Determination of $|V_{cb}|$, m_b and m_c from Inclusive B Decay Distributions

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Overview

- Will cover results of a fit to lepton, hadron and photon moments using calculations in kinetic scheme and discuss some applications (Phys. Rev. D 73, 073008 (2006))

- Motivation
  - Short Introduction to theoretical framework
- Experimental Measurements
  - hadron and lepton moments from $B \to X_c l\nu$
  - photon energy moments from $B \to X_s \gamma$
- Results of the combined fit
  - Extraction of $|V_{cb}|$, m_b, m_c and higher order heavy quark parameters
- Applications and prospects for improving on current status
- Conclusions

Heavy Quark Expansions

The Operator Product Expansion separates perturbative from non-perturbative scales in a systematic way:

$$\Gamma_{SL}(B \to X_c l\nu) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2(1+A_{ew})A_{pert}(r,\mu)$$

$$\times\left[z_0(r)\left(1-\frac{\mu_\pi^2-\mu_G^2+\frac{\rho_D^3+\rho_{LS}^3}{m_b}}{2m_b^2}\right)-2(1-r)^4\frac{\mu_G^2+\frac{\rho_D^3+\rho_{LS}^3}{m_b}}{m_b^2}+d(r)\frac{\rho_D^3}{m_b^3}+O(1/m_b^4)\right]$$

kinetic expec. value

chromomagnetic expec. value

Darwin term

spin-orbit

kinetic scheme

$r \equiv (m_c/m_b)^2$

Need to determine non-pert. parameters!

Benson, Bigi, Mannel & Uraltsev, hep-ph/0410080
Gambino & Uraltsev, Eur.Phys.J. C34, 181 (2004)

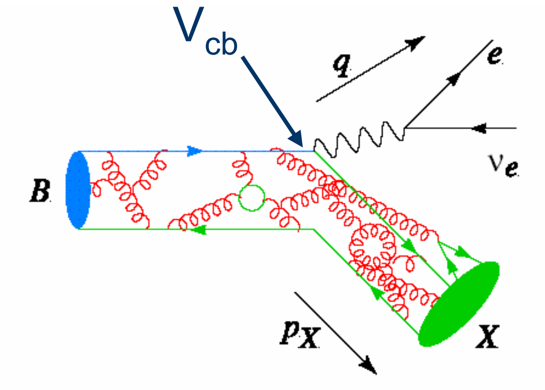
⟶ Use moments of inclusive distributions where same parameters appear:

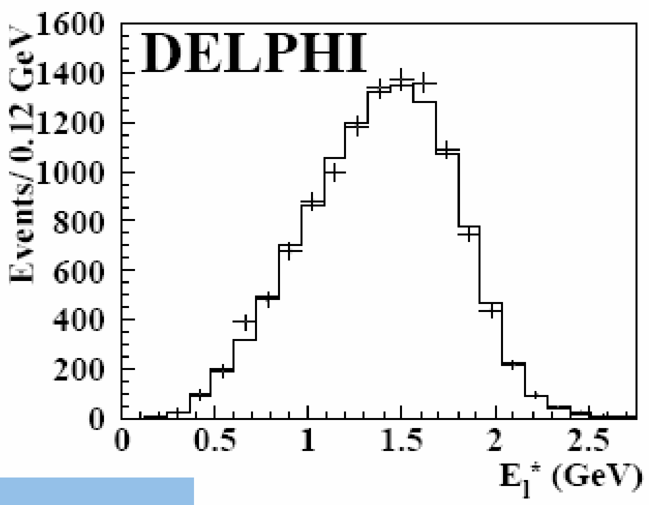
$$<X^n>(E_{cut}) = \left.\frac{\int (X-X^0)^n \frac{d\Gamma}{dX}\,dX}{\int \frac{d\Gamma}{dX}\,dX}\right|_{E_l>E_{cut}} \cong f'_{OPE}(m_b, m_c, a_i)$$

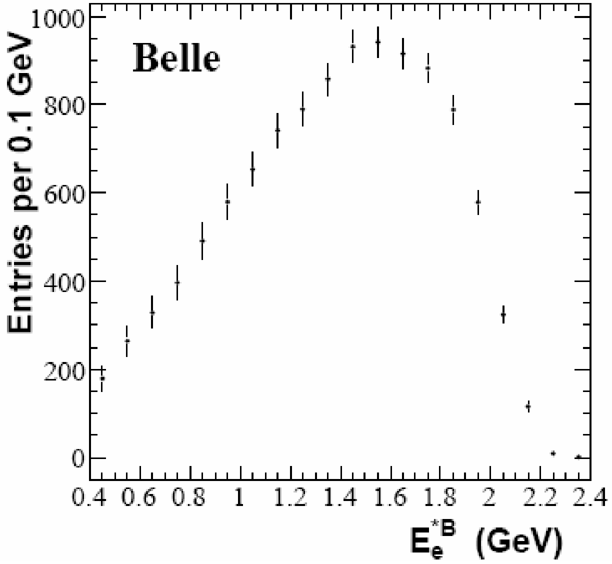
m_b and μ_π^2 are used to parameterise both $B\to X_s\gamma$ and $B\to X l\nu$ spectra

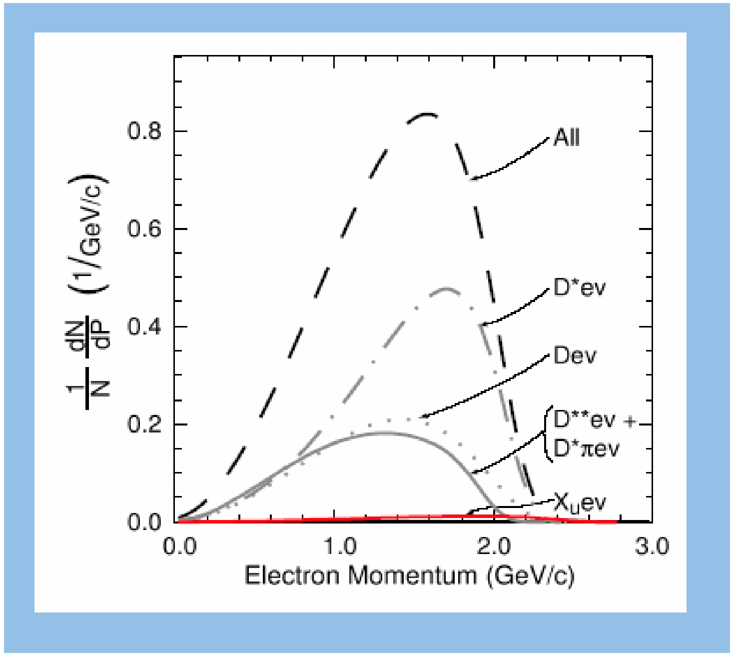
- Hadronic Mass distribution $\langle M_X^n\rangle \to \langle M_X\rangle(m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \alpha_s)$
- Lepton Energy spectrum $\langle E_\ell^n\rangle \to \langle E_\ell^n\rangle(m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \alpha_s)$
- Photon Energy spectrum $\langle E_\gamma^n\rangle \to \langle E_\gamma^n\rangle(m_b, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \alpha_s)$.

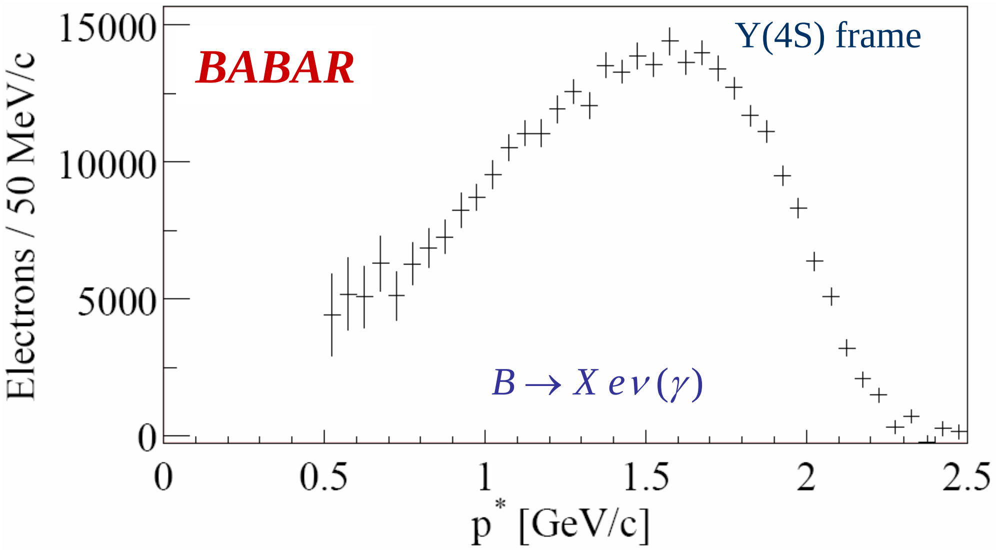
Lepton Energy Moments







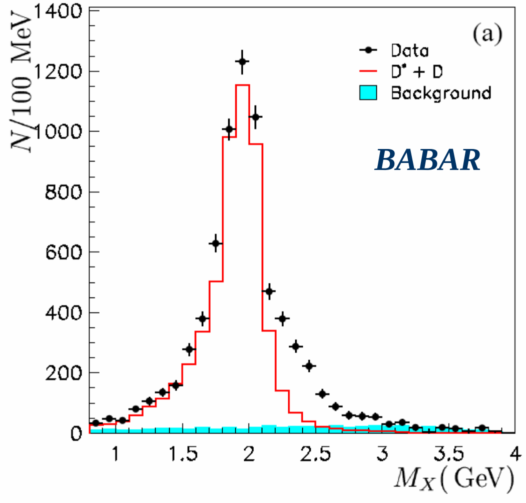




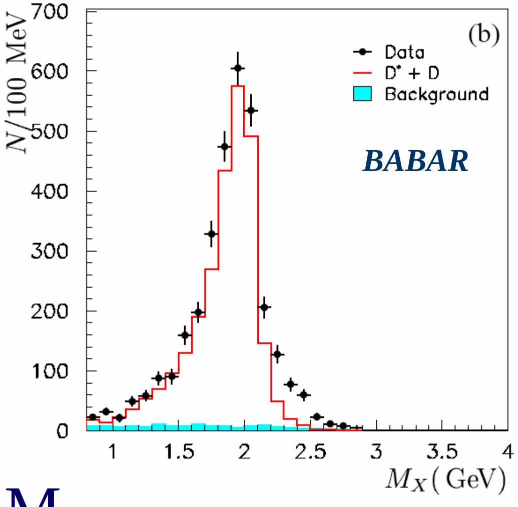
Hadronic Mass Moments

BaBar, Belle and CLEO measure full spectrum

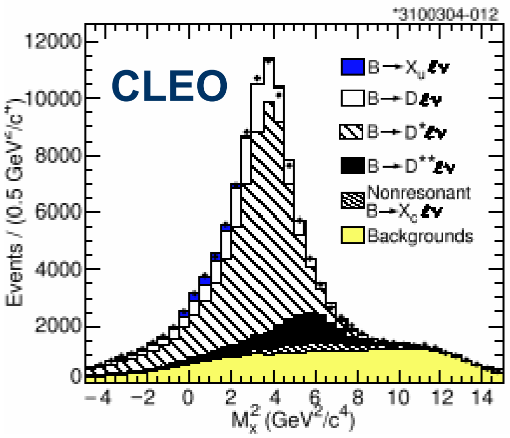
p* > 0.9 GeV

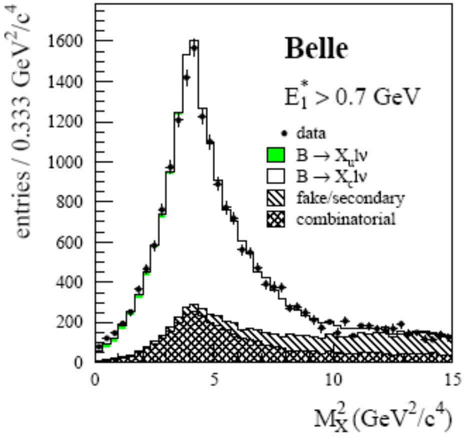


p* > 1.6 GeV

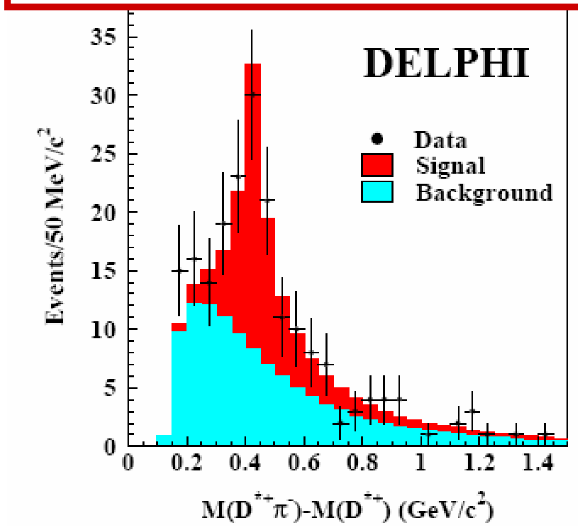


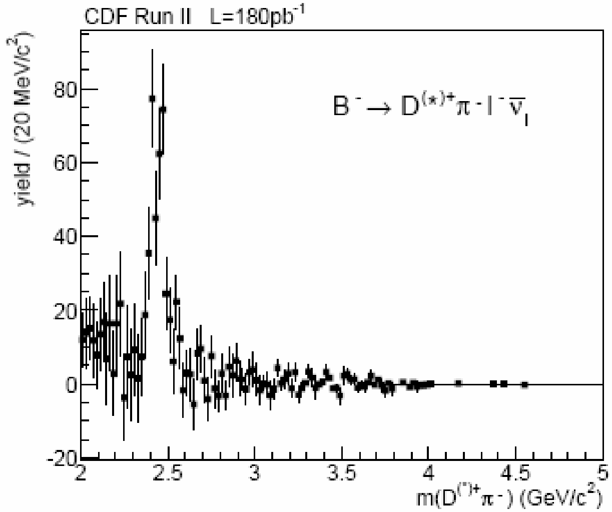
M_X





Delphi and CDF only measure higher resonances





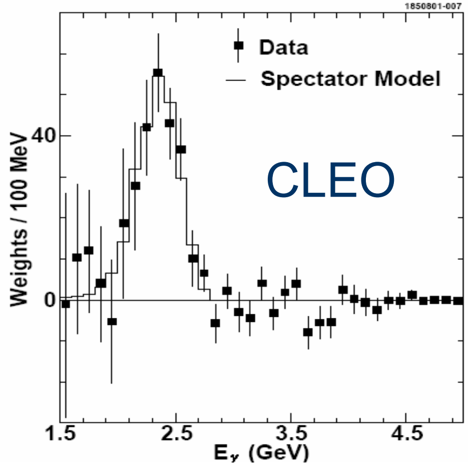
b→sγ Spectra and Moments

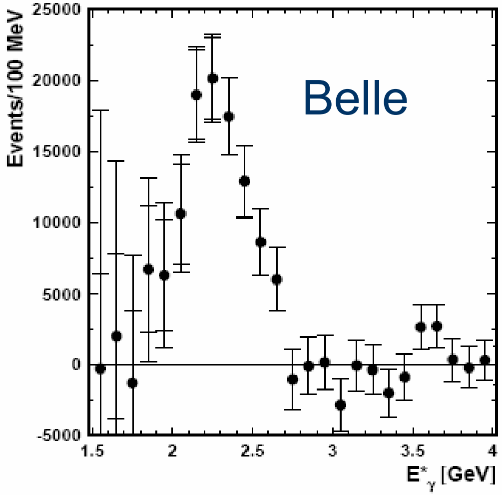
Measure photon spectrum in b→sγ decays:

<u>Two main approaches:</u>

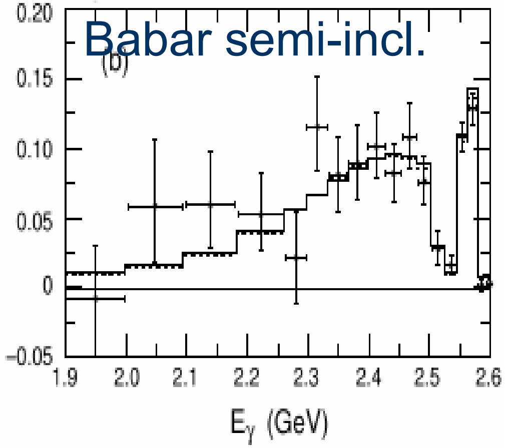
- Inclusive:
  - identify photon
- Semi-Inclusive:
  - reconstruct many exclusive final states (up to 38!)

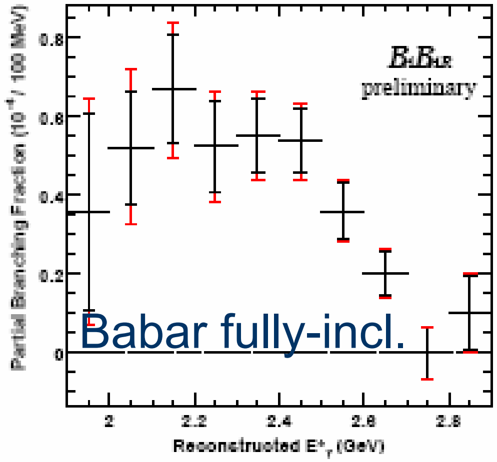
Difficult measurement:
Overwhelming background
from π^0s for $E_\gamma < 1.8$ GeV





Measurement of photon spectrum and its moments gives information about inner structure of B meson:
- **b quark mass**
- **Fermi momentum**





Available moment measurements

Legend:

n = order of (central) moment of observable M_X, E_l and E_γ

l = min. lepton momentum

γ = min. photon energy

☑ published with covariance matrix and used in fit

☒ not used in fit

	Hadron Moments		Lepton Moments		Photon Moments	
BaBar	n=1,2,3,4 l=0.9-1.6	☑	n=0,1,2,3 l=0.6-1.5	☑	n=1,2,3 γ=1.9-2.3	☑
Belle	n= 1,2 l=0.9-1.6	☒	n=1,2 l=0.6-1.5	☒	n=1,2 γ=1.8	☑
CLEO	n=2,4 l=1.0-1.5	☑	n=1,2 l=0.6-1.5	☒	n=1,2 γ=2.0	☑
Delphi	n=2,4,6 l=0.0	☑	n= 1,2,3 l=0.0	☑		
CDF	n=2,4 l=0.7	☑				

Total of 51 measurements!

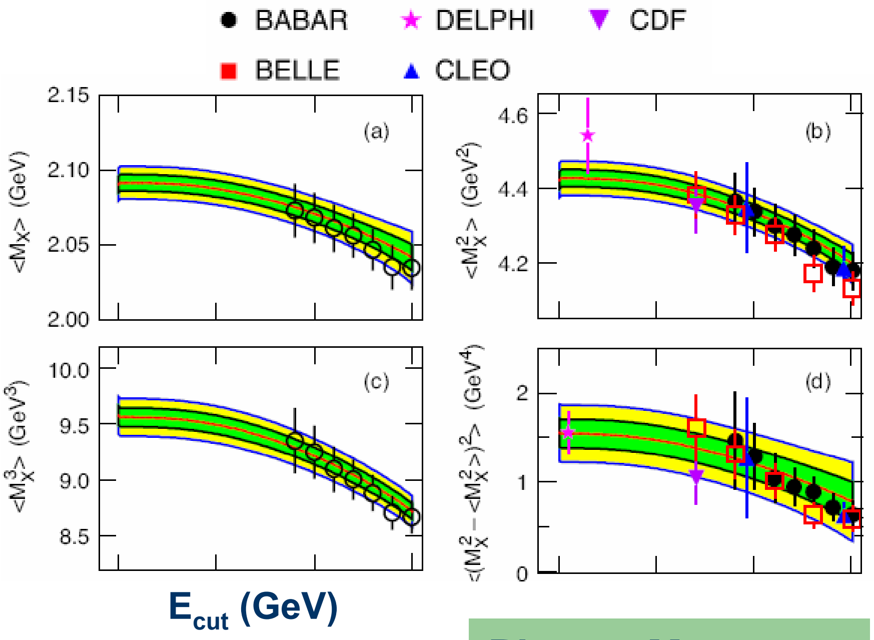
Important to take correlations between moments with different min. lepton/photon energies into account

Inclusive $|V_{cb}|$ - Fit to Moments

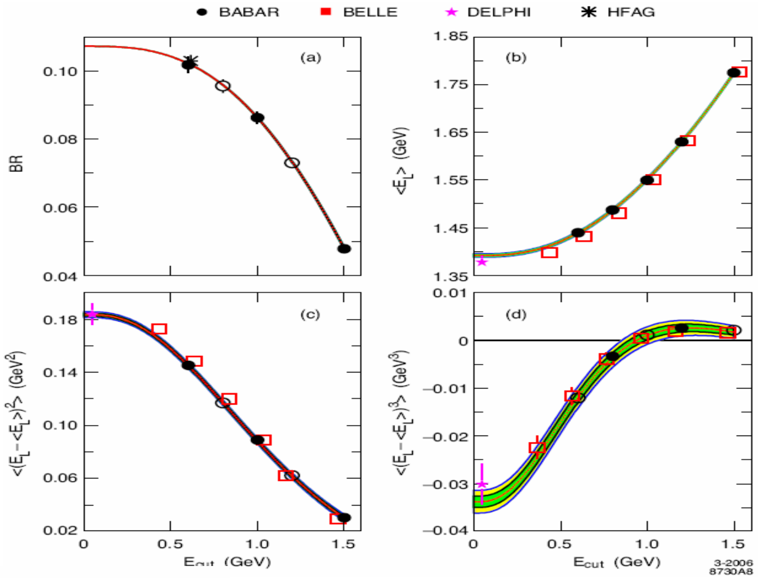
Based on calculations in kinetic scheme:

Benson, Bigi, Mannel & Uraltsev, hep-ph/0410080
Gambino & Uraltsev, hep-ph/0401063
Benson, Bigi & Uraltsev, hep-ph/0410080

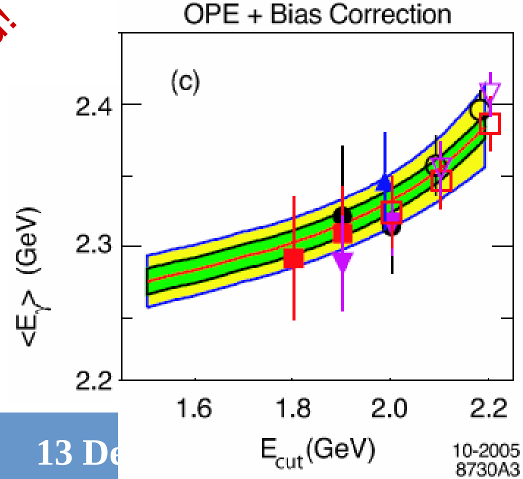
Hadron Moments

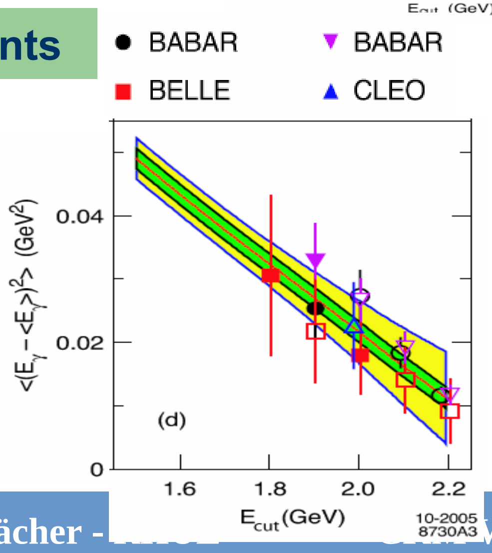


Lepton Moments



Photon Moments





Measurements highly correlated!

O. Buchmueller,H.F.
hep-ph/0507253

Inclusive $|V_{cb}|$

Result of fit to all moment measurements:

		exp	HQE	Γ_{SL}
$\|V_{cb}\|$ =	(41.96	± 0.23	± 0.35	± 0.59) 10^{-3}
m_b =	4.590	± 0.025	± 0.030	GeV
m_c =	1.142	± 0.037	± 0.045	GeV
μ_π^2 =	0.401	± 0.019	± 0.035	GeV²
μ_G^2 =	0.297	± 0.024	± 0.046	GeV²
ρ_D^3 =	0.174	± 0.009	± 0.022	GeV³
ρ_{LS}^3 =	-0.183	± 0.054	± 0.071	GeV³
BR_{clv} =	10.71	± 0.10	± 0.08	%

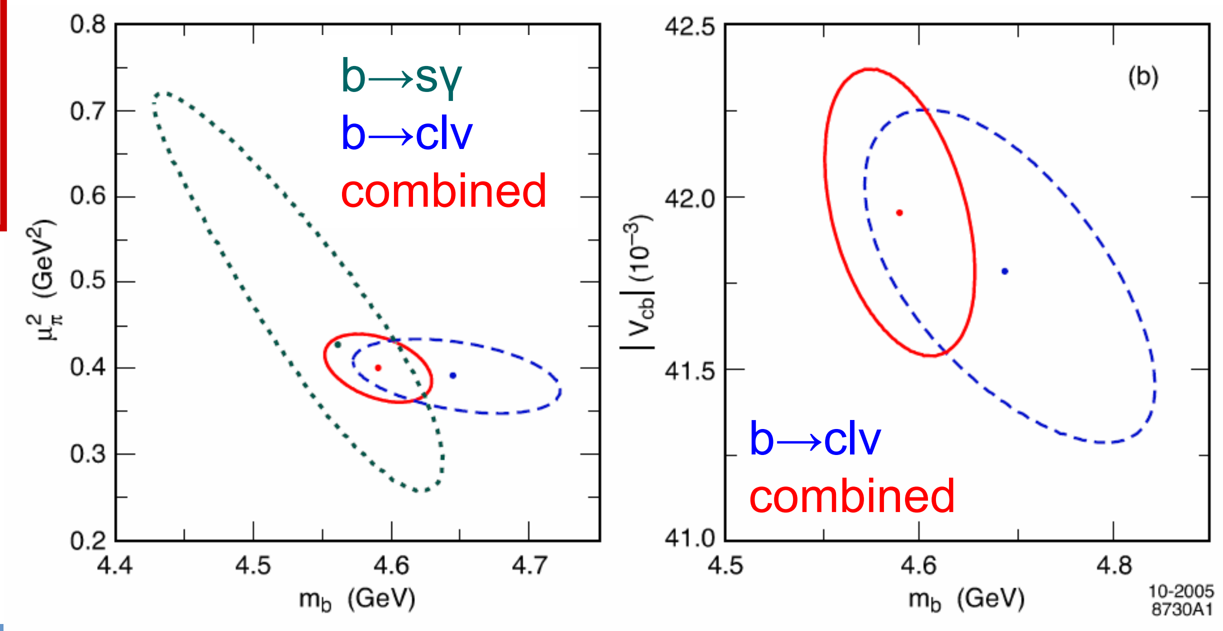
χ^2= 19.3/44dof

hep-ph/0507253

$|V_{cb}|$ @ 2%
m_b < 1%
m_c @ 5%

In $\overline{MS}$ scheme:
$\overline{m}_b(\overline{m}_b)$ = 4.20 ± 0.04 GeV
$\overline{m}_c(\overline{m}_c)$ = 1.24 ± 0.07 GeV
$\overline{m}_c(\mu)/\overline{m}_b(\mu)$= 0.235 ± 0.012

courtesy of N.Uraltsev



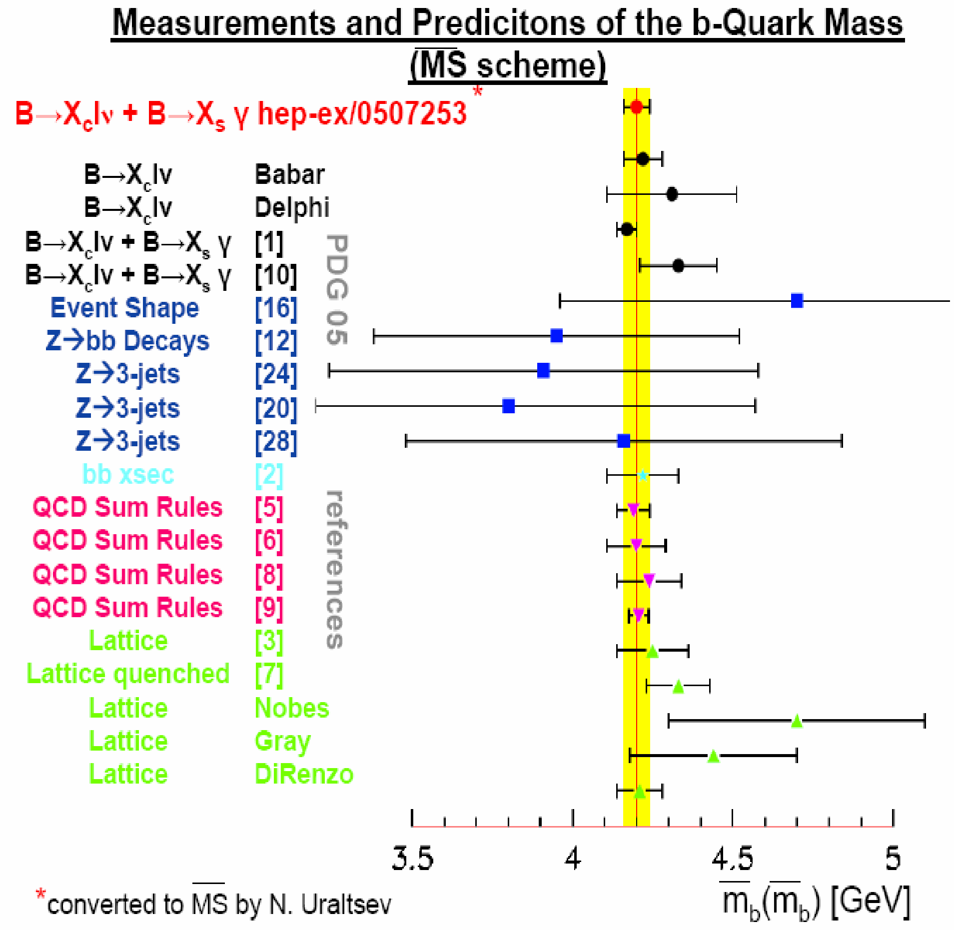
Good agreement with other similar analyses:

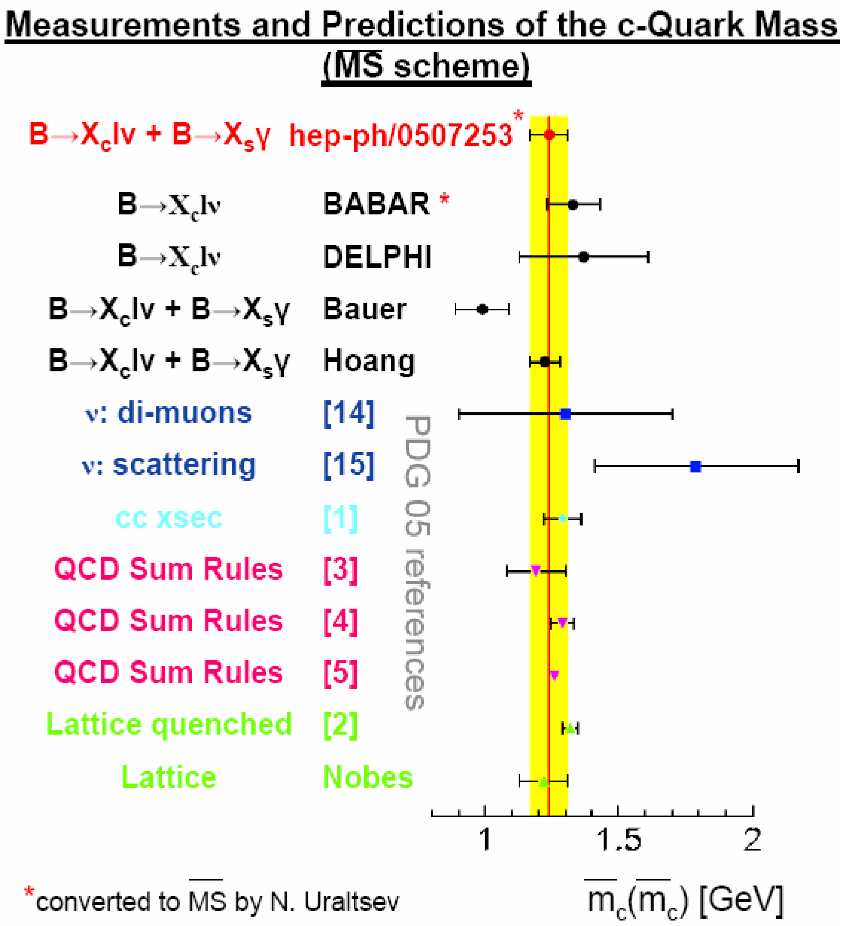
Bauer et al. hep-ph/0408002
DELPHI hep-ex/0510024
Belle hep-ex/0611047

Treatment of Theory Errors

- Construct combined experimental and theoretical covariance matrix
  - $C_{tot} = C_{stat} + C_{syst} + C_{theo}$
- Uncertainties in moment expansions
  - to evaluate perturbative uncertainties on Wilson coefficients vary
    - 2nd order terms by 20%
    - 3rd order terms by 30%
  - also assume 20 MeV uncertainty on m_b and m_c in corresponding terms
  - additional uncertainties for E_γ moments due to higher E_{cut}
  - vary α_s around 0.22 ± 0.04 for lepton and 0.3 ± 0.1 for hadron moments
  - errors considered fully correlated for varying E_{cut} and uncorrelated between moments of different order
- Uncertainties in Γ_{SL} (hep-ph/0302262)
  - pert. uncertainty in Wilson coefficient of leading bb operator - 0.5%
  - pert. corrections to Wilson coefficients of chromomagnetic and Darwin operators - 1.2%
  - non-perturbative soft charm effects (hep-ph/0511158) - 0.35%
  - higher order power suppressed contributions - 0.4%
  - “Fixed” uncertainty of 1.4% - not included in fit
- α_s uncertainty
  - Use “best value” for Γ_{SL} ($\alpha_s = 0.22 \pm 0.01$)

Comparison with other determinations





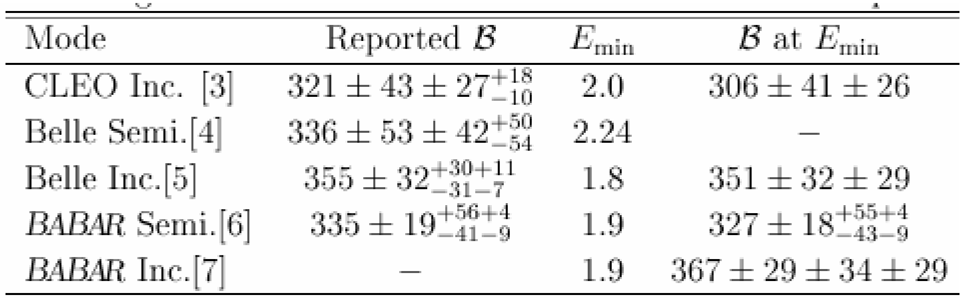
$$\overline{m}_b(\overline{m}_b) = 4.20 \pm 0.04 \text{ GeV}$$

$$\overline{m}_c(\overline{m}_c) = 1.24 \pm 0.07 \text{ GeV}$$

Conversion from kinetic mass scheme to $\overline{\text{MS}}$ scheme with hep-ph/9708372, hep-ph/0302262
See also report from CKM WS hep-ph/0304132

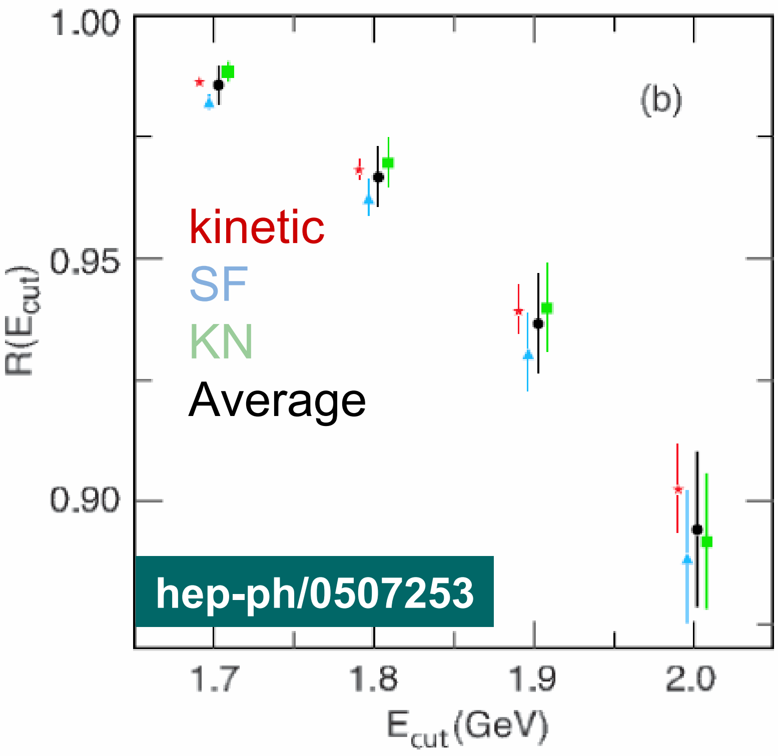
$B \rightarrow X_s \gamma$ Branching Fraction Average

- Partial branching fractions are measured above different photon energies
- Need to be extrapolated to $E_\gamma > 1.6$ GeV to compare with theory
- Extrapolation factors based on HQE fit to $b \rightarrow c l \nu$ and $b \rightarrow s \gamma$ moments



Mode	Reported $\mathcal{B}$	$E_{\min}$	$\mathcal{B}$ at $E_{\min}$
CLEO Inc. [3]	$321 \pm 43 \pm 27^{+18}_{-10}$	2.0	$306 \pm 41 \pm 26$
Belle Semi.[4]	$336 \pm 53 \pm 42^{+50}_{-54}$	2.24	–
Belle Inc.[5]	$355 \pm 32^{+30+11}_{-31-7}$	1.8	$351 \pm 32 \pm 29$
BABAR Semi.[6]	$335 \pm 19^{+56+4}_{-41-9}$	1.9	$327 \pm 18^{+55+4}_{-43-9}$
BABAR Inc.[7]	–	1.9	$367 \pm 29 \pm 34 \pm 29$

Extrapolation Factors for BF



hep-ph/0507253

<ins>To obtain spectra in other schemes:</ins>

predict 1st and 2nd E_γ moment at $E_{cut} = 1.6$GeV based on result of fit in kinetic scheme.

Then fit the moments in the new schemes

→ good agreement between calculations if comparing on basis of moments

Use consistent extrapolation factors for all measurements

BR($B \to X_s\gamma$) Average

SM Prediction (NLO)

$3.57 \pm 0.30 \times 10^{-4}$

Nucl.Phys.B631:219-238,2002

NNLO

CLEO PRL 87, 251807 (2001)

BELLE Phys.Lett. B 511, 151 (2001)

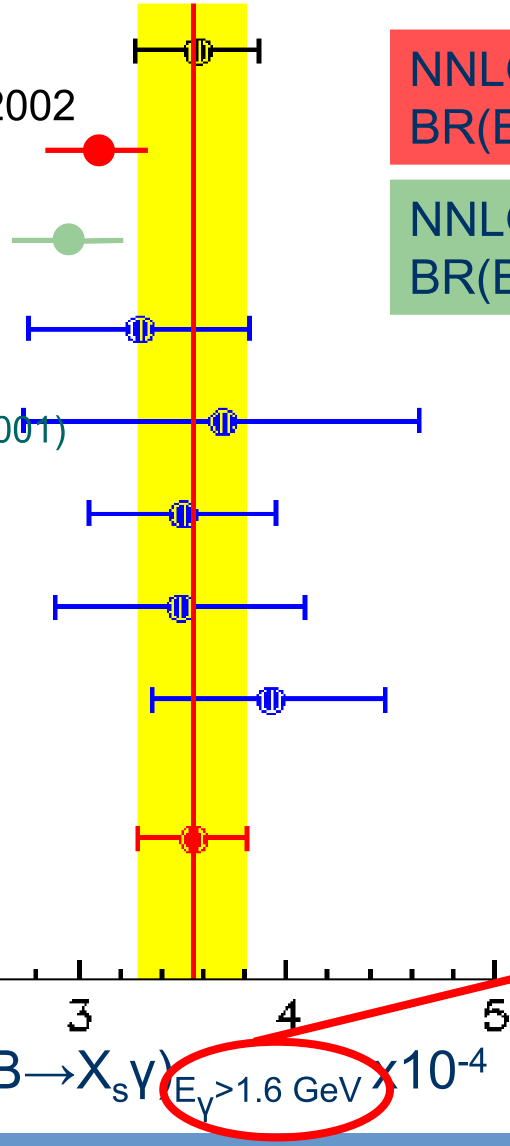
BELLE PRL.93:061803,(2004)

BABAR PRD 72, 052004 (2005)

BABAR PRL 97, 171803 (2006)

HFAG Average

$3.55 \pm 0.26 \times 10^{-4}$



NNLO: M.Misiak et al,hep-ph/0609232
$BR(B\to X_s\gamma)_{E_\gamma>1.6\ GeV} = (3.15 \pm 0.23) \times 10^{-4}$

NNLO: Becher & Neubert, hep-ph/0610067
$BR(B\to X_s\gamma)_{E_\gamma>1.6\ GeV} = (2.98 \pm 0.26) \times 10^{-4}$

Two NNLO calculations
1.2 and 1.4 σ below
World Average

7 – 9% uncertainty for both theory and experiment

involves E_γ extrapolation!

Prospects for the Future

- Include Belle measurements (see previous talk by P. Urquijo)

- Additional Experimental Measurements
  - possibly measure higher order moments → higher sensitivity to higher order parameters
  - Recent effort to calculate $1/m_b^4$ terms in SL rate

    Mannel et al (hep-ph/0611168)
  - fourth central moment in M_X^2 is dominated by fourth order terms
  - enough experimental sensitivity?
- Measure "modified moments"

  $N_x^2 = M_x^2 - 2\Lambda E_x + \Lambda^2$ $\Lambda \sim 500\text{MeV}$
  - under better theoretical control, leading to reduction in theory errors
- Measure q^2 moments in $B \to X_c l\nu$
  - will constrain - or measure – non-perturbative soft charm effects ("intrinsic charm")
  - helps in reducing theory uncertainty on $|V_{cb}|$

- Revisit theory uncertainties
  - perturbative uncertainties of Wilson Coefficients
  - 20% and 30% variations of 2nd and 3rd order terms

Conclusions

- Consistency for moment measurements between experiments
- Good agreement of results from semileptonic and radiative B decays
- Precison measurements of SM parameters:
  - $|V_{cb}|$ at 2% level
  - Helping to reduce uncertainty on $|V_{ub}|$ to ~7%
    - probing consistency with sin(2β) and hence SM
  - m_b (<1%) and m_c (5%)
  - limited by theory uncertainties in many cases
- Radiative B decays
  - Consistent extrapolation for all experimental measurements
  - BR($B \to X_s \gamma$) @ 7%
  - important constraint on many NP models

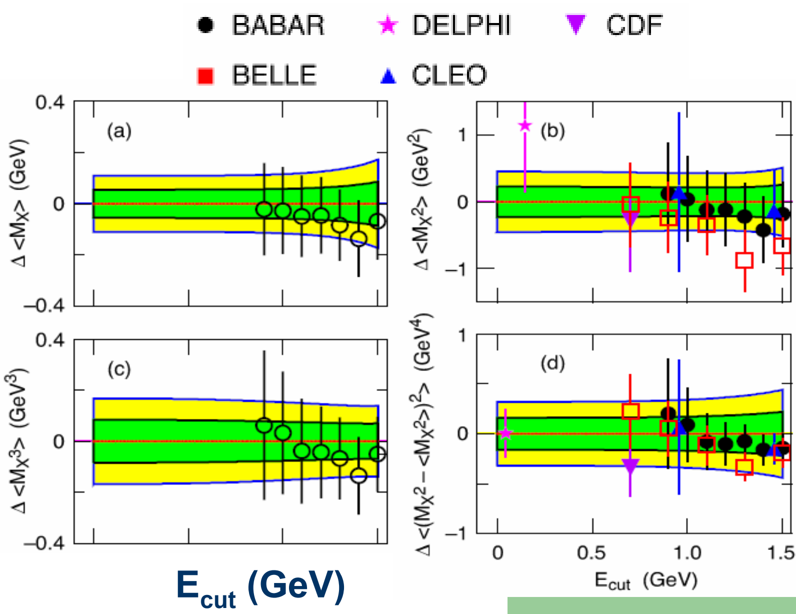
Backup Slides

Inclusive $|V_{cb}|$ - Fit to Moments

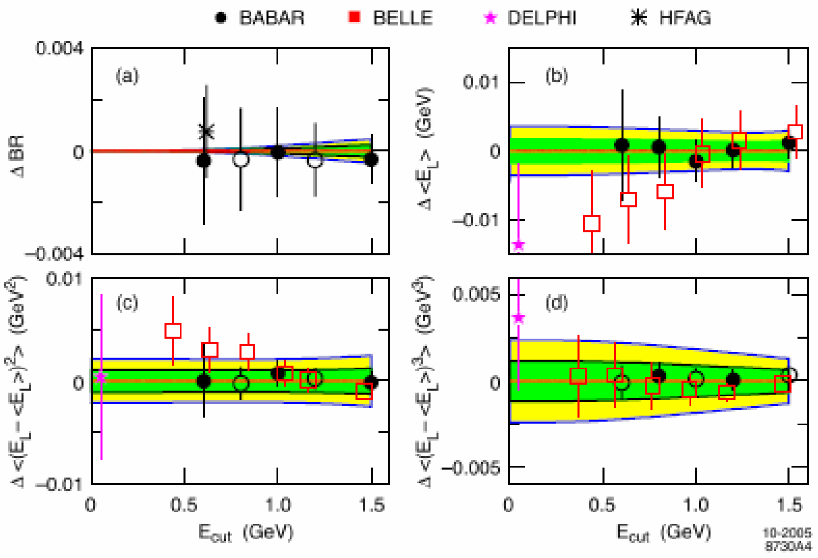
Based on calculations in kinetic scheme:

Benson, Bigi, Mannel & Uraltsev, hep-ph/0410080
Gambino & Uraltsev, hep-ph/0401063
Benson, Bigi & Uraltsev, hep-ph/0410080

Hadron Moments

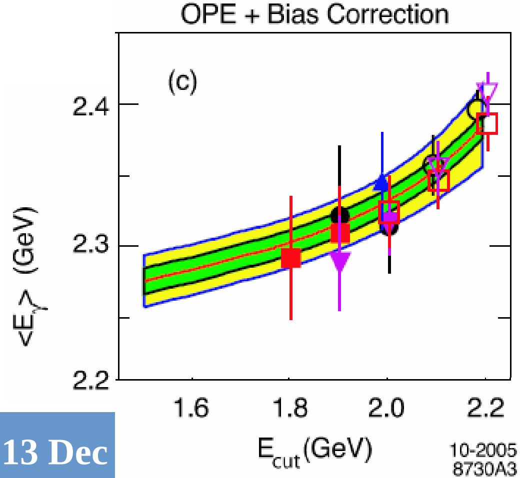


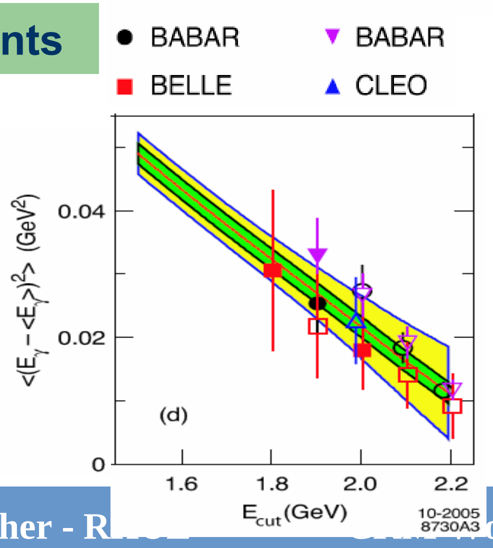
Lepton Moments



Photon Moments

Measurements highly correlated!





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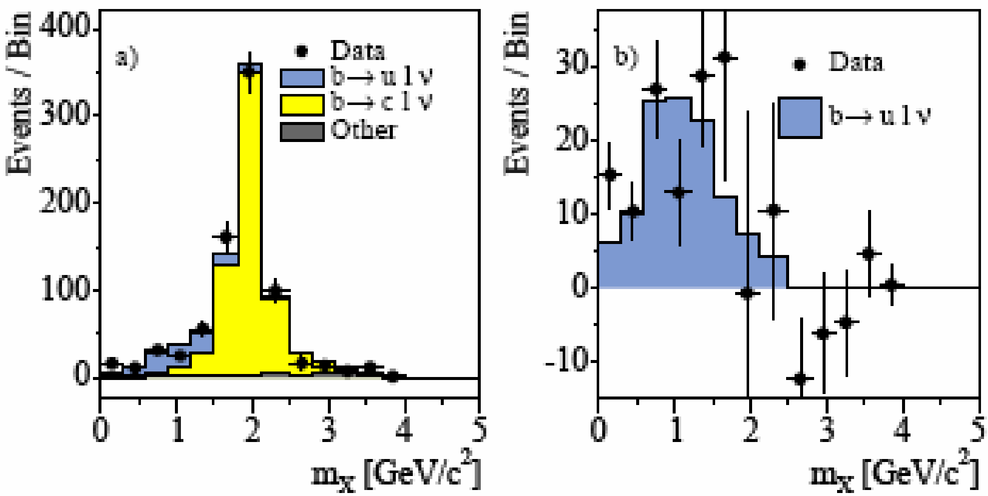
hep-ph/0507253

Inclusive $|V_{ub}|$

Measurement of hadronic mass spectrum in B->Xu lv decays:

- Measure hadronic mass spectum on the recoil of fully reconstructed B mesons
- Measurement of spectrum up to 2.5 GeV includes 96% of total rate

hep-ex/0601046



Standard local OPE for full rate:
Uraltsev
hep-ph/9905520
Hoang,Ligeti, Manohar
hep-ph/9811239

$$|V_{ub}| = 4.268\cdot 10^{-3}\cdot\sqrt{\frac{BR(B\to X_u\ell\nu)}{0.002}\,\frac{1.61ps}{\tau_B}}$$
$$\times(1\pm 0.012_{QCD}\pm 0.022_{HQE})\,.$$

reduced theory error as no extrapolation to full rate necessary

OPE: M_X < 2.50 GeV:

$|V_{ub}| = (3.84 \pm 0.70_{stat} \pm 0.30_{syst} \pm 0.10_{theo})\ 10^{-3}$