$|V_{ub}|$ from shape-function free relations

CKM workshop in Nagoya, Japan

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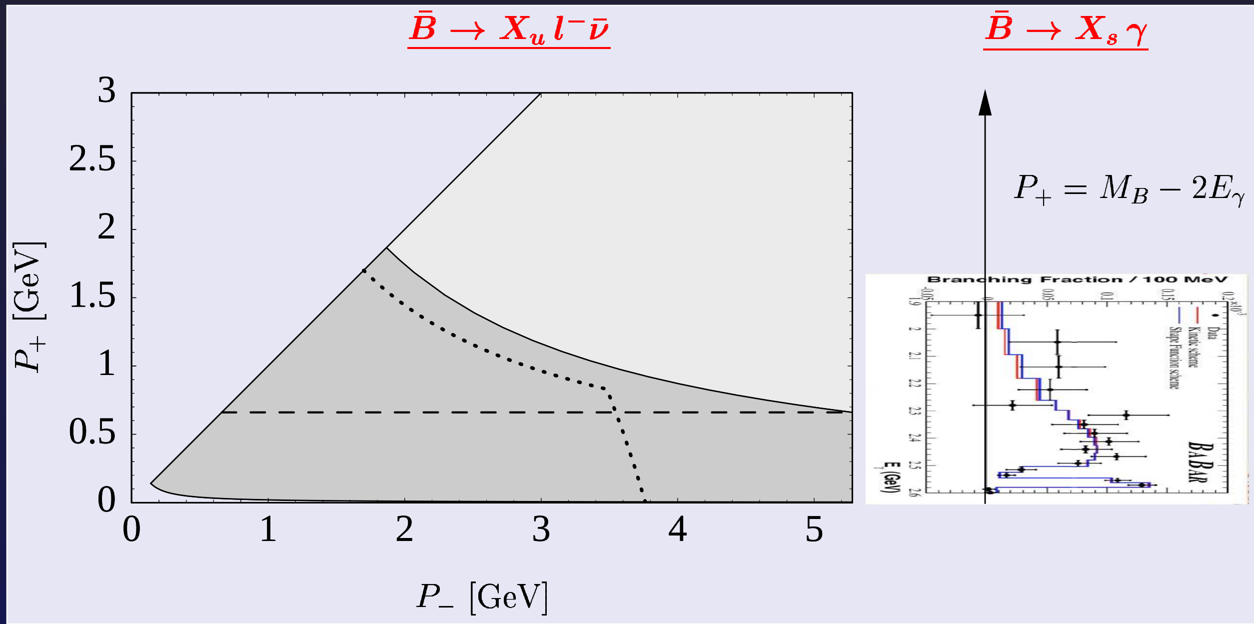
Introduction



$$\underbrace{\Gamma_u\Big|_{\rm cut}}_{\text{Exp. Input}} = |V_{ub}|^2 \int_0^\Delta dP_+ \, W(\Delta, P_+) \underbrace{\frac{1}{\Gamma_s}\frac{d\Gamma_s}{dP_+}}_{\text{Exp. Input}} + |V_{ub}|^2 \, \Gamma_{\rm rhc}\Big|_{\rm cut} .$$

Biggest advantage: No need to extract the leading shape function, i.e. model independent, no complications from resonances, etc.

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- *any cut with "SF events"* [B.L., hep-ph/0511098].

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Factorization

- From $H_{u,s}\, J \otimes S$ for both $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_u\, l^-\bar{\nu}$ at leading power follows $W^{(0)} = H \otimes Y$.
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$$\mathcal{F}_\gamma(P_+) = \hat{S}(P_+) + \frac{1}{M_B - P_+}\mathcal{F}_\gamma^{(1)}(P_+)\,,$$

$$\mathcal{F}_1(P_+, y) = S(P_+) + \frac{1}{M_B - P_+}\mathcal{F}_1^{(1)}(P_+, y)\,,$$

$$\mathcal{F}_3(P_+, y) = \frac{1}{M_B - P_+}\mathcal{F}_3^{(1)}(P_+, y)\,.$$

$$\Gamma_{\rm rhc} = \frac{G_F^2\, U(\mu_h, \mu_i)}{16\pi^3} \int_0^\Delta dP_+\, (M_B - P_+)^4 \int_0^{\varepsilon_{\rm max}(\Delta, P_+)} d\varepsilon \int_\varepsilon^1 dy\, y^{-2a_\Gamma}$$

$$\times \left[(y-\varepsilon)(1-y+\varepsilon)(\mathcal{F}_1^{(1)} - \mathcal{F}_\gamma^{(1)}) + (y-\varepsilon)\varepsilon\, \mathcal{F}_3^{(1)} \right]\,.$$

Examples

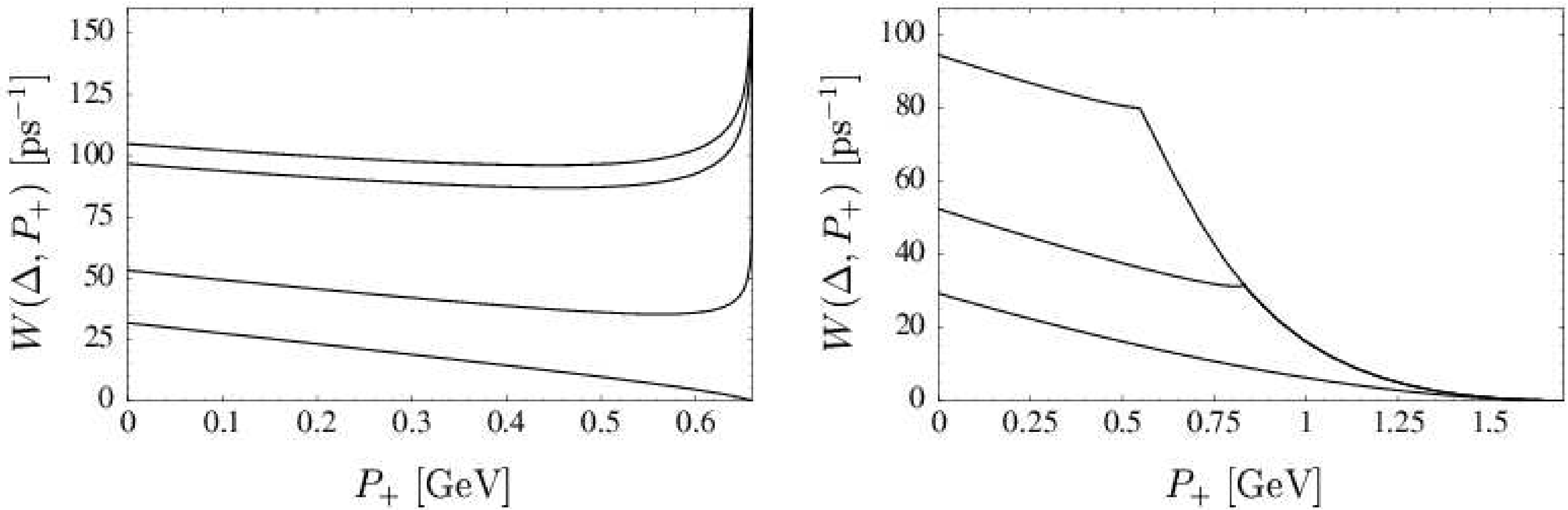

Figure 2: Examples of the weight function for different kinematic cuts. LEFT: Cutting on $P_+ \leq \Delta = 0.66$ GeV and $E_l > E_0$. From top to bottom the four functions are for $E_0 = 0$, $E_0 = 1$ GeV, $E_0 = 2$ GeV, and $E_0 = (M_B - \Delta)/2$. RIGHT: Cutting on $M_X \leq M_0 = 1.7$ GeV, $q^2 > q_0^2$, and $E_l > 1$ GeV. The three functions are for $q_0^2 = 0$ (top), $q_0^2 = 8$ GeV^2 (middle), and $q_0^2 = (M_B - M_0)^2$ (bottom).

Examples

$$\left.\frac{W\otimes d\Gamma_s+\Gamma_{\rm rhc}}{\rm ps^{-1}}\right|_{\substack{P_+\le 0.66\ \mathrm{GeV},\\ E_l\ge 1\ \mathrm{GeV}}} = 38.93\ ^{+2.23}_{-1.96}\,[\text{pert.}]\ \pm 1.42\,[\text{hadr.}]\ ^{+1.71}_{-1.67}\,[m_b]\ ^{+0.46}_{-0.63}\,[\text{pars.}]$$

$$\Rightarrow \quad \delta|V_{ub}^{\rm theory}|/|V_{ub}| = ^{+5.7}_{-5.0}\ \%$$

$$\left.\frac{W\otimes d\Gamma_s+\Gamma_{\rm rhc}}{\rm ps^{-1}}\right|_{\substack{M_X\le 1.7\ \mathrm{GeV},\\ E_l\ge 1\ \mathrm{GeV}}} = 48.14\ ^{+1.60}_{-1.82}\,[\text{pert.}]\ \pm 0.47\,[\text{hadr.}]\ ^{+1.93}_{-1.88}\,[m_b]\ ^{+1.18}_{-0.99}\,[\text{pars.}]$$

$$\Rightarrow \quad \delta|V_{ub}^{\rm theory}|/|V_{ub}| = ^{+4.5}_{-4.0}\ \%$$

$$\left.\frac{W\otimes d\Gamma_s+\Gamma_{\rm rhc}}{\rm ps^{-1}}\right|_{\substack{M_X\le 1.7\ \mathrm{GeV},\\ q^2\ge 8\ \mathrm{GeV}^2,\\ E_l\ge 1\ \mathrm{GeV}}} = 23.21\ ^{+1.33}_{-1.51}\,[\text{pert.}]\ \pm 0.43\,[\text{hadr.}]\ ^{+1.10}_{-1.07}\,[m_b]\ ^{+0.71}_{-0.70}\,[\text{pars.}]$$

$$\Rightarrow \quad \delta|V_{ub}^{\rm theory}|/|V_{ub}| = ^{+6.3}_{-5.0}\ \%$$

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- For cuts near the charm threshold, $E_0 = 2.31$ GeV, partial rate is power suppressed, i.e. has low efficiency.
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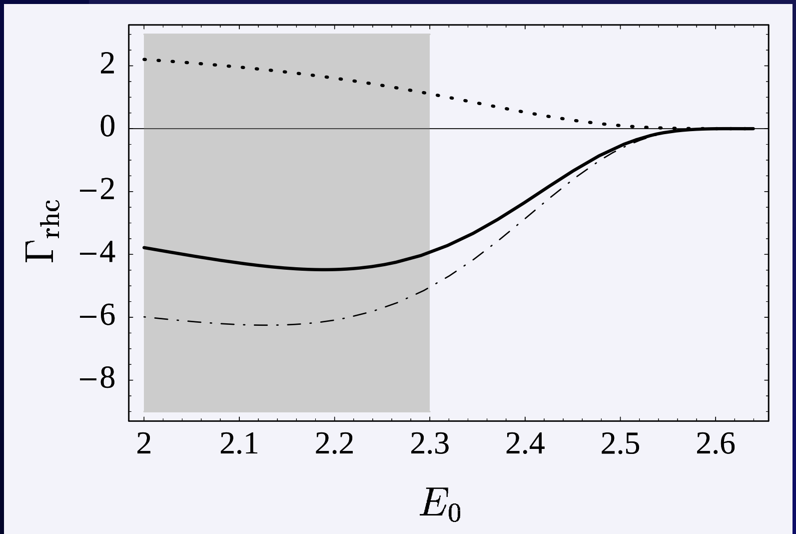
- These terms are giving rise to $\Gamma_{\rm rhc}$, which spoils the usefulness of the approach.
- Need to relax the cut far into the charm background region, near 2.0 GeV, to regain control.
- Can we learn something about SSFs by assuming stability of the resulting $|V_{ub}|$?

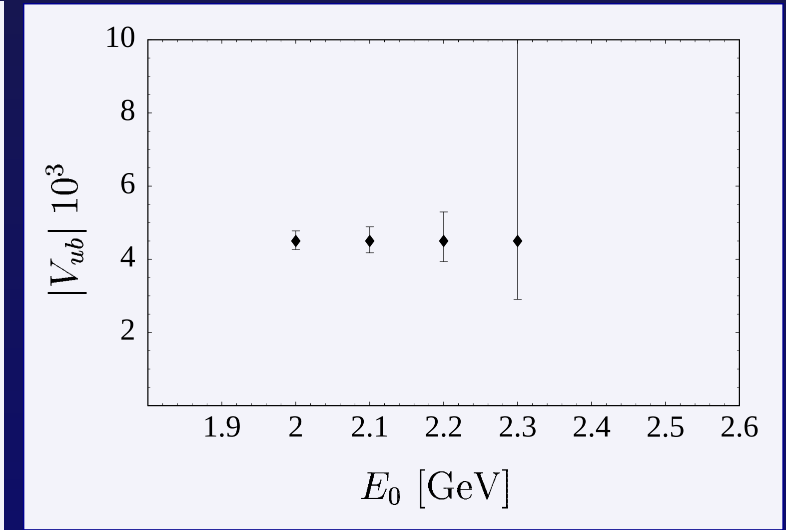
Lepton endpoint

[Thanks to V.B.Golubev, Y.I.Skovpen, V.Lüth for bringing this to our attention again.]

E_0 [GeV]	$W \otimes d\Gamma_s$	$\Gamma_{\rm rhc}$	$\Delta\Gamma_{\rm rhc}$
2.0	22.5	-5.6	$\pm$ 0.6
2.1	17.1	-6.3	$\pm$ 0.7
2.2	11.8	-6.7	$\pm$ 0.8
2.3	7.0	-6.0	$\pm$ 1.0

[Below, left Figure for $\mu_h = \mu_i$]





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- $\Gamma_u = |V_{ub}|^2 (W \otimes d\Gamma_s + \Gamma_{\rm rhc})$
  $W = H \otimes Y + \ldots$
- Typically $\sim 5\%$ theory error on $|V_{ub}|$ for cuts with high efficiency.
- Lepton endpoint: large SSF contributions and uncertainty
  Error acceptable for $E_0 \sim 2.0$ GeV, but increases rapidly for higher E_0!
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Thank you for your attention.