# Inclusive determinations of |Vcb| and the b quark mass

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# Theory:Scales,OPE

b quark decay: short distance process, distance scale $1/m_b$

Hadronization: a long distance process, distance scale $1/\Lambda_{QCD}$

The scale separation $m_b >> \Lambda_{QCD}$ and unit probability to Hadronize justifies the OPE of the T-product of currents (and HQET fields)

$$d\,\Gamma \sim \sum_X \langle B|J^{\mu\,\dagger}|X\rangle\,\langle X|J^{\nu}|B\rangle\,L_{\mu\,\nu}$$

$$\sim \mathrm{Im}\langle B|T\left(J^{\mu\,\dagger},\,J^{\nu}\right)|B\rangle\,L_{\mu\,\nu}$$

(duality violation ?)

$$T\left(J^{\dagger},J\right) \sim C_0(\mu)\bar{h}_v\,h_v + \frac{C_1(\mu)}{m_b^2}\,\bar{h}_v\,(iD)^2\,h_v \;\; + \frac{C_2(\mu)}{m_b^2}\,\bar{h}_v\,\sigma^{\alpha\,\beta}\,G_{\alpha\,\beta}\,h_v + \ldots$$

Giving the decay width

$$\Gamma(\bar{B}\to X_c\,\ell\,\nu_\ell) = \frac{G_F^2\,|V_{cb}|^2\,m_b^5}{192\,\pi^3}\left(f(\frac{m_c^2}{m_b^2})\left(1+\frac{\lambda_1}{2\,m_b^2}\right)+g(\frac{m_c^2}{m_b^2})\,\frac{\lambda_2}{2\,m_b^2}+\cdots\right)$$

Alternative definitions: $\mu_\pi^2 = -\lambda_1 + \mathcal{O}(\alpha_s)$

$$\mu_G^2 = 3\,\lambda_2$$

# Trouble with the Pole Mass

The pole mass $m_b^{pole}$ appears in the observables, if one relates it to the meson mass in the simplest way

$$m_B = m_b + \bar{\Lambda} + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m_b^2}\right)$$

the resulting perturbation theory is poorly behaved.

# 1S Mass Scheme

Hoang, Ligeti, Manohar

The 1S scheme relates $m_b^{pole}$ to a mass parameter $m_b^{1S}$

$$\frac{m_b^{1S}}{m_b^{pole}} = 1 - \frac{(\alpha_s C_F)^2}{8}\Big\{1\epsilon + \frac{\alpha_s}{\pi}\Big[\Big(\ell + \frac{11}{6}\Big)\beta_0 - 4\Big]\epsilon^2 + \mathcal{O}\big(\epsilon^3\big)\Big\},$$

exploit the fact that $\frac{m_\Upsilon}{2} - m_b^{1S} = \Lambda_{1S}$. $\qquad \ell = \ln\left(\frac{\mu}{m_b\,\alpha_s\,C_F}\right)$

The $\ell$ terms exponentiate so that $\alpha_s^{n+1}$ terms in this expression count as $\epsilon^n$

The $\alpha_s^n$ terms in the B decay rate in terms of $m_b^{pole}, m_c^{pole}$ count as $\epsilon^n$

Combine terms that are the same order in $\epsilon^n$

# Mass Scheming

Perturbative behavior improved:

Formally $\mathcal{O}\left(\Lambda_{QCD}/m_b\right)$ renormalon in $m_b^{pole}$ cancels $m_b^{1S}/m_b^{pole}$

Practically for $\bar{B} \to X_c\, e^-\, \bar{\nu}$

$1 - 0.12\,\epsilon - 0.07\,\epsilon^2 (\mathrm{BLM})$ Improved to $1 - 0.096\,\epsilon - 0.029\,\epsilon^2 (\mathrm{BLM})$

Consider the size of the corrections we are including to get |Vcb|, in the 1S scheme:

$$\Gamma = \frac{G_F^2\,|V_{cb}|^2}{192\,\pi^3}\,(0.534)\left(\frac{m_\Upsilon}{2}\right)^5 \Big[1 - 0.22\frac{\Lambda_{1S}}{500\,\mathrm{Mev}} - 0.011\frac{\Lambda_{1S}^2}{(500\,\mathrm{Mev})^2} - 0.052\frac{\lambda_1}{(500\,\mathrm{Mev})^2} - 0.071\frac{\lambda_2}{(500\,\mathrm{Mev})^2}$$

$$-0.006\frac{\lambda_1\,\Lambda_{1S}}{(500\,\mathrm{MeV})^3} + 0.011\frac{\lambda_2\,\Lambda_{1S}}{(500\,\mathrm{MeV})^3} - 0.006\frac{\rho_1}{(500\,\mathrm{MeV})^3} + 0.008\frac{\rho_2}{(500\,\mathrm{MeV})^3}$$

$$+0.011\frac{T_1}{(500\,\mathrm{MeV})^3} + 0.002\frac{T_2}{(500\,\mathrm{MeV})^3} - 0.017\frac{T_3}{(500\,\mathrm{MeV})^3} - 0.008\frac{T_4}{(500\,\mathrm{MeV})^3}$$

$$-0.096\,\epsilon - 0.030\,\epsilon_{BLM}^2 + 0.015\,\epsilon\,\frac{\Lambda_{1S}}{500\mathrm{MeV}}\Big]$$

At this level of precision we can't afford bad perturbative behavior.

# Choices

1. Other mass schemes exist, the PS scheme, the kinetic scheme.

   All threshold mass schemes seek to remove $\mathcal{O}\left(\Lambda_{QCD}/m_b\right)$ renormalon

   Scheme dependence exists. Try several. Make sure it is under control.

2. Have 7 parameters in a global fit up to $\mathcal{O}\left(\Lambda_{QCD}^3/m_b^3\right)$ in order to extract |Vcb| and $m_b^{scheme}$. Expand in $\Lambda_{QCD}/m_c$ or not?

   Different parameters fit to if you expand or do not expand.
   No evidence of a problem.

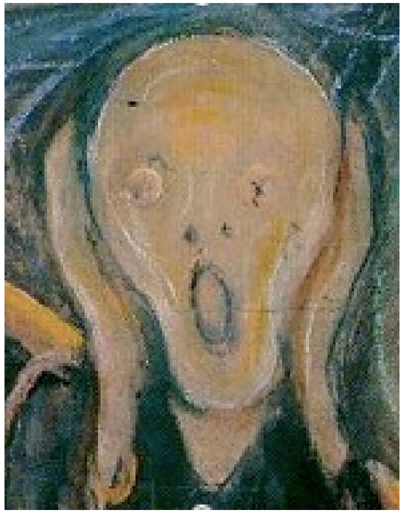
3. How to treat theoretical error.

   What is the error due to the full $\alpha_s^2$ perturbative terms?
   We have the $\alpha_s^2\,\beta_0$ piece as an estimate of these terms.

   $\alpha_s^2(m_b) \sim 0.05 \qquad \frac{\alpha_s^2(m_b)}{(4\,\pi)^2} \sim 0.0003 \qquad \epsilon_{BLM}^2/2 \sim 0.015$

   This issue is a cost of success. Make a reasonable choice, clearly state it.

# Fears



Munch, The Scream

What if theory error is underestimated? Expressions are truncated series, neglected terms,

$$\alpha_s \frac{\Lambda_{QCD}^2}{m_b^2},\ (full)\alpha_s^2,\ \frac{\Lambda_{QCD}^4}{m_b^2\, m_c^2}, \frac{\Lambda_{QCD}^4}{m_b^4}$$

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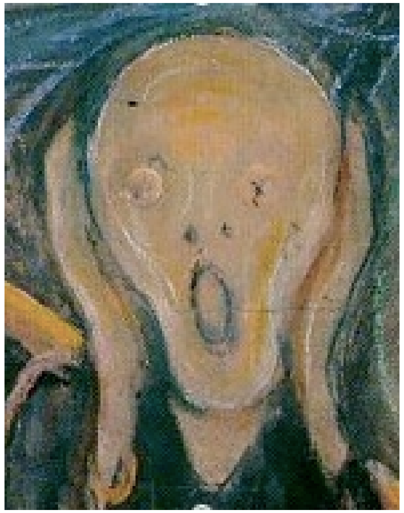

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## Reasons Not to Fear

Poor $\chi^2/\nu$ could indicate either of these. No evidence of a problem.

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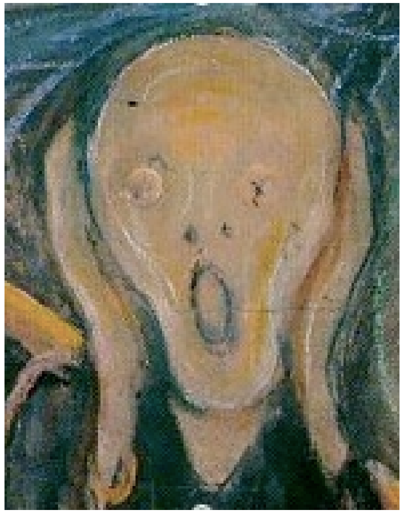

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$$D_3 = \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{0.7}\,\frac{d\,\Gamma}{d\,E_\ell}\,d\,E_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{1.5}\,\frac{d\,\Gamma}{d\,E_\ell}\,d\,E_\ell}$$

Theory Prediction: $0.5190 \pm 0.0007$
Measurement: $0.5193 \pm 0.0008$

$$D_4 = \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{2.3}\,\frac{d\,\Gamma}{d\,E_\ell}\,d\,E_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{2.9}\,\frac{d\,\Gamma}{d\,E_\ell}\,d\,E_\ell}$$

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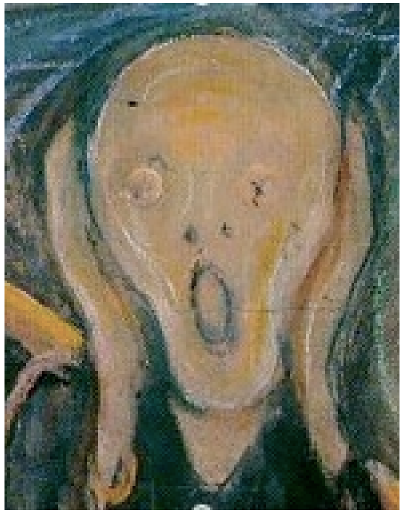

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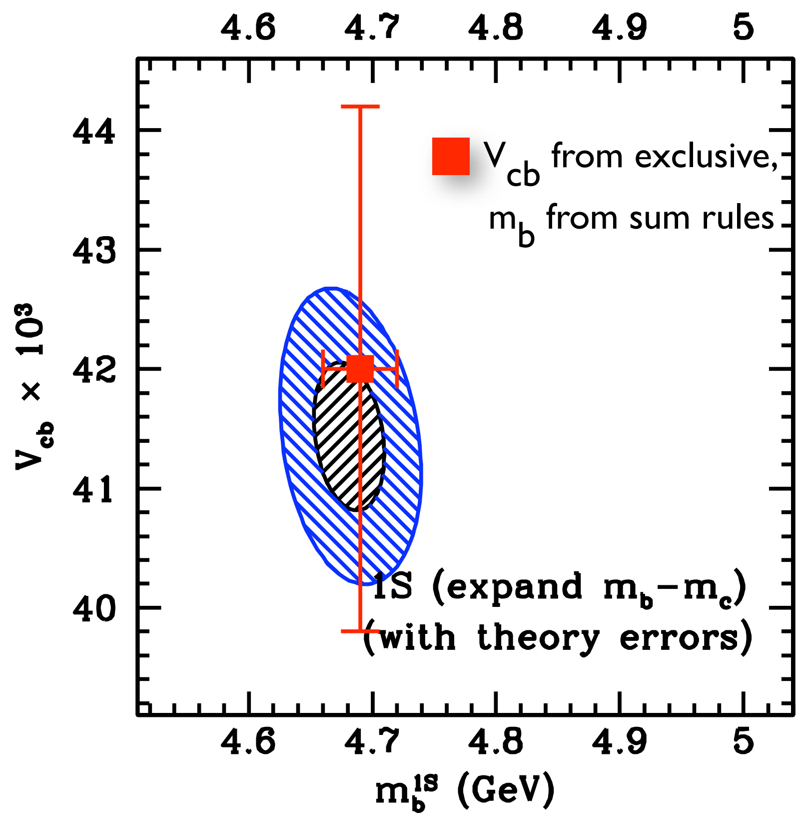
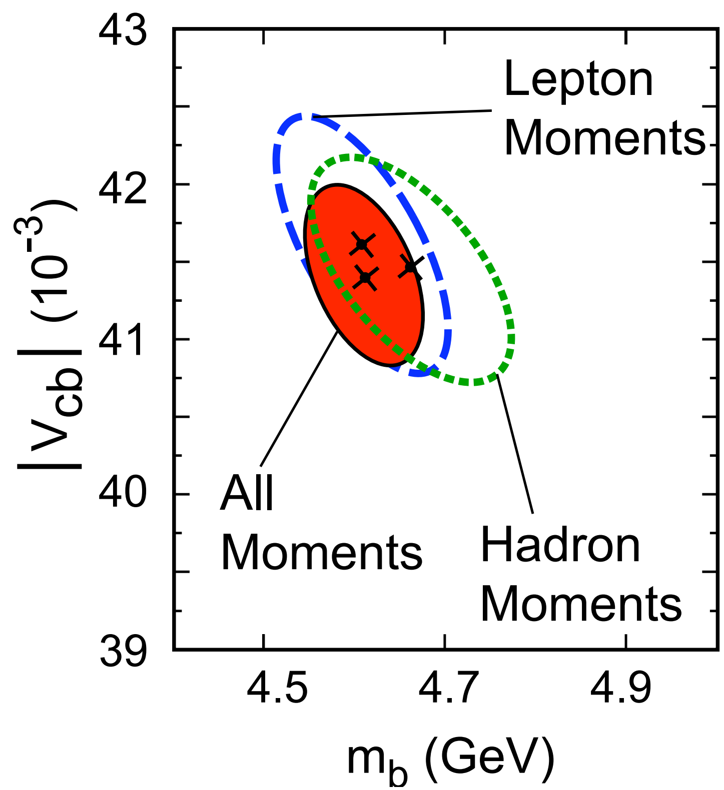
Another, independent determination of $m_b^{1S}$ agrees. Hoang's determination

# Results




Bauer, Ligeti, Luke, Manohar, Trott




BaBar using Gambino & Uraltsev

$$|V_{cb}| = (41.4 \pm 0.6 \pm 0.1\,\tau_B) \times 10^{-3}$$

$$m_b^{1S} = 4.68 \pm 0.03\,\mathrm{GeV}$$

$$|V_{cb}| = (41.4 \pm \underset{\text{EXP}}{0.4} \pm \underset{\text{HQE}}{0.4} \pm \underset{\text{TH}}{0.6}) \times 10^{-3}$$

$$m_b^{kin}(1\mathrm{GeV}) = (4.61 \pm 0.05 \pm 0.04 \pm 0.02)\,\mathrm{GeV}$$

## Hopes

To improve the theory side of the determinations:

| | ~ error | Assessment |
|---|---|---|
| Full $\alpha_s^2$ dependence for observables | 1 % | Very, very challenging. |
| Determine the $\alpha_s \frac{\Lambda_{QCD}^2}{m_b^2}$ terms | < 1 % | Easier that $\alpha_s^2$ |
| Determine the $\frac{\Lambda_{QCD}^4}{m_b^2 m_c^2}$ terms | 0.1% | Recent progress, see Dassinger, Mannel, Turczyk |

# Conclusions

The fits work remarkably well. $|V_{cb}| = (41.4 \pm 0.6 \pm 0.1\,\tau_B) \times 10^{-3}$

$m_b^{1S} = 4.68 \pm 0.03\,\mathrm{GeV}$

Testing error assigned by crosschecks is important to increase confidence in the result.

To push the theory error in |Vcb| and $m_b^{scheme}$ lower one needs to fulfill some of the hopes above.