Threshold Resummation in Semi-Inclusive B decays

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The work presented in this talk is based on the following papers:

U. Aglietti, G. Ferrera and G. Ricciardi, Phys. Rev. D74 (2006), [hep-ph/0507285].

U. Aglietti, G. Ferrera and G. Ricciardi, Phys. Rev. D74 (2006), [hep-ph/0509095].

U. Aglietti, G. Ferrera and G. Ricciardi, Phys. Rev. D74 (2006), [hep-ph/0509271].

U. Aglietti, G. Ferrera and G. Ricciardi, [hep-ph/0608047].

U. Aglietti, G. Corcella and G. Ferrera, [hep-ph/0610035].

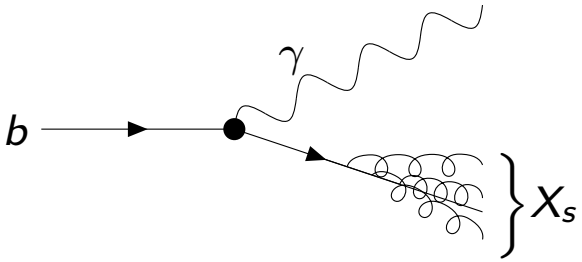
U. Aglietti, L. Di Giustino, G. Ferrera and L. Trentadue, [hep-ph/0612073].

Outline

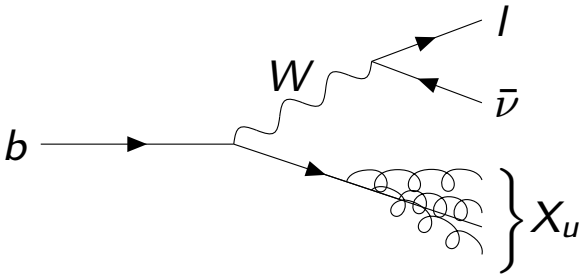
1. Semi-inclusive B decays
2. Analytic QCD coupling
3. Phenomenological Analysis
4. Conclusions and Perspectives

Semi-inclusive B decays

$$b \to X_q + \langle \text{ non QCD partons } \rangle, \quad q \equiv u, s, d$$



Radiative decay:
$b \to X_s + \gamma$



Semileptonic charmless decay:
$b \to X_u + l + \bar{\nu}$

- threshold region: $E_X \gg m_X, \quad Q = 2\,E_X$
- the perturbative expression involves logarithmically enhanced coefficients depending on the kinematics:

$$\sigma(y) = \sum_{n=1}^{\infty} \sum_{k=1}^{2n} c_{n,k}\, \alpha_S^n(Q)\, \ln^k y\ , \quad y = \frac{m_X^2}{Q^2}\ .$$

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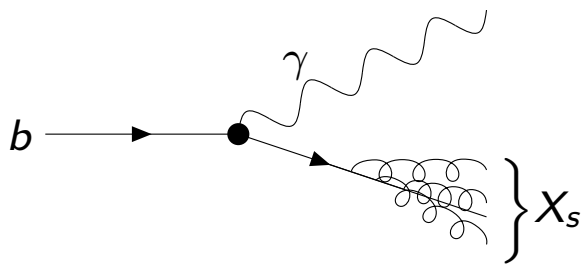

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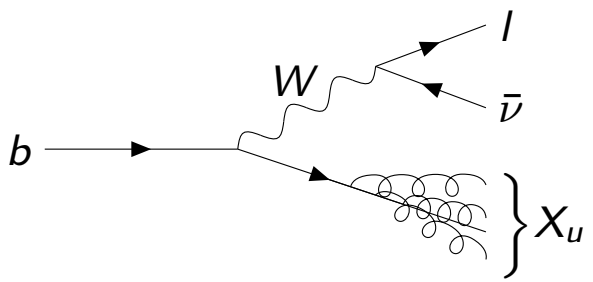

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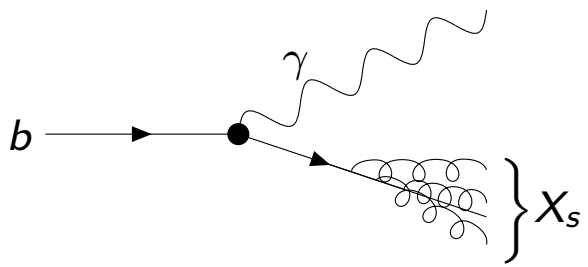

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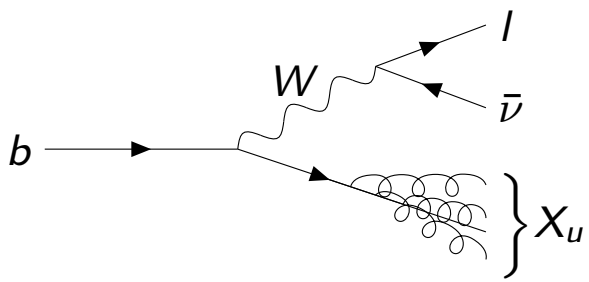

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In the Mellin space the threshold resummed form factor reads [Sterman ('87), Catani & Trentadue ('89)]:

$$\ln\sigma_N = \int_0^1 dy \frac{(1-y)^{N-1}-1}{y} \left\{ \int_{Q^2y^2}^{Q^2y} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] + B[\alpha_S(Q^2y)] + D[\alpha_S(Q^2y^2)] \right\}$$

$$\text{where} \quad A(\alpha_S) = \sum_{n=1}^{\infty} A_n \, \alpha_S^n, \quad B(\alpha_S) = \sum_{n=1}^{\infty} B_n \, \alpha_S^n, \quad D(\alpha_S) = \sum_{n=1}^{\infty} D_n \, \alpha_S^n \, .$$

When $y = \frac{m_X^2}{Q^2} \to 0$ the integration in $k_\perp^2$ involves the QCD coupling evaluated at the Landau pole: it is necessary a prescription, for example the minimal prescription (MP) [Catani, Mangano, Nason & Trentadue ('96)].

Our approach is different, we treat the unphysical Landau pole from the very beginning using the analytic QCD coupling [Aglietti, Ricciardi ('04)].

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Analytic QCD coupling

- Standard QCD coupling: physical cut at $\mu^2 < 0$ and unphysical pole at $\mu^2 = \Lambda^2_{QCD}$:

$$\alpha_S^{lo}(\mu^2) = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda^2_{QCD}}} \, .$$

- Analytic QCD coupling: same discontinuity along the cut but analytic elsewhere in the complex plane [Shirkov & Solovtsov ('97)]:

$$\bar{\alpha}_S^{lo}(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{\ln Q^2/\Lambda^2_{QCD}} - \frac{\Lambda^2_{QCD}}{Q^2 - \Lambda^2_{QCD}}\right] \, , \quad LO \quad space-like$$

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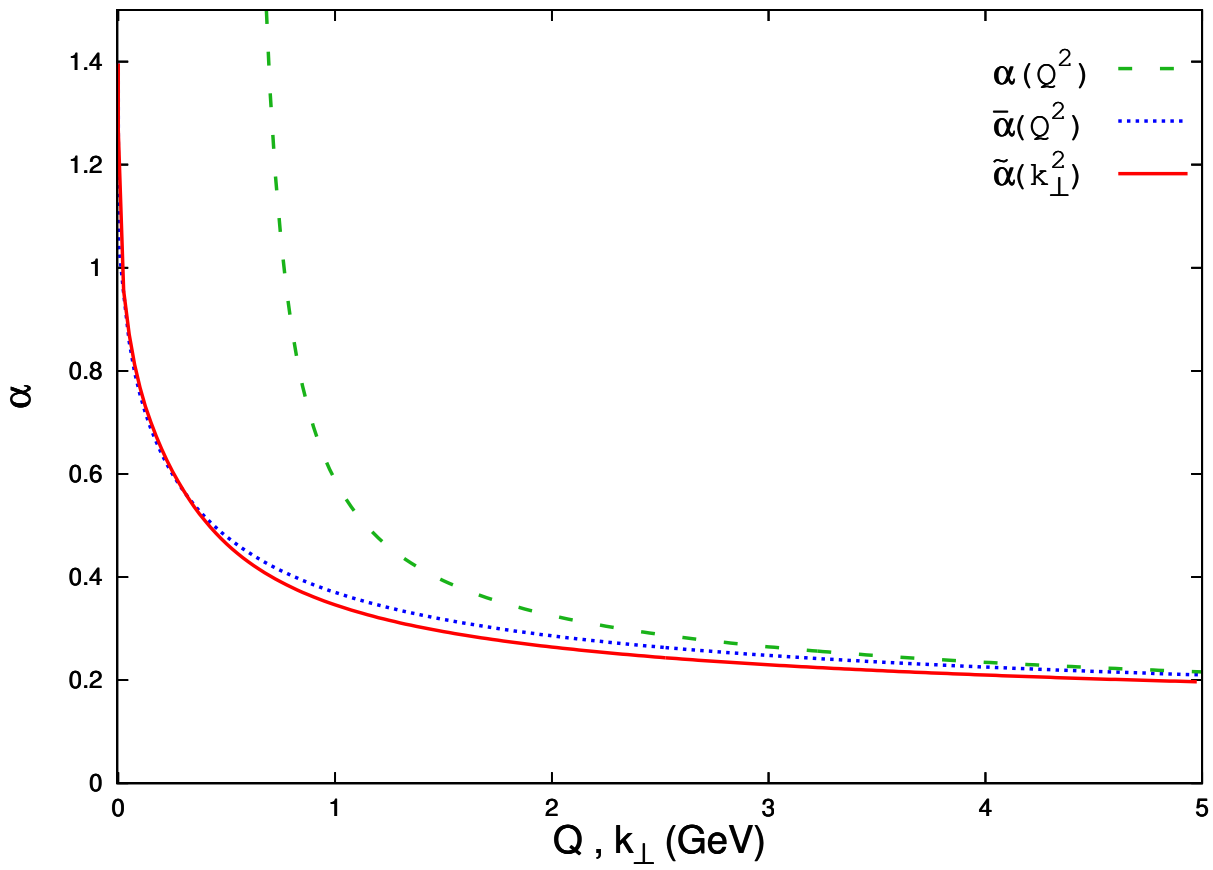

Figure 1: Time-like and space-like analytic couplings compared with the standard one.

Phenomenological Analysis

b-quark fragmentation: $e^+e^- \to Z^0 \to B + X, \qquad x_b = \frac{2E_b}{m_Z}$

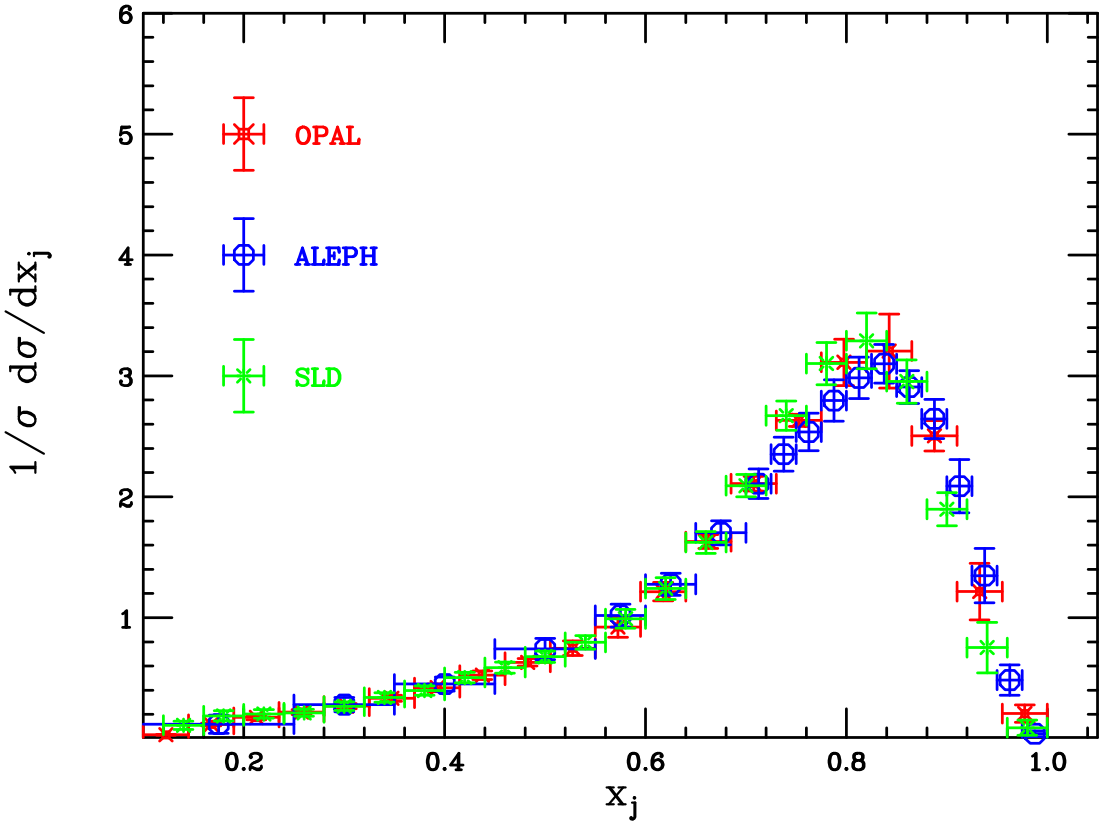

Figure 2: B-hadron spectrum in e^+e^- annihilation at Z^0 peak: prevision of the model compared with experimental data [Aleph ('01), Delphi ('02), SLD ('00)].

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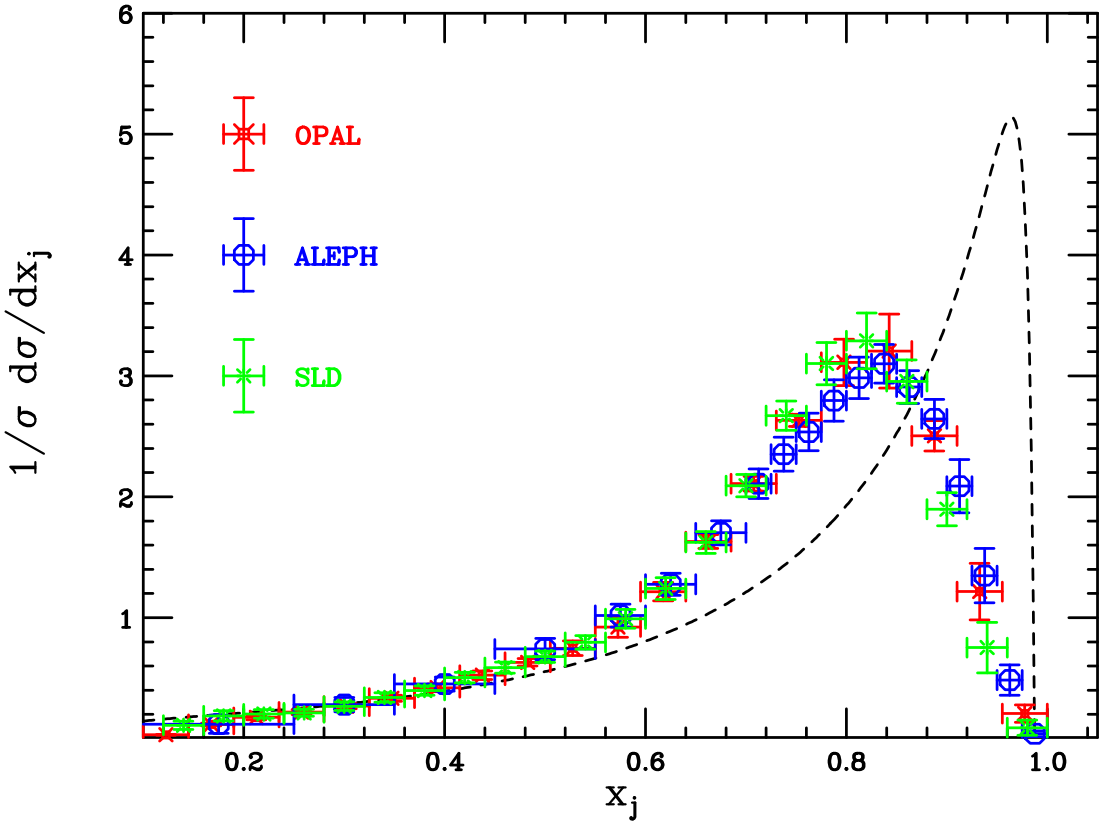

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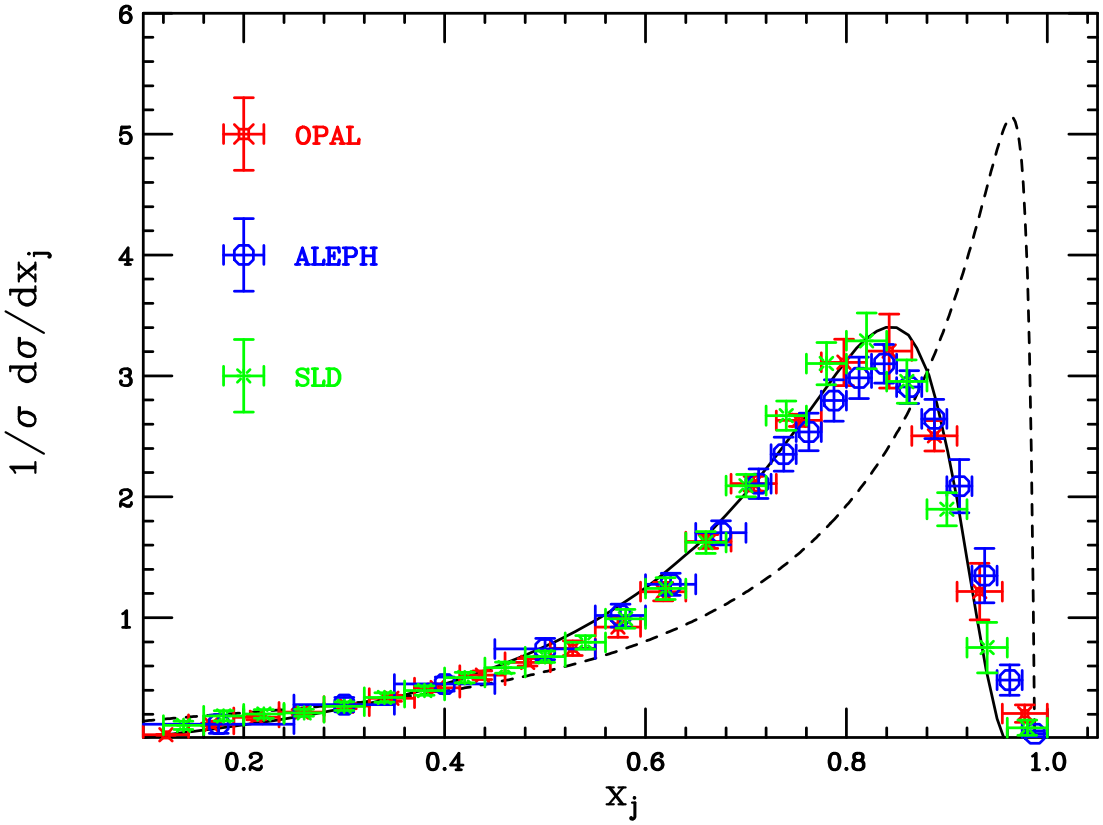

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Radiative decay: hadron mass distribition

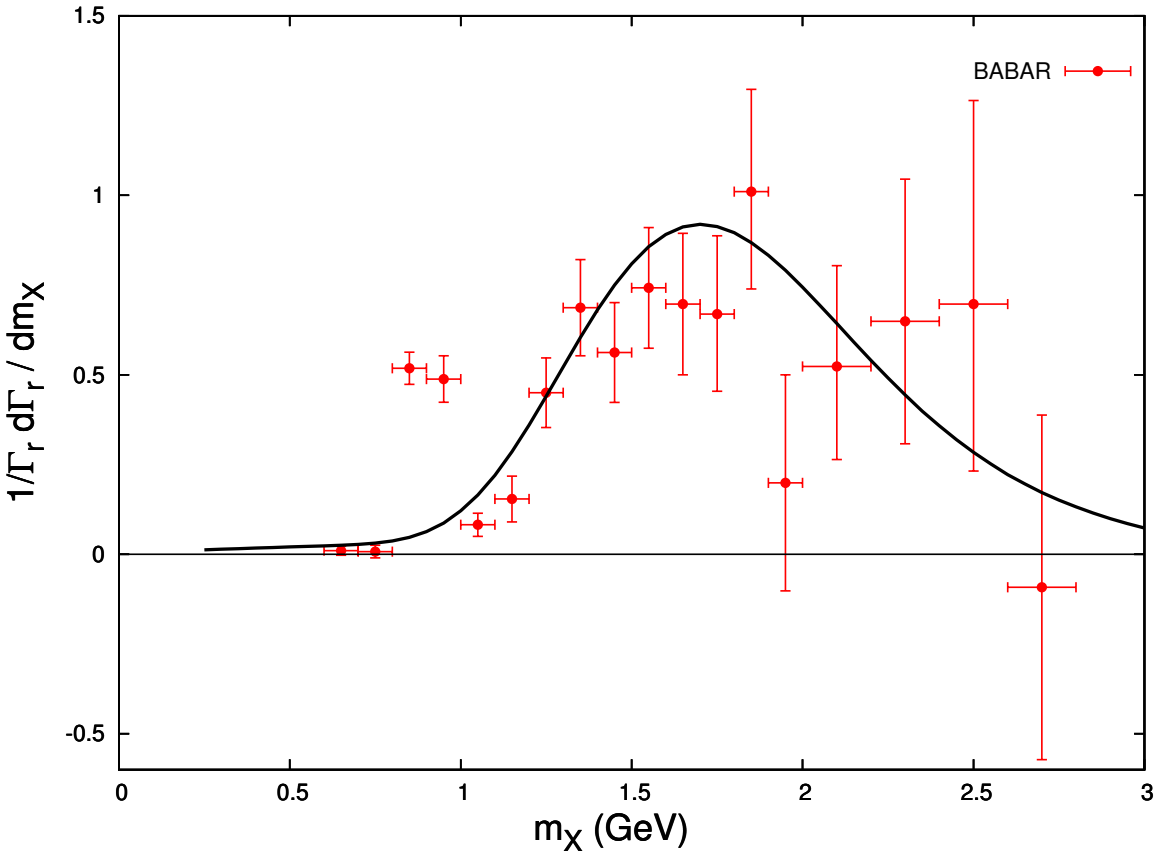

Figure 3: Invariant hadron mass distribution in the radiative decay: prevision of the model compared with the experimental data [BaBar ('05)]. The K^* peak cannot clearly be accounted for in a perturbative QCD framework.

Radiative decay: photon energy distribution

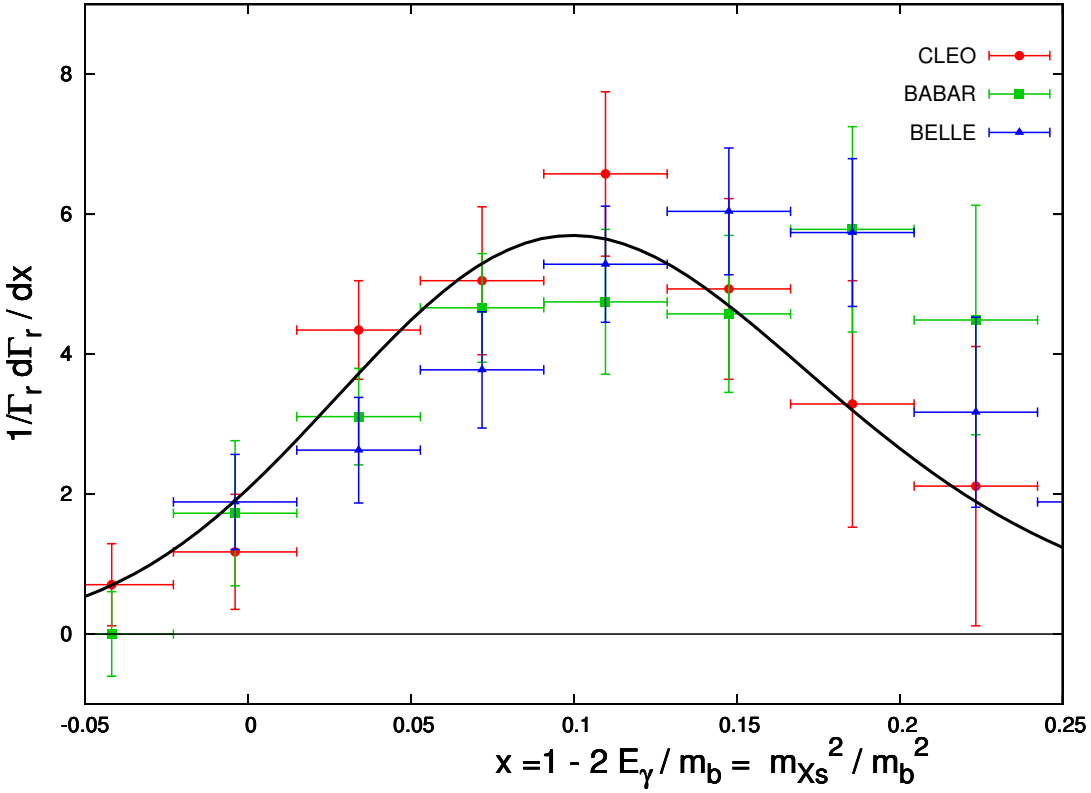

Figure 4: Photon energy spectrum in the radiative decay: prevision of the model compared with data [Cleo ('01), BaBar ('05), Belle ('05)]. To model the Doppler effect related to the motion of the B mesons , we have convoluted the theoretical curve with a Gaussian distribution with $\sigma \sim 180$ MeV.

Semileptonic decay: hadron mass distribution

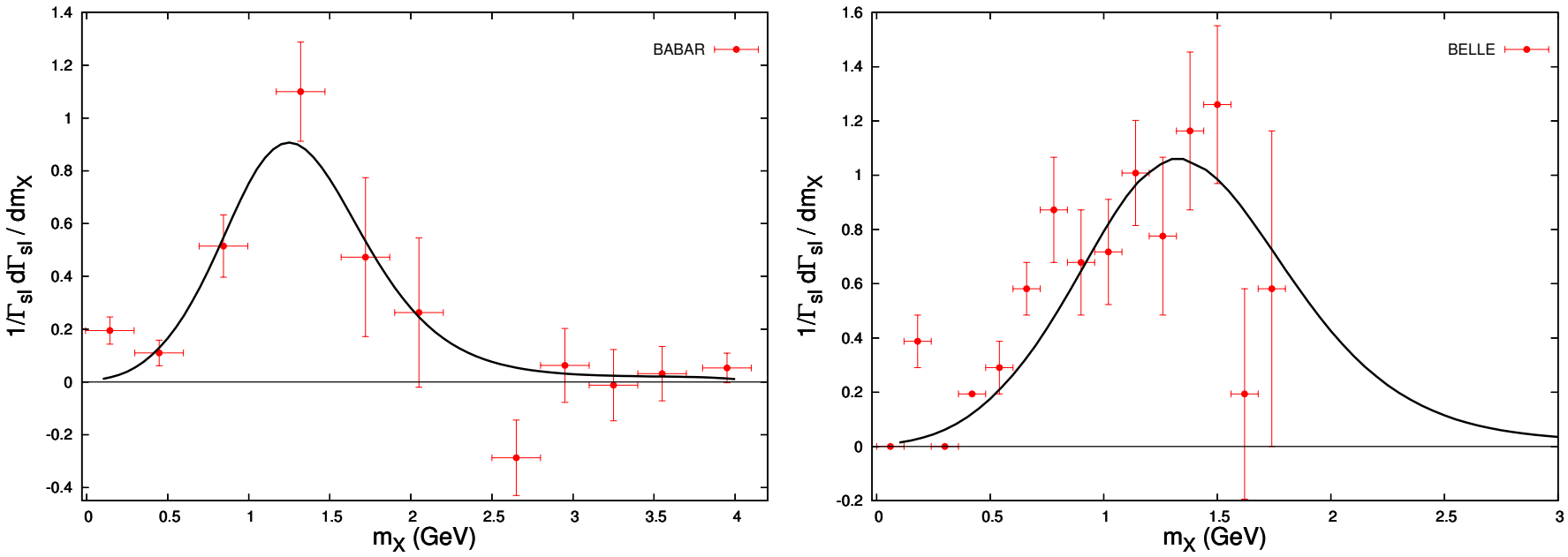

Figure 5: Invariant hadron mass distribution in the semileptonic decay: prevision of the model compared with the experimental [Belle ('04), BaBar ('05)]. Note the π and the ρ peaks at small hadron masses.

Semileptonic decay: electron energy distribution

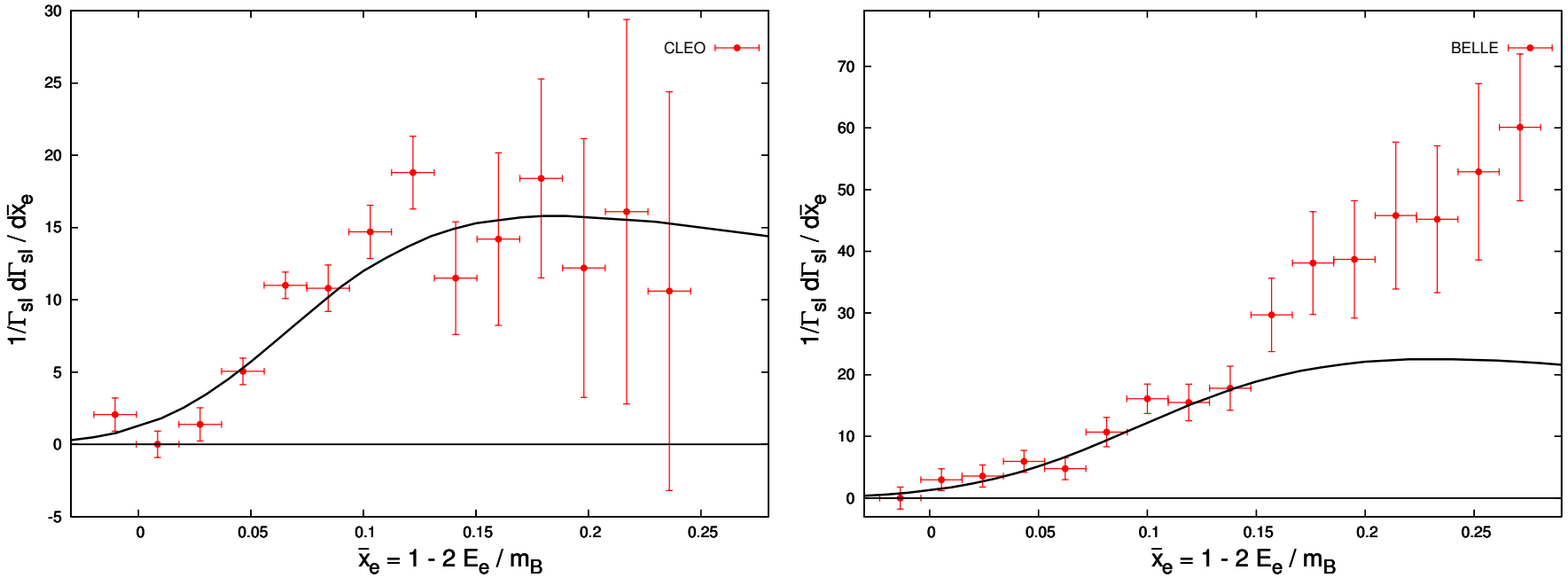

Figure 6: Inclusive charmless electron spectrum in the semileptonic decay: prevision of the model compared with data [Cleo ('01) and Belle ('04)]. To include the Doppler effect, we have convoluted our spectrum with a Gaussian distribution with $\sigma \sim 100$ MeV. Our model predicts a maximum around the energy $E_e = 2$ GeV, below which data are not available.

Semileptonic decay: electron energy distribution

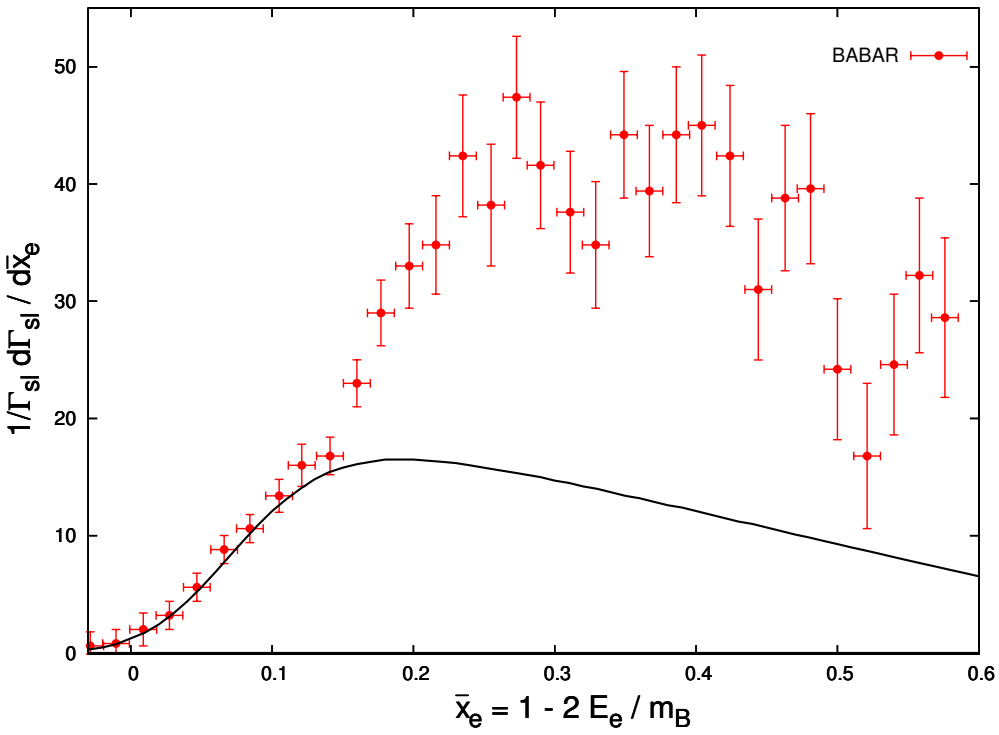

Figure 7: Inclusive charmless electron spectrum in the semileptonic decay: prevision of the model compared with preliminary data [Babar ('05)]. To include the Doppler effect, we have convoluted our spectrum with a Gaussian distribution with $\sigma \sim 100$ MeV. We do not known whether the discrepancy is related to a deficiency of our model or to an under-estimate charm background.

Extraction of $\alpha_S(m_Z)$ and V_{ub} from the data

$b \to s\,\gamma$

$$\alpha_S(m_Z) = 0.117 \pm 0.004 \qquad (E_\gamma : CLEO,\ \sigma_\gamma = 150\ MeV)$$
$$\alpha_S(m_Z) = 0.129 \pm 0.005 \qquad (E_\gamma : BABAR,\ \sigma_\gamma = 200\ MeV)$$
$$\alpha_S(m_Z) = 0.130 \pm 0.005 \qquad (E_\gamma : BELLE,\ \sigma_\gamma = 200\ MeV)$$

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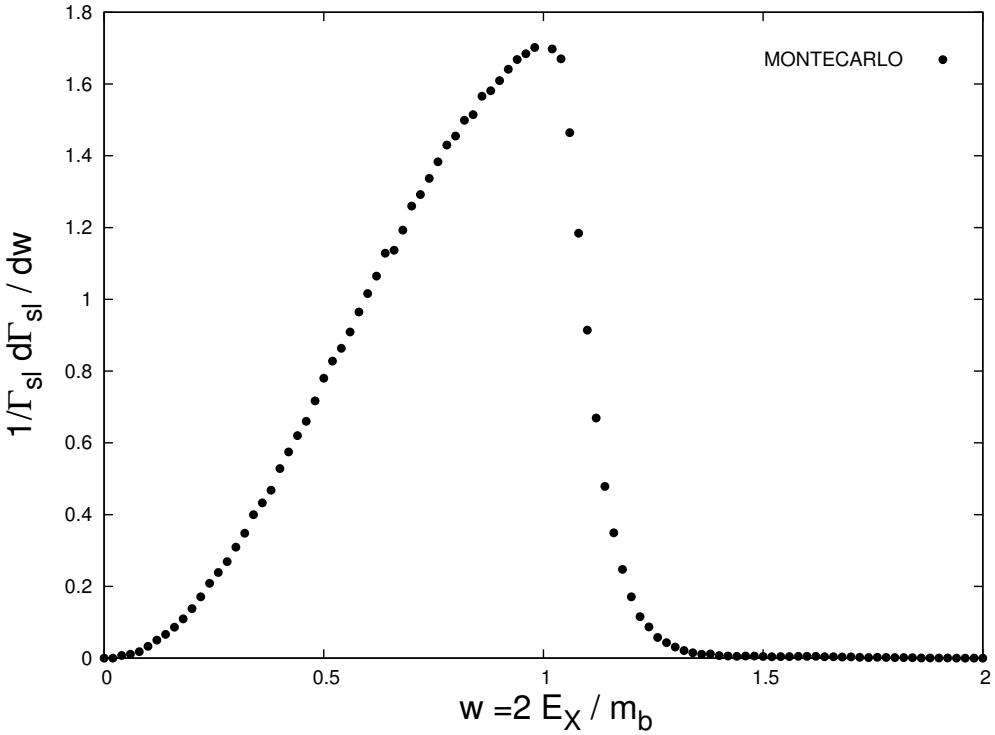

Figure 8: Hadron energy distribution in the radiative dacay generated by a montecarlo based on the model.

Semi-leptonic charmed decay and V_{cb}

To describe the process $B \to X_c\ l\ \nu$ we need a new formalism to take in account the non-vanishing charm mass m_c: [Aglietti, Di Giustino, G.F. and Trentadue ('06)]

$$\sigma_N(Q^2, m^2) = \sigma_N(Q^2)\ \delta_N(Q^2, m^2), \qquad r \equiv \frac{m^2}{Q^2} \simeq 0.1$$

$$\ln\delta_N = \int_0^1 dy \frac{(1-y)^{r(N-1)}-1}{y} \left\{ -\int_{m^2y^2}^{m^2y} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] - B[\alpha_S(m^2y)] + D[\alpha_S(m^2y^2)] \right\}$$

Using this formula, the full $O(\alpha_S)$ triple differential distribution [Trott ('04), Aquila, Gambino, Ridolfi & Uraltsev ('05)] and the model previously described we are confident that we can provide quantitative description of the data.

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Conclusions and Perspectives

- Through the analytic QCD coupling and NNLL threshold resummation we have developed a model that includes the large non-perturbative effects in the semi-inclusive B decays: it describes with good accuracy the measured spectra without introducing any ad hoc non-perturbative component.
- We have found a remarkable disagreement with the electron spectrum in the semileptonic charmless decay (BaBar and Belle). We suppose a possible underestimate of the charm background.
- It would be interesting if the experimentalist made a statistical analysis of the compatibility of our model with the data to extract V_{ub}.
- Using a new resummation formalism recently developed together with available fixed order calculation we will carry out a phenomenological analysis of semileptonic charmed decays soon.

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Conclusions and Perspectives

- Through the analytic QCD coupling and NNLL threshold resummation we have developed a model that includes the large non-perturbative effects in the semi-inclusive B decays: it describes with good accuracy the measured spectra without introducing any ad hoc non-perturbative component.
- We have found a remarkable disagreement with the electron spectrum in the semileptonic charmless decay (BaBar and Belle). We suppose a possible underestimate of the charm background.
- It would be interesting if the experimentalist made a statistical analysis of the compatibility of our model with the data to extract V_{ub}.
- Using a new resummation formalism recently developed together with available fixed order calculation we will carry out a phenomenological analysis of semileptonic charmed decays soon.