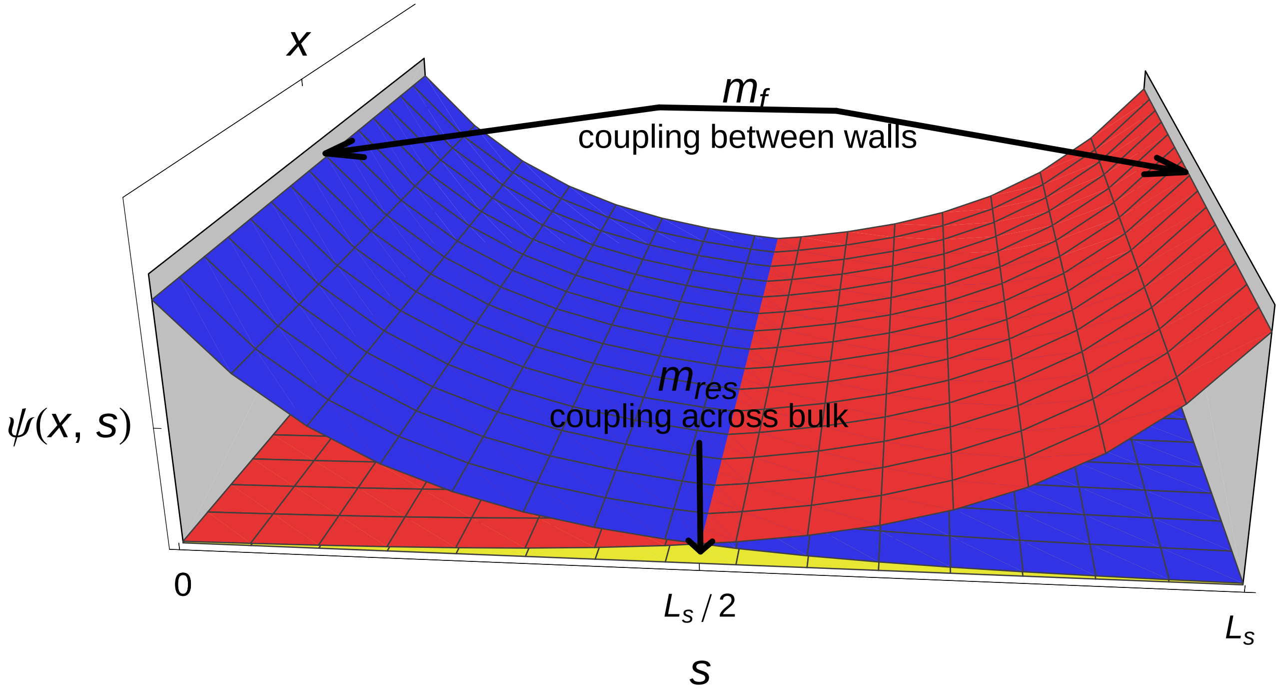
Domain-Wall Fermions

Domain-wall fermions allow us to control chiral symmetry breaking, but we need to keep our residual masses small, since the unwanted terms may contribute as $O(m_{\mathrm{res}}^2)$. In this framework, we may also use non-perturbative renormalization to find our Z factors.

Our Lattices

Controlled Chiral Symmetry Breaking

The domain-wall fermion action is constructed in such a way that low-lying eigenstates are chiral modes bound to the edges of the fifth dimension depending on their chirality. The overlap (exponentially suppressed) acts like an additive renormalization to fermion mass called the residual mass.



Our Lattices

Lattice Scale

The lattice scale may be computed from the static quark potential, vector meson (ρ) mass, or the method of planes. All agree:

$$a^{-1} = 1.62(4)\ \text{GeV},\ a = 0.121(3)\ \text{fm}$$
$$V \approx (2\ \text{fm})^3$$

Residual Mass

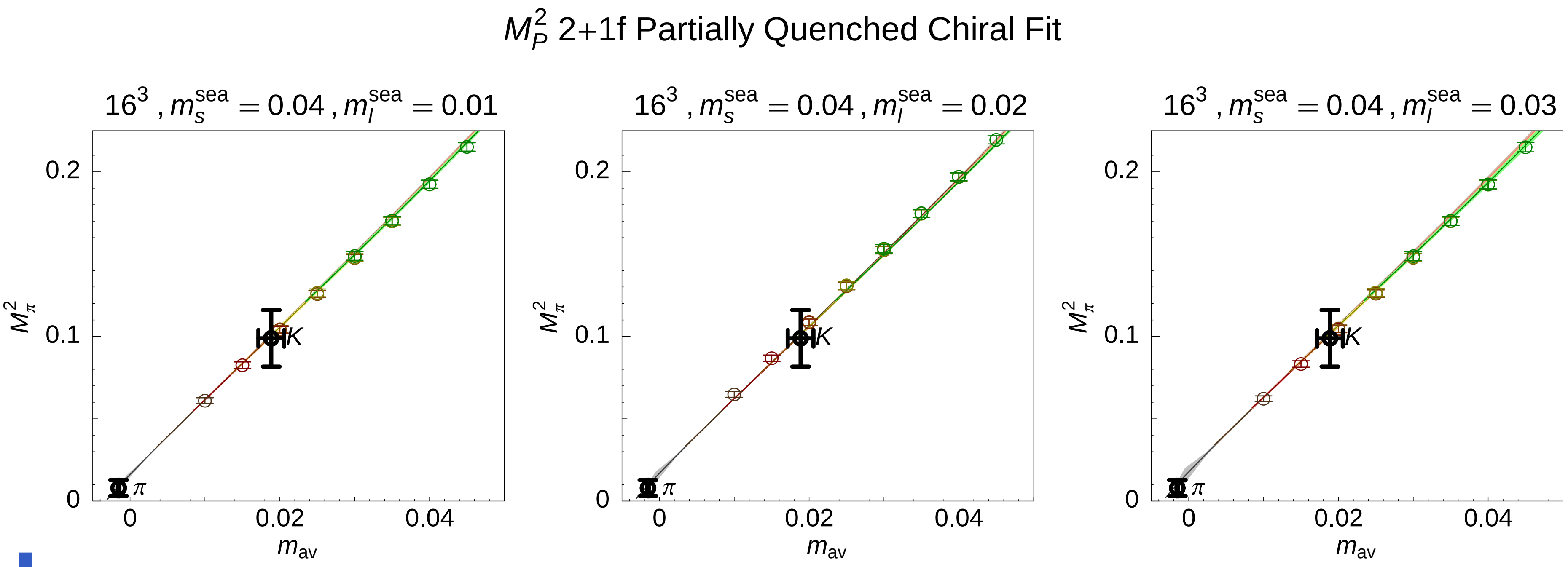
The residual mass is determined from the midpoint correlator, and is also the (negative) quark mass at which the pseudoscalar mass extrapolates to zero.

$$am_{\text{res}} = 0.00308(4)$$

Lattice Results

Quark Masses

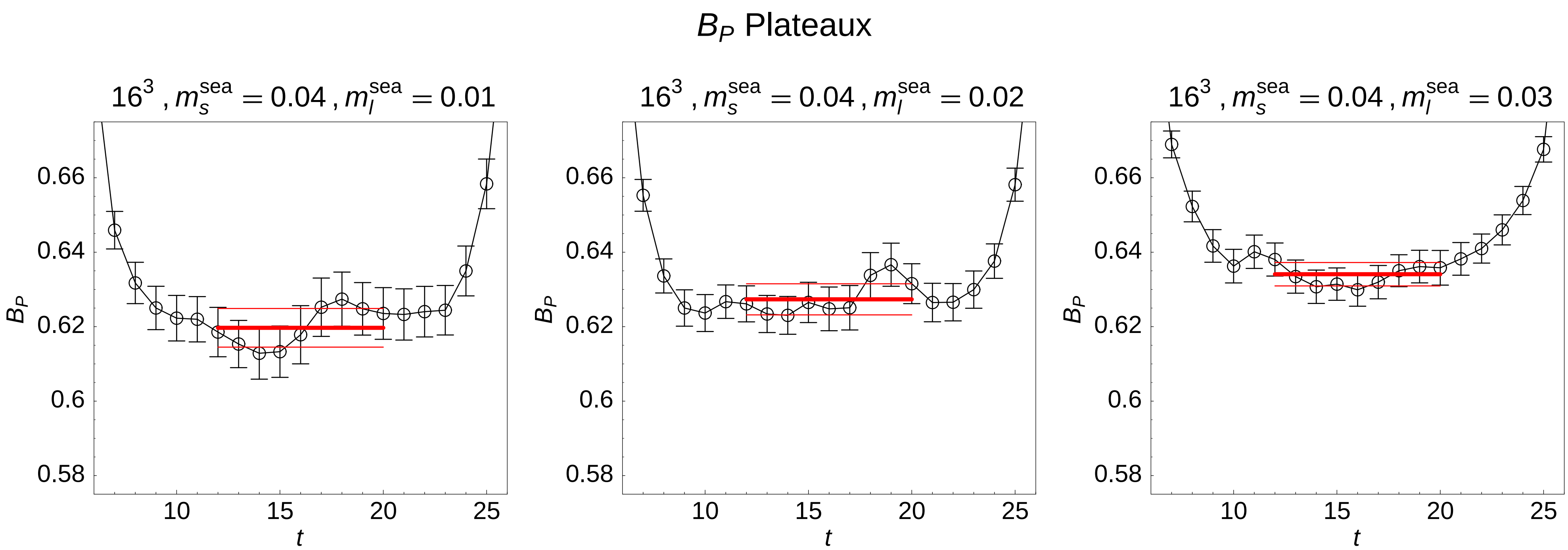
We set the bare light and strange quark masses by fixing the pseudoscalar masses to the physical pion and kaon masses. We find am_l = 0.00100(6), am_s = 0.0408(44). We use the lowest-order fit from chiral perturbation theory: $M_P^2 = 2\mu(m_{\rm av} + m_{\rm res})$.



Lattice Results

Three-Point Plateaux

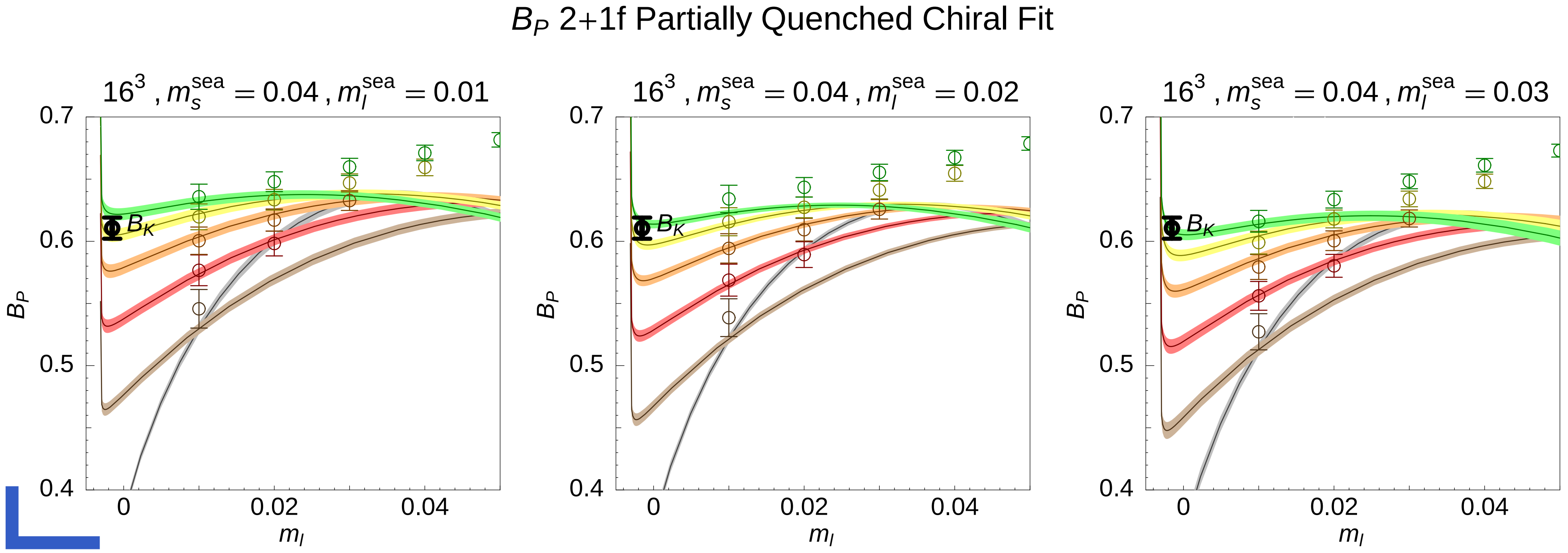
We expect the matrix element to approach its asymptotic value far from the source and sink. Depicted here are the unitary quark masses (where $m_{\rm val} = m_{\rm sea}$).



Lattice Results

Kaon Bag Parameter

B_K is computed for 15 nondegenerate combinations of valence strange and light masses. Light mass runs along the x-axis; strange mass is denoted by color. We fit to the partially quenched 2+1 flavor NLO chiral perturbation theory form of Sharpe and Van de Water. $B_K^{\mathrm{bare}} = 0.607(9)$



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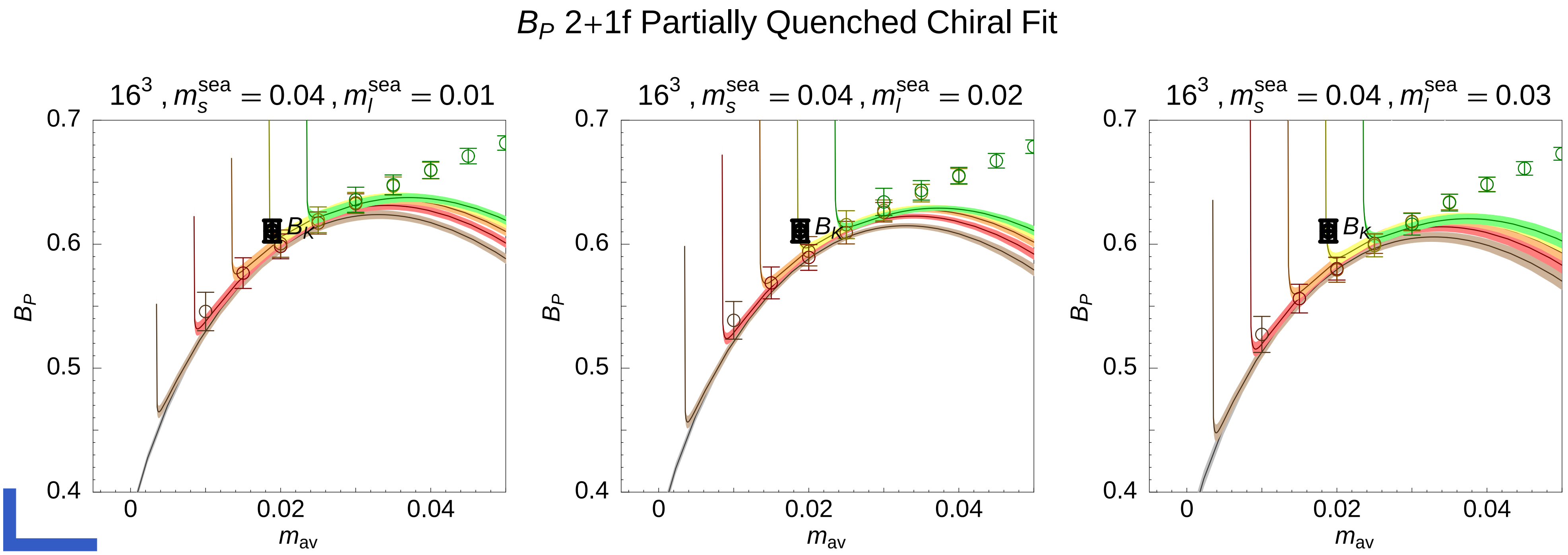
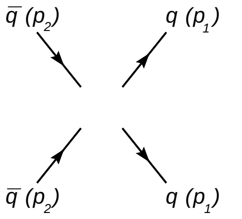


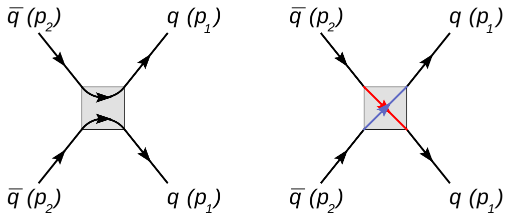
Nonperturbative Renormalization

Method

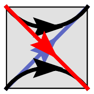
1. Calculate propagators with fixed momentum.



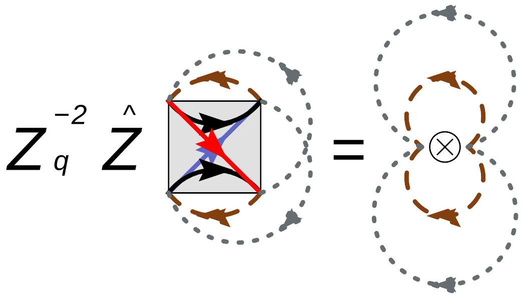
2. Combine the propagators according to the structure of the matrix element.



3. Amputate the external legs.



4. After projection, this matching (including quark field renormalization Z_q) gives the renormalization.

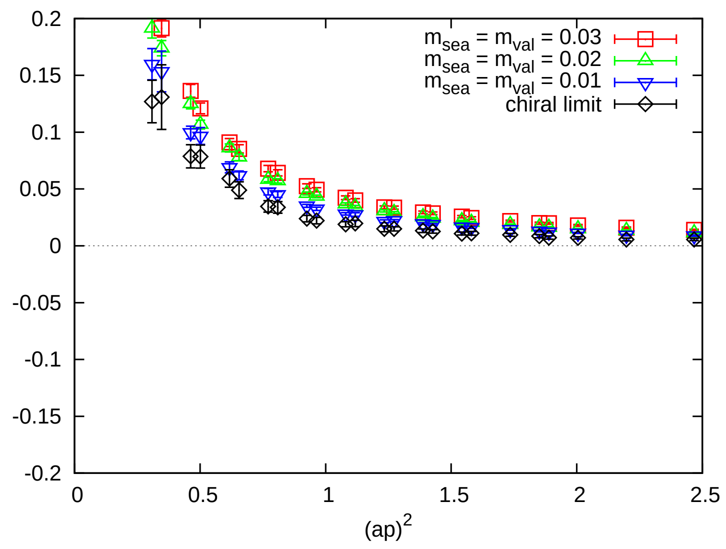


Nonperturbative Renormalization

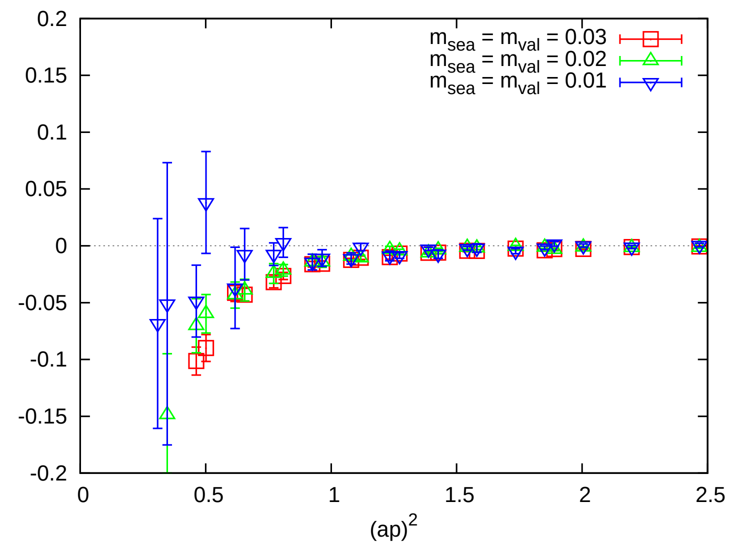
Operator Mixing

First, we verify that the undesired mixings are indeed small.

$Z_{VV+AA,VV-AA}$



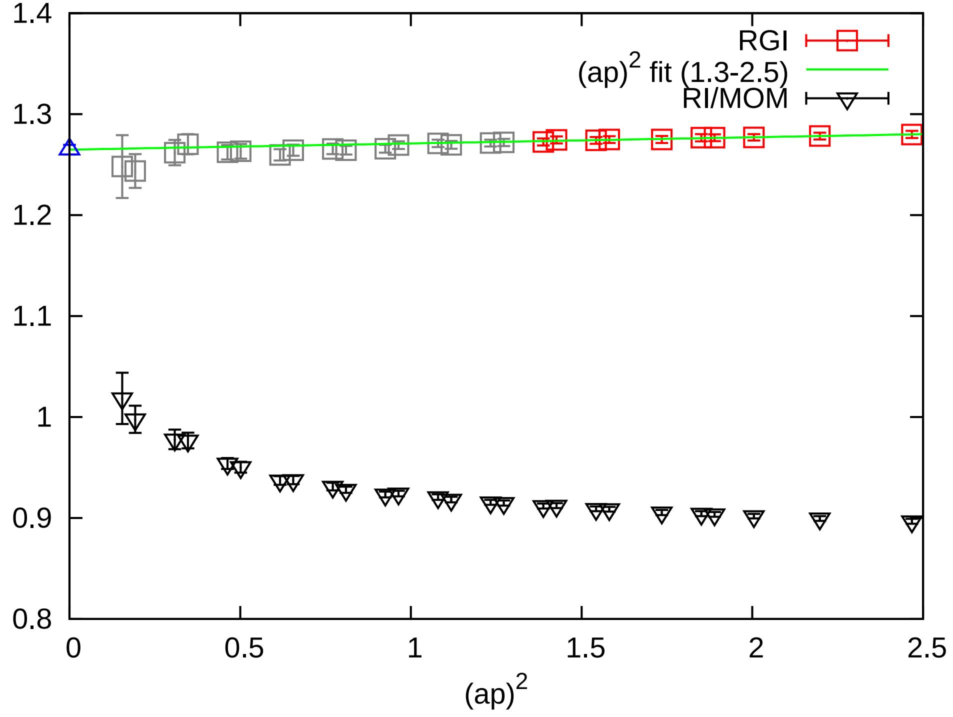
$Z_{VV+AA,SS-PP}$



Nonperturbative Renormalization

Lattice-to-Continuum Matching

$Z_{VV+AA,VV+AA}$ can be divided into a constant Z^{RGI} and a running $f(\mu)$.

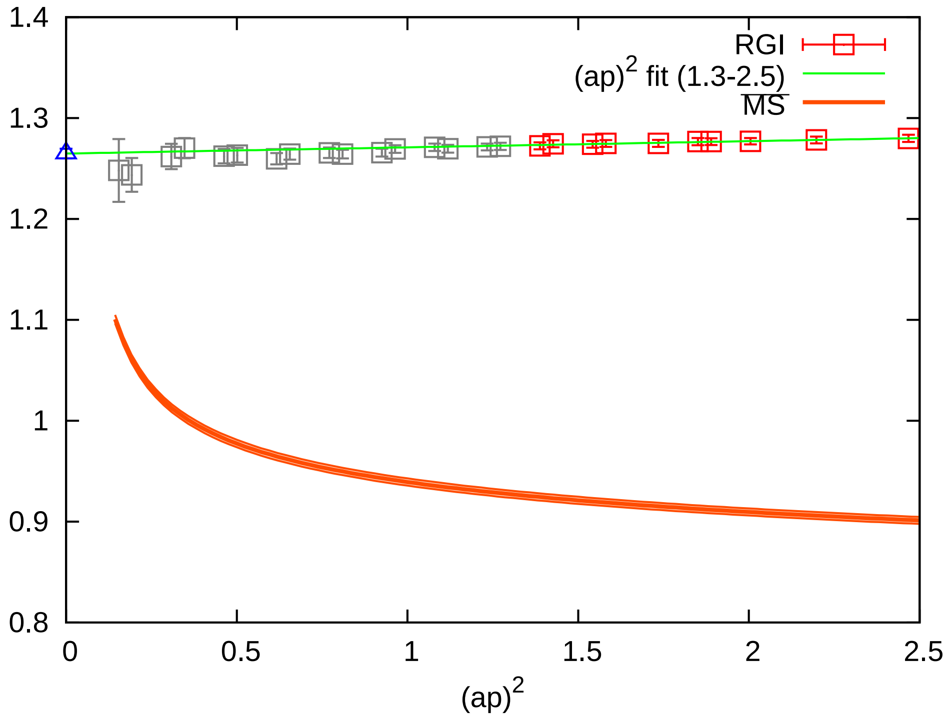


Nonperturbative Renormalization

Lattice-to-Continuum Matching

$Z_{VV+AA,VV+AA}$ can be divided into a constant Z^{RGI} and a running $f(\mu)$.

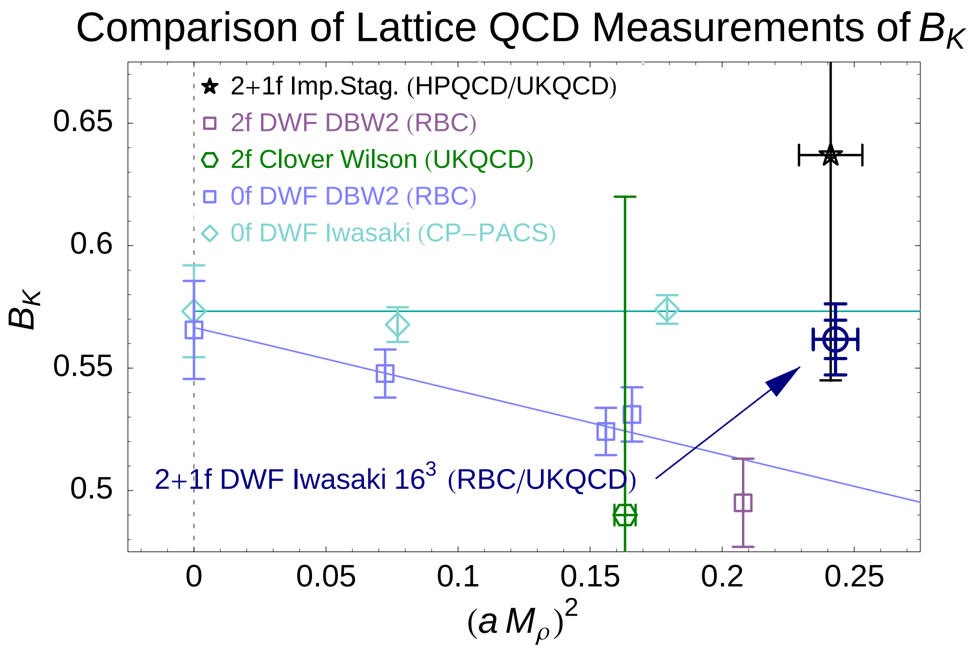
We may simply apply perturbation theory to find $Z_{B_K}^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$.



Lattice Results

Comparisons

Here we compare our 2+1 flavor domain-wall fermion B_K result with other chiral fermion and unquenched results.



With $Z_{B_K} = 0.92(2)$

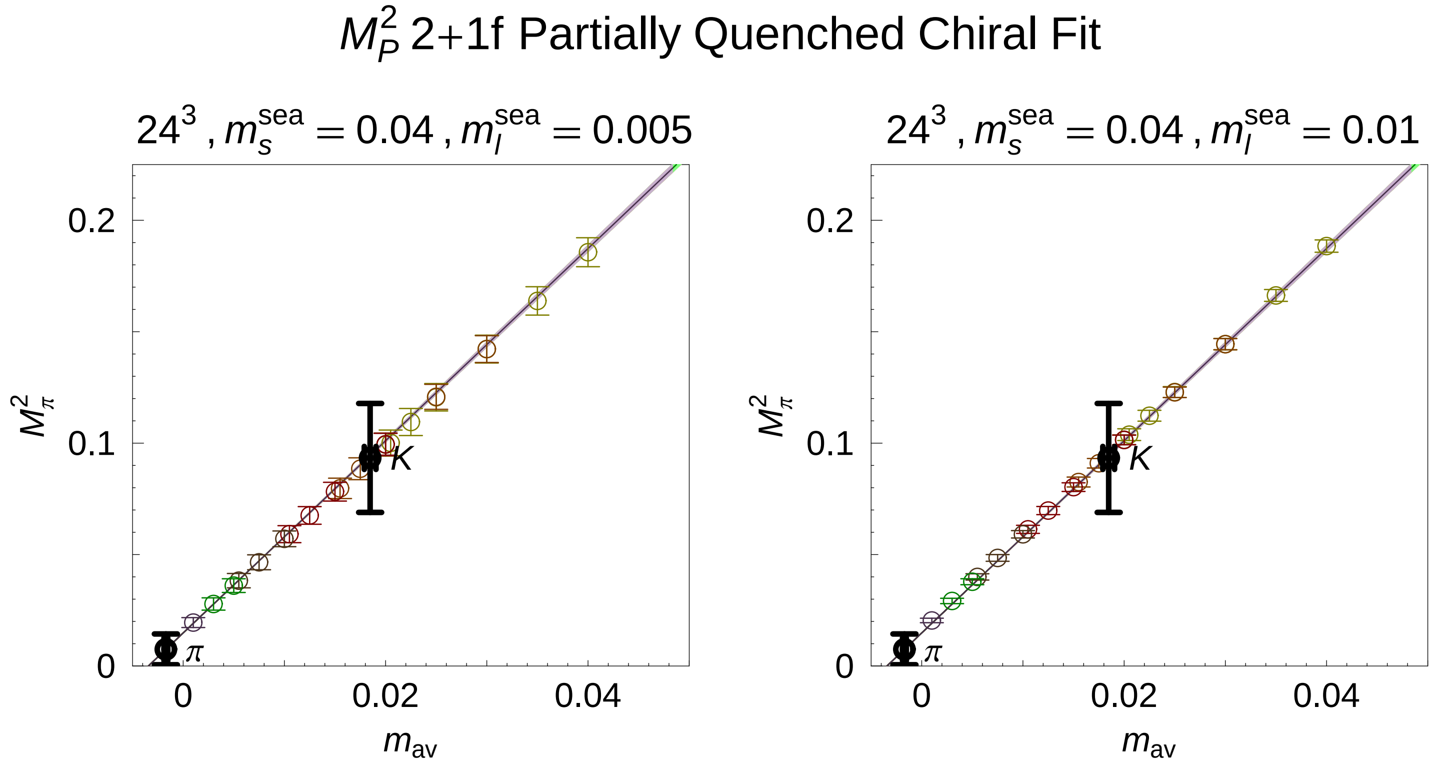
in $\overline{\mathrm{MS}}$ at 2 GeV

$$B_K = 0.558(8)(11)$$

Lattice Results (Larger Volume)

Quark Masses

Analysis is in progress on a larger volume $(3\text{ fm})^3$ at the same scale, using even smaller valence and sea masses, down to $am = 0.001$.

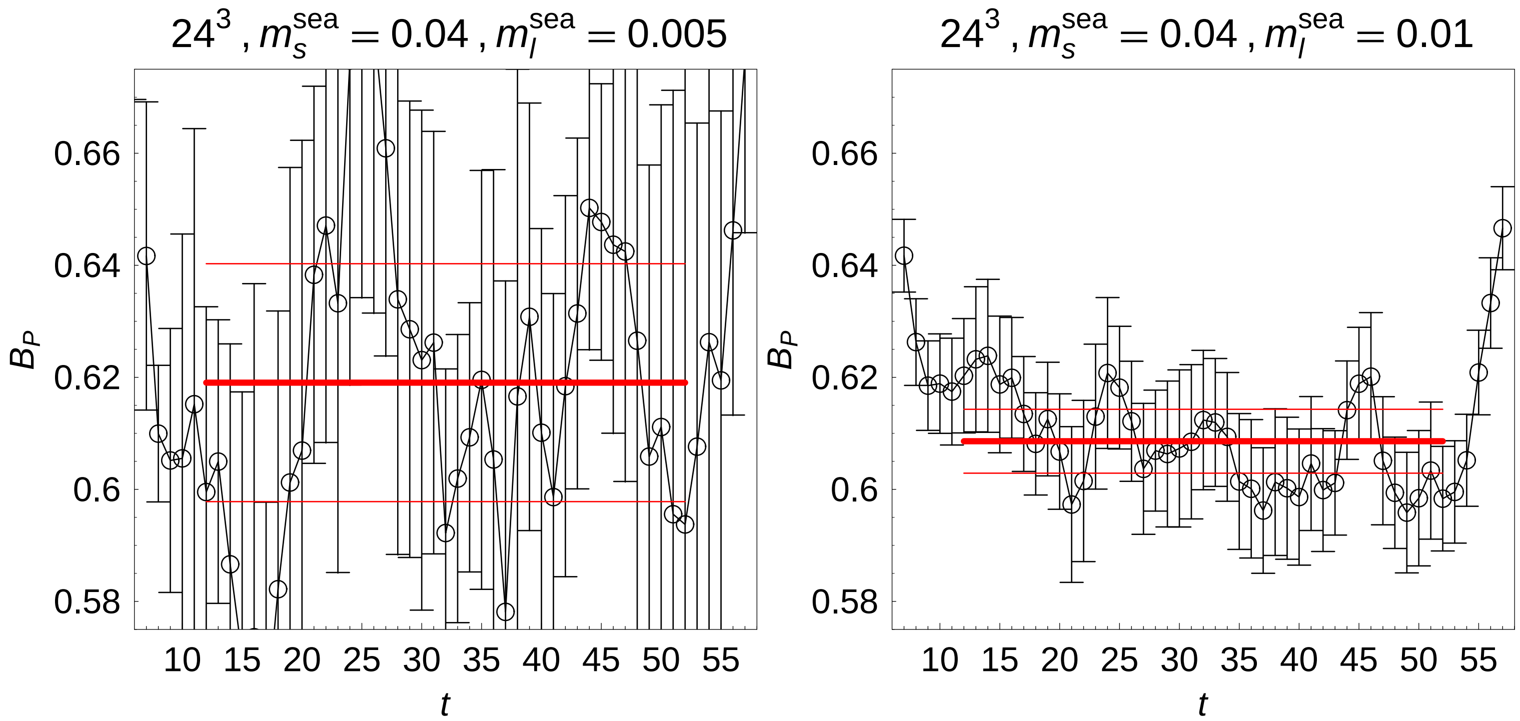


Lattice Results (Larger Volume)

Three-Point Plateaux

The much longer length of the time-dimension allows us to average over more fluctuations in the gauge field, yielding smaller statistical errors.

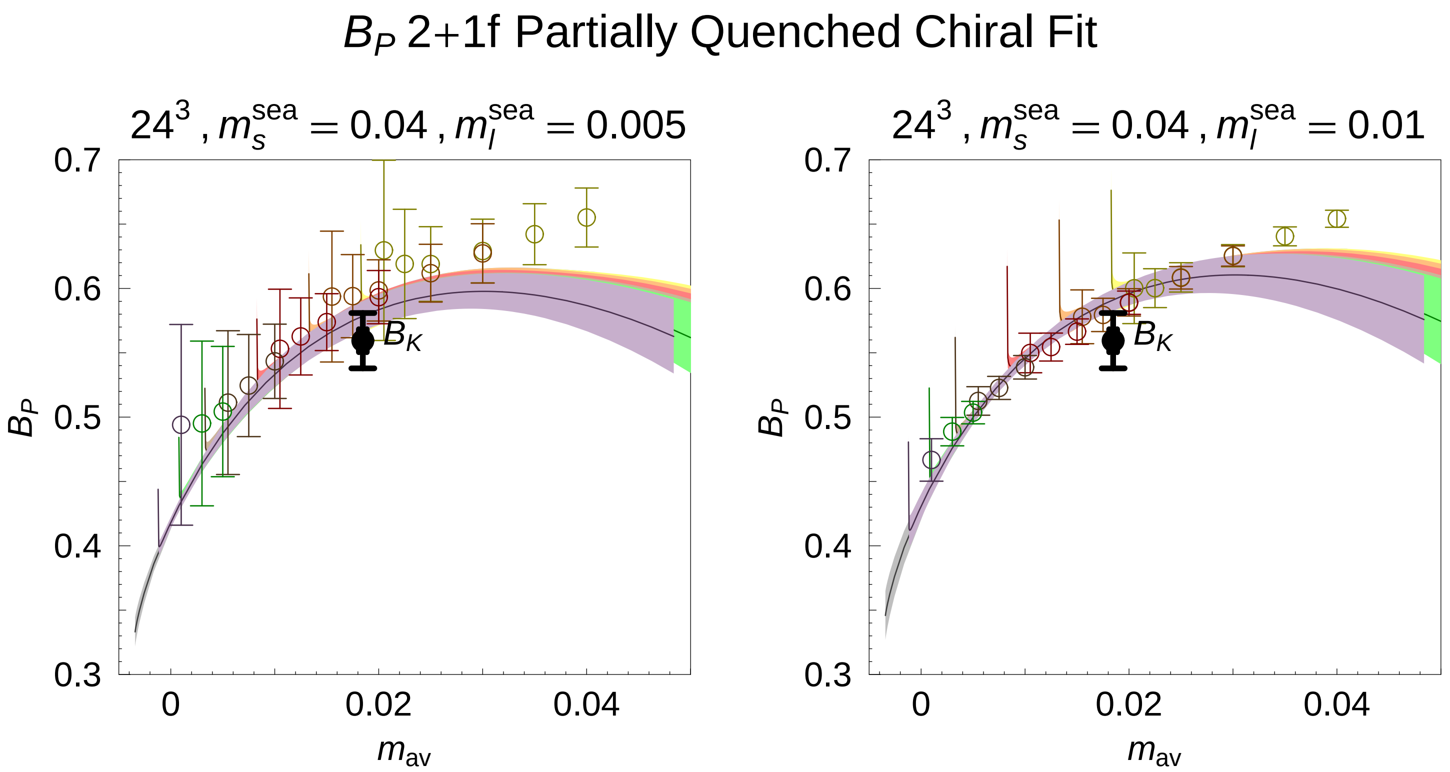
B_P Plateaux



Lattice Results (Larger Volume)

Kaon Bag Parameter

Already, on only 31 and 5 configurations, we are able to see preliminary results. This new data will allow us to test the chiral form at very low mass and take into account finite-volume effects.



More Information

Our Publications

For further details on our methodology using older $L_s = 8$ data, please see:

- First results from 2+1-flavor domain-wall QCD: mass spectrum, topology change and chiral symmetry with $L_s = 8$, `hep-lat/0612005`

For more details on the data presented here, please see the forthcoming:

- Light meson spectroscopy with 2+1 flavors of dynamical domain-wall fermions
- Neutral kaon mixing from 2+1 flavor domain-wall QCD

Bonus Slides

Bonus Slides

Bonus Slides

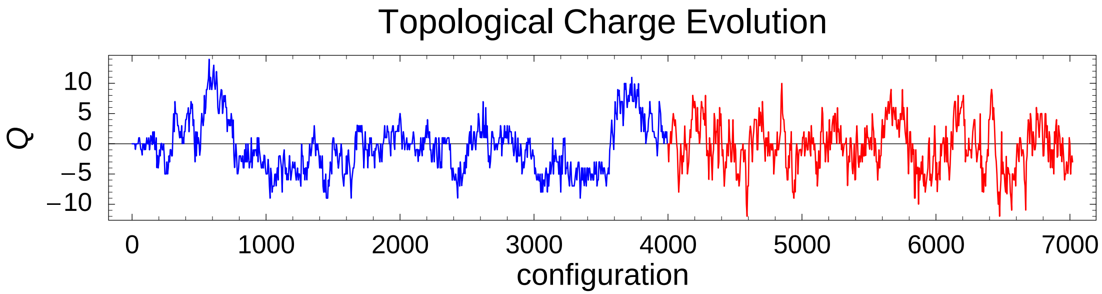
Our Lattices

Our Machine

We generated lattices using 4k-node QCDOC machines located in Brookhaven National Labs and the University of Edinburgh.

Our Program

We avoid having to extrapolate in step-size by using an exact algorithm, the Rational Hybrid Monte Carlo (RHMC). Further improvement to the algorithm (QRHMC) allows the topology of our configurations to move more easily.



Our Lattices

Our Action

We choose domain-wall fermions to control chiral symmetry breaking. Since our lattices are rather coarse, an improved action will help lower the residual mass. Adding a rectangle term to the gauge action accomplishes this by smoothing the gauge field at short distances. We use the Iwasaki rectangle coefficient ($c_1 = -0.331$).

$$S_g = -\frac{\beta}{3}\,\mathrm{ReTr}\left((1 - 8\,c_1)\left\langle \mathbb{1} - \square \right\rangle + c_1 \left\langle \mathbb{1} - \square\!\square \right\rangle \right)$$

Our Lattices

Our Parameters

Our parameters are tuned to select the desired lattice scale and to ensure the strange sea quark mass is near the physical strange mass. The smaller 16^3 volume run is now completed; analysis is in progress for the larger 24^3 volume.

β	2.13	c_1	-0.331		
L_s	16			$16^3 \times 32$	$24^3 \times 64$
am_s	0.04	am_l	0.005	–	(5 confs)
			0.010	(150 confs)	(31 confs)
			0.020	(150 confs)	(0 confs)
			0.030	(150 confs)	–