INSTITUTE *for* ADVANCED STUDY

Precision Determination *of* $|V_{ub}|$

Gil Paz

Institute *for* Advanced Study, Princeton

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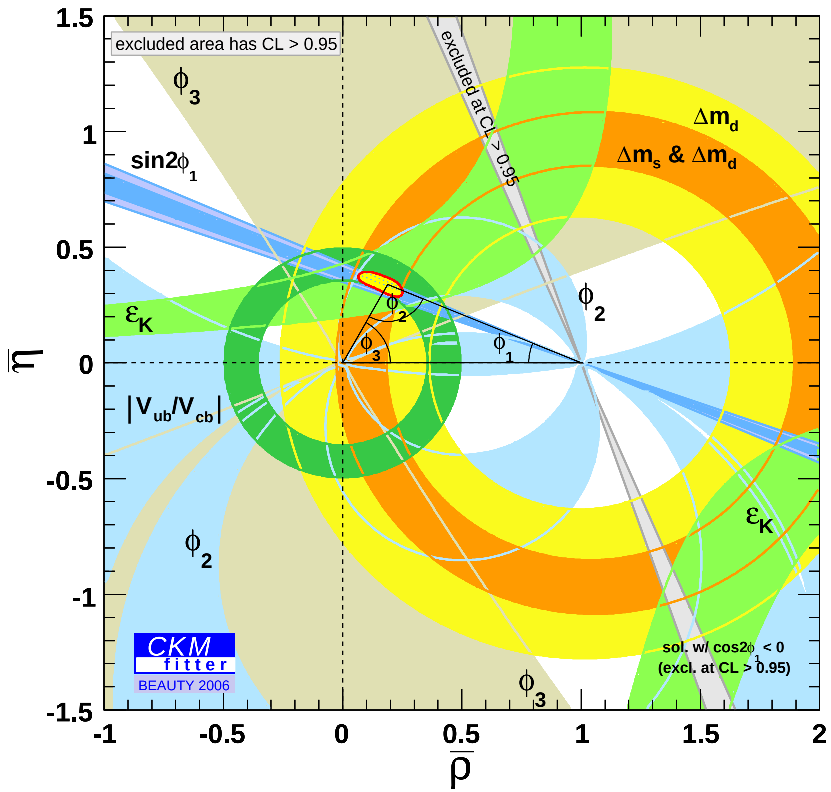
Motivation



2σ "tension" between $\sin 2\phi_1$ and $|V_{ub}|$:

$$\text{Measured} \quad |V_{ub}| = (4.10 \pm 0.09 \pm 0.39) \cdot 10^{-3}$$
$$\text{Fit} \quad |V_{ub}| = (3.59^{+0.17}_{-0.18}) \cdot 10^{-3}$$

Inclusive $|V_{ub}|$ gives the smallest error

How is $|V_{ub}|$ determined from $\bar{B} \to X_u l \bar{\nu}$ decays?

Kinematics

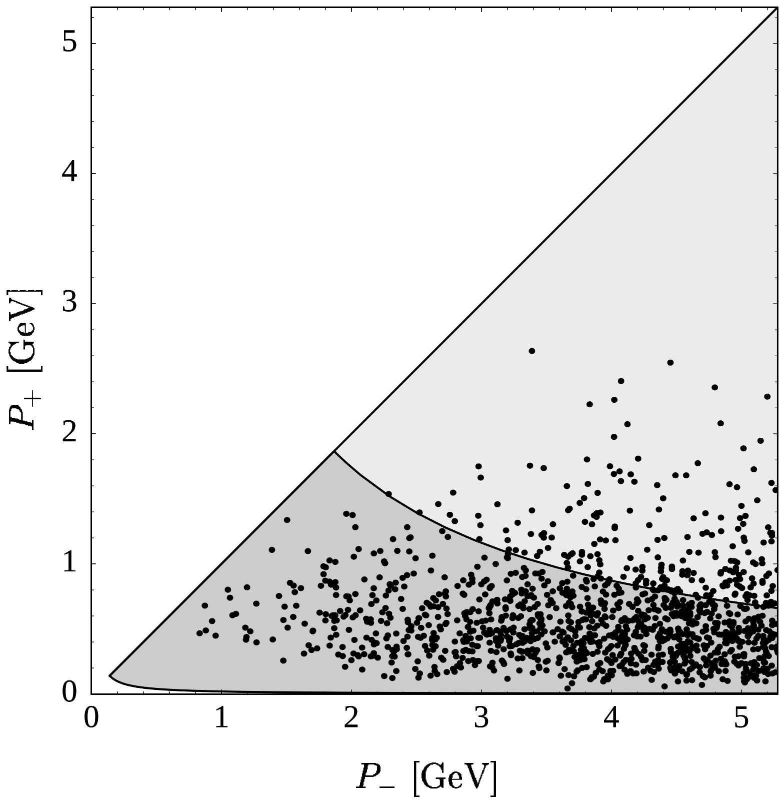
- Hadronic tensor $W^{\mu\nu}$ in (v, n) basis:
  (Lange, Neubert, GP [PRD 72, 073006 (2005)]):
  $v = (1, 0, 0, 0) \quad n = (1, 0, 0, 1) \quad [\bar{n} = 2v - n = (1, 0, 0, -1)]$
- Motivates:
  $P_l = M_B - 2E_l, \quad \bar{n} \cdot P = P_- = E_X + |\vec{P}_X|, \quad n \cdot P = P_+ = E_X - |\vec{P}_X|$
- Exact triple rate: $y = (P_- - P_+)/(M_B - P_+)$

$$\frac{d^3\Gamma_u}{dP_+ \, dP_- \, dP_l} = \frac{G_F^2 |V_{ub}|^2}{16\pi^3} \, (M_B - P_+) \Big[(P_- - P_l)(M_B - P_- + P_l - P_+) \, \tilde{W}_1$$
$$+ (M_B - P_-)(P_- - P_+) \, \frac{\tilde{W}_2}{2} + (P_- - P_l)(P_l - P_+) \, \left(\frac{y}{4} \tilde{W}_3 + \tilde{W}_4 + \frac{1}{y} \tilde{W}_5 \right) \Big]$$

- Simplest phase space: $\frac{M_\pi^2}{P_-} \leq P_+ \leq P_l \leq P_- \leq M_B$
- No explicit dependence on m_b ! Can predict partial rates instead of fractions

(Pedestrian introduction to inclusive $|V_{ub}|$, chapter 1 of GP hep-ph/0607217)

Kinematics



- $P_+ P_- = M_X^2 \quad q^2 = (M_B - P_-)(M_B - P_+)$
- Experimental cuts ⇒
  $P_+ \sim \Lambda_{\rm QCD} \sim 0.5$ GeV $\quad P_- \sim m_b \sim 5$ GeV
- In order to calculate $d^3\Gamma$ we need to know $\tilde{W}_i$

Dynamics - OPE region

- If we had no charm background...
  Integrate over P_+, P_- up to M_B, and use HQET based OPE

$$\tilde{W}_i \sim c_0\langle O_0\rangle + c_2\frac{\langle O_2\rangle}{m_b^2} + c_3\frac{\langle O_3\rangle}{m_b^3} + \cdots$$

- c_i calculable in PT:
  - c_0 known at $\mathcal{O}(\alpha_s)$ (De-Fazio, Neubert '99)
  - c_2 known at $\mathcal{O}(\alpha_s^0)$ (Blok, Koyrakh, Shifman, Vainshtein ; Manohar, Wise '93)
  - c_3 known at $\mathcal{O}(\alpha_s^0)$ (Gremm, Kapustin '96)
  - c_4 known at $\mathcal{O}(\alpha_s^0)$ (Dassinger, Mannel, Turczyk '06)
- $\langle O_i\rangle$ are HQ parameters, taken from experiment:
  - $\langle O_0\rangle = 1$
  - $\langle O_2\rangle \to \mu_\pi^2, \mu_G^2 = [(M_B^*)^2 - (M_B)^2]/4$
  - $\langle O_3\rangle \to \rho_{LS}^3, \rho_D^3$
- OPE works very well for $\bar{B} \to X_c\, l^- \bar{\nu}$
  $\Rightarrow$ Error on $|V_{cb}|$ is 2%, know HQ parameters
- Similar OPE for total $\bar{B} \to X_s \gamma$ rate (almost..), which we can't measure.

Dynamics - SF Region

- Because of the charm background, forced into regions of phase space where HQET based OPE is not valid ("OPE breaks down")
- We do have a systematic $1/m_b$ expansion, calculated using SCET:

$$\tilde{W}_i \sim H_u \cdot J \otimes S + \frac{1}{m_b} \sum_k h_u^k \cdot j_u^k \otimes s_u^k + \cdots$$

- H - physics at scale $\mu_h \geq m_b$ - Calculable in PT
  J - physics at scale $\mu_i \sim \sqrt{m_b \Lambda_{\rm QCD}}$ - Calculable in PT
  S - physics at scale $\mu_0 \sim \Lambda_{\rm QCD}$ - Non perturbative function
- For $\bar{B} \to X_s \gamma$ near endpoint:

$$\frac{d\Gamma}{dE} \sim H_s \cdot J \otimes S + \frac{1}{m_b} \sum_k h_s^k \cdot j_s^k \otimes s_s^k + \cdots$$

Dynamics - SF Region

- Currently:
  - H_u known at $\mathcal{O}(\alpha_s)$ (Bauer, Manohar '03; Bosch, Lange, Neubert, GP '04)
  - H_s known at $\mathcal{O}(\alpha_s)$ (Neubert '04)
  - J known at $\mathcal{O}(\alpha_s^2)$ (Becher, Neubert '06)
  - $h_u^k \cdot j_u^k$ known at $\mathcal{O}(\alpha_s^0)$ and s_u^k classified (K.S.M. Lee, Stewart '04; Bosch, Neubert, GP '04; Beneke, Campanario, Mannel, Pecjak '04; Earlier partial studies)
  - $Q_{7\gamma} - Q_{7\gamma}$ contribution: $h_s^k \cdot j_s^k$ known at $\mathcal{O}(\alpha_s^0)$ and s_s^k classified (Loc. cit.)
  - The rest of s_s^k are being calculated (S.J. Lee, Neubert, GP in preparation) preliminary results in hep-ph/0609224
- Relation between the two regions:
  - Moments of SFs related to HQ parameters, e.g.:
    First moment of $S \leftrightarrow m_b$, known at $\mathcal{O}(\alpha_s^2)$ (Neubert '04)
    Second moment of $S \leftrightarrow \mu_\pi^2$, known at $\mathcal{O}(\alpha_s^2)$ (Loc. cit.)

  $\Rightarrow$ Good knowledge of HQ parameters, constrain the SFs
  - Integrate over large enough regions of phase space, recover OPE result

BLNP Approach (2005): Principles

- BLNP approach (Lange, Neubert, GP [PRD 72, 073006 (2005)]):
  Use all that we know (2005) about $\bar{B} \to X_u l \bar{\nu}$ and $\bar{B} \to X_s \gamma$:
  - LO in $1/m_b$: H_u, H_s, J at $\mathcal{O}(\alpha_s)$:

$$\tilde{W}_1^{(0)}(P_+, y) = U_y(\mu_h, \mu_i) H_u(y, \mu_h) \int_0^{P_+} d\hat{\omega}\, y m_b J(y m_b (P_+ - \hat{\omega}), \mu_i)\, \hat{S}(\hat{\omega}, \mu_i)$$

  - $1/m_b$ subleading SFs at $\mathcal{O}(\alpha_s^0)$:

$$\tilde{W}_1^{\text{hadr}(1)}(P_+, y) = \frac{U_y(\mu_h, \mu_i)}{M_B - P_+} \left[(P_+ - \bar{\Lambda}) \hat{S}(P_+) + 2\, \hat{t}(P_+) + \frac{(\hat{u}(P_+) - \hat{v}(P_+))(1-y)}{y} \right]$$

  - Known $1/m_b \cdot \alpha_s$ terms from OPE (convoluted with $\hat{S}$):

$$\tilde{W}_1^{\text{kin}(1)}(P_+, y) = \frac{U_y(\mu_h, \mu_i)}{(M_B - P_+)} \frac{C_F \alpha_s(\bar{\mu})}{4\pi} \int_0^{P_+} d\hat{\omega}\, \hat{S}(\hat{\omega}, \mu_i) f\left(\frac{P_+ - \hat{\omega}}{M_B - P_+}, y\right)$$

  - Known $1/m_b^2$ terms from OPE (convoluted with $\hat{S}$):

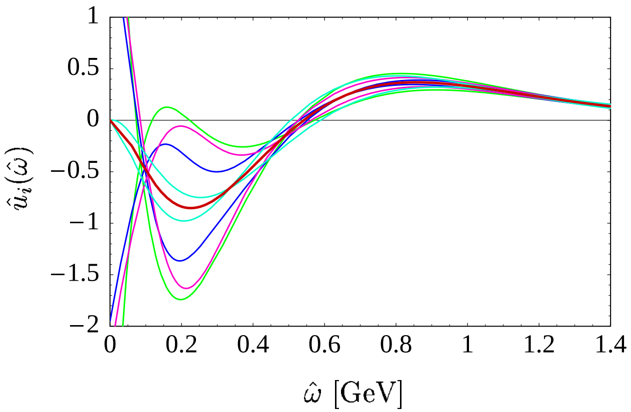
$$\tilde{W}_1^{\text{hadr}(2)}(P_+, y) = \frac{U_y(\mu_h, \mu_i)}{(M_B - P_+)^2} \left[\frac{4\lambda_1 - 6\lambda_2}{3y^2} - \frac{\lambda_1 + 3\lambda_2}{3} \right] \hat{S}(P_+, \mu_i)$$

BLNP Approach (2005): Principles

- Similar expansion can be constructed for $\bar{B} \to X_s \gamma$
- Absorb the SSF into the LO SF without changing the moment expansion:

$$\hat{\mathcal{S}}(\hat{\omega}) \equiv \hat{S}(\hat{\omega}) + \frac{2(\bar{\Lambda} - \hat{\omega})\hat{S}(\hat{\omega}) - \hat{t}(\hat{\omega}) + \hat{u}(\hat{\omega}) - \hat{v}(\hat{\omega})}{m_b} \Rightarrow \frac{d\Gamma_s}{dE_\gamma} = \cdots \hat{\mathcal{S}}(\hat{\omega}, \mu_i)$$

- Extract $\hat{\mathcal{S}}$ from $\bar{B} \to X_s \gamma$ and use as input for $\bar{B} \to X_u \, l^- \bar{\nu}$
- Model subleading SFs using moment constraints
- Subleading SFs: 3 functions, 9 models each, scan over $9^3 = 729$ combinations



- BLNP formalism smoothly and unambiguously interpolates between OPE and SF regions

BLNP Approach (2005): Experimental Cuts

- Lepton Energy endpoint: $E_l > 2.31$ GeV

$$\begin{array}{lcccccccccc} & \Gamma_u^{(0)} & + & \Gamma_u^{\rm kin(1)} & + & \Gamma_u^{\rm hadr(1)} & + & \Gamma_u^{\rm kin(2)} & + & \Gamma_u^{\rm hadr(2)} & \\ = & 6.810 & + & 0.444 & - & 3.967 & + & 0.042 & - & 0.555 & |V_{ub}|^2\,{\rm ps}^{-1} \end{array}$$

- P_+ spectrum: $P_+ < (M_D^2/M_B) \approx 0.66$ GeV

$$\begin{array}{lcccccccccc} & \Gamma_u^{(0)} & + & \Gamma_u^{\rm kin(1)} & + & \Gamma_u^{\rm hadr(1)} & + & \Gamma_u^{\rm kin(2)} & + & \Gamma_u^{\rm hadr(2)} & \\ = & 53.225 & + & 4.646 & - & 11.862 & + & 0.328 & - & 0.227 & |V_{ub}|^2\,{\rm ps}^{-1} \end{array}$$

- M_X spectrum: $M_X < M_D \approx 1.87$ GeV

$$\begin{array}{lcccccccccc} & \Gamma_u^{(0)} & + & \Gamma_u^{\rm kin(1)} & + & \Gamma_u^{\rm hadr(1)} & + & \Gamma_u^{\rm kin(2)} & + & \Gamma_u^{\rm hadr(2)} & \\ = & 58.541 & + & 8.027 & - & 9.048 & + & 2.100 & - & 0.318 & |V_{ub}|^2\,{\rm ps}^{-1} \end{array}$$

BLNP Approach (2005): Results

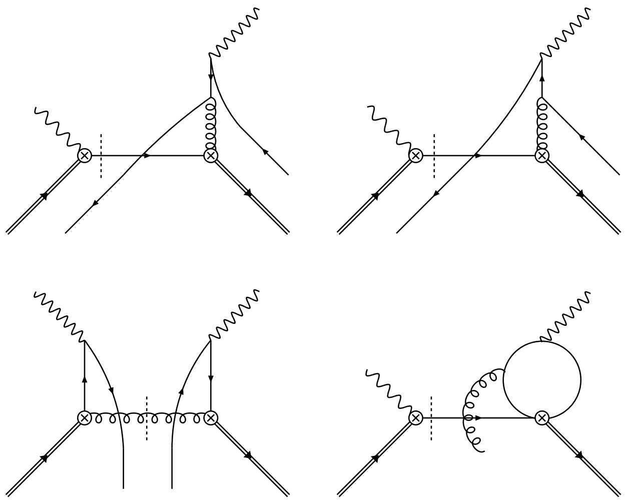
- Error Analysis
  - LO SF taken from experiment
  - Perturbative error
  - SSF error by varying > 700 models
  - WA: take as fixed % of rate
- Experimental implementation:
  - Belle: E_l cut, M_X cut, M_X & q^2 cut, P_+ cut
  - BaBar: $S_H^{\max}$ & E_l cut, M_X & q^2 cut, E_l cut
- HFAG average (ICHEP 2006): $|V_{ub}| = (4.49 \pm 0.19 \pm 0.27) \cdot 10^{-3}$ with
  - 4.2% HQ error
  - 3.8% Theory error (Perturbative + Subleading SFs)
  - 1.9% WA

Improved $|V_{ub}|$

- Today! Eliminate WA error
  - Cut on **high** $q^2 < q^2_{\max}$ e.g. $q^2_{\max} = (M_B - M_D)^2$, combined with M_X or P_+ cut (Lange, Neubert, GP '05)
  - Loose efficiency but also the WA error and its uncertainty, Preliminary study gives smaller error with such a cut
  - **Still waiting for experimental implementation!**
- Today! High precision weight functions
  - See talk by B.O. Lange (WG 2)
  - **Still waiting for experimental implementation!**
- Future:
  - $Q_{7\gamma}$ for $\bar{B} \to X_s\gamma$ is known at $\mathcal{O}(\alpha_s^2)$, other ops. are being calculated
    Once they are known, want $\bar{B} \to X_u\, l^- \bar{\nu}$ at $\mathcal{O}(\alpha_s^2)$:
    "Only" need H_u at $\mathcal{O}(\alpha_s^2) \Rightarrow$ full 2 loop inclusive $|V_{ub}|$
  - Subleading SFs at order $\mathcal{O}(\alpha_s) \Leftrightarrow$ OPE at $\mathcal{O}(\alpha_s)$
  - Can we find a way to extract subleading SFs from data?
  - Complete subleading SF basis for $\bar{B} \to X_s\gamma$:

Complete SSF Basis for $\bar{B} \to X_s\gamma$

- $Q_{7\gamma} - Q_{7\gamma}$ for $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_u\, l^- \bar{\nu}$ SSF:
  - $1/m_b$ correction for $d\Gamma$
  - SSF integrate to zero
- Recent new result: $\alpha_s \cdot 1/m_b$ corrections to $\Gamma(\bar{B} \to X_s\gamma)$! (Lee, Neubert, GP: hep-ph/0609224)



- See talk by M. Neubert (WG 2/3/6 joint session)
- What is the impact on inclusive $|V_{ub}|$?

New SSF Impact on $|V_{ub}|$ - Preliminary!

- New SSF for $\bar{B} \to X_s \gamma$, e.g.

$$\int d\omega\, e^{-\frac{i}{2}\omega \bar{n}\cdot x} f_{78}(\omega) = \sum_q e_q \int_{-\infty}^{0} ds \int_{-\infty}^{0} dt\, \langle \bar{B}|\bar{h}(0)...\Gamma_i h(x_-)\bar{q}(s\bar{n})...\Gamma_i q(t\bar{n})|\bar{B}\rangle$$

  Contribution to $d\Gamma_s/dE_\gamma$:

$$\frac{d\Gamma_s}{dE_\gamma} \propto \frac{4\pi\alpha_s}{m_b} \cdot \hat{f}_{78}(M_B - 2E\gamma)$$

  - New SSF do not integrate to zero
  - Harder to estimate moments
  - E.g. zeroth moment of f_{78}: non local matrix element of a $4-q$ op.

- New contributions to the rate $\Gamma(\bar{B} \to X_s\gamma)$ are important
  They are the **only** $1/m_b$ correction to the rate

- New contributions to $d\Gamma_s/dE_\gamma$ have to "compete" with known SSF not suppressed by α_s (t, u, v) and other $\alpha_s \cdot 1/m_b$ corrections

- **Maybe** more important for cuts with high values of P_+

- For cuts with lower values of P_+ effect **probably** already included in SSF error

Inclusive $|V_{ub}|$: Comparison of Approaches

- BLNP approach is based on more than 12 years of heavy quark expansion(s) (HQET, SCET). It is the **most comprehensive approach**
- There are other approaches used by HFAG: "BLL", "DGE", "LLR"
  - $|V_{ub}| = (5.02 \pm 0.26 \pm 0.37) \cdot 10^{-3}$ (Bauer, Ligeti, Luke '01)
  - $|V_{ub}| = (4.46 \pm 0.20 \pm 0.20) \cdot 10^{-3}$ (Andersen, Gardi '05)
  - $|V_{ub}| = (4.43 \pm 0.45 \pm 0.29) \cdot 10^{-3}$ (Leibovich, Low, Rothstein '00)
  - $|V_{ub}| = (4.49 \pm 0.19 \pm 0.27) \cdot 10^{-3}$ (Lange, Neubert, GP '05)
- All approaches seem to agree (is it a result of $\Gamma_u \approx \Gamma_u^{(0)}$?)
- But central values are not the whole story...
- It is time to take a critical look at the error bars!
  - BLL: Considering theoretical advances in control over LO SF and SSF, should reevaluate SF(s) sensitivities(s)
  - DGE: No power corrections are included or estimated!
  - (LLR measurement would become obsolete with experimental implementation of new weight functions)
- Important to resolve considering the 2σ "tension" between $\sin 2\phi_1$ and $|V_{ub}|$

Inclusive $|V_{ub}|$: Summary

- Impressive improvement in determination of $|V_{ub}|$
  Result of hard experimental and theoretical work
- Error on inclusive $|V_{ub}|$: 18% in PDG 2004 $\Rightarrow$ 8% in PDG 2006
- Improve $|V_{ub}|$ today!
  - Cut on **high** q^2 to eliminate WA
  - Advanced two loop relations between $\bar{B} \to X_u\, l^- \bar{\nu}$ and $\bar{B} \to X_s \gamma$
- New SSF for $\bar{B} \to X_s \gamma$
- Need to compare approaches: assumptions, perturbative corrections, non perturbative corrections
- More room for theoretical improvement