Inclusive $\bar{B} \to X_u l \bar{\nu}$ spectra by DGE

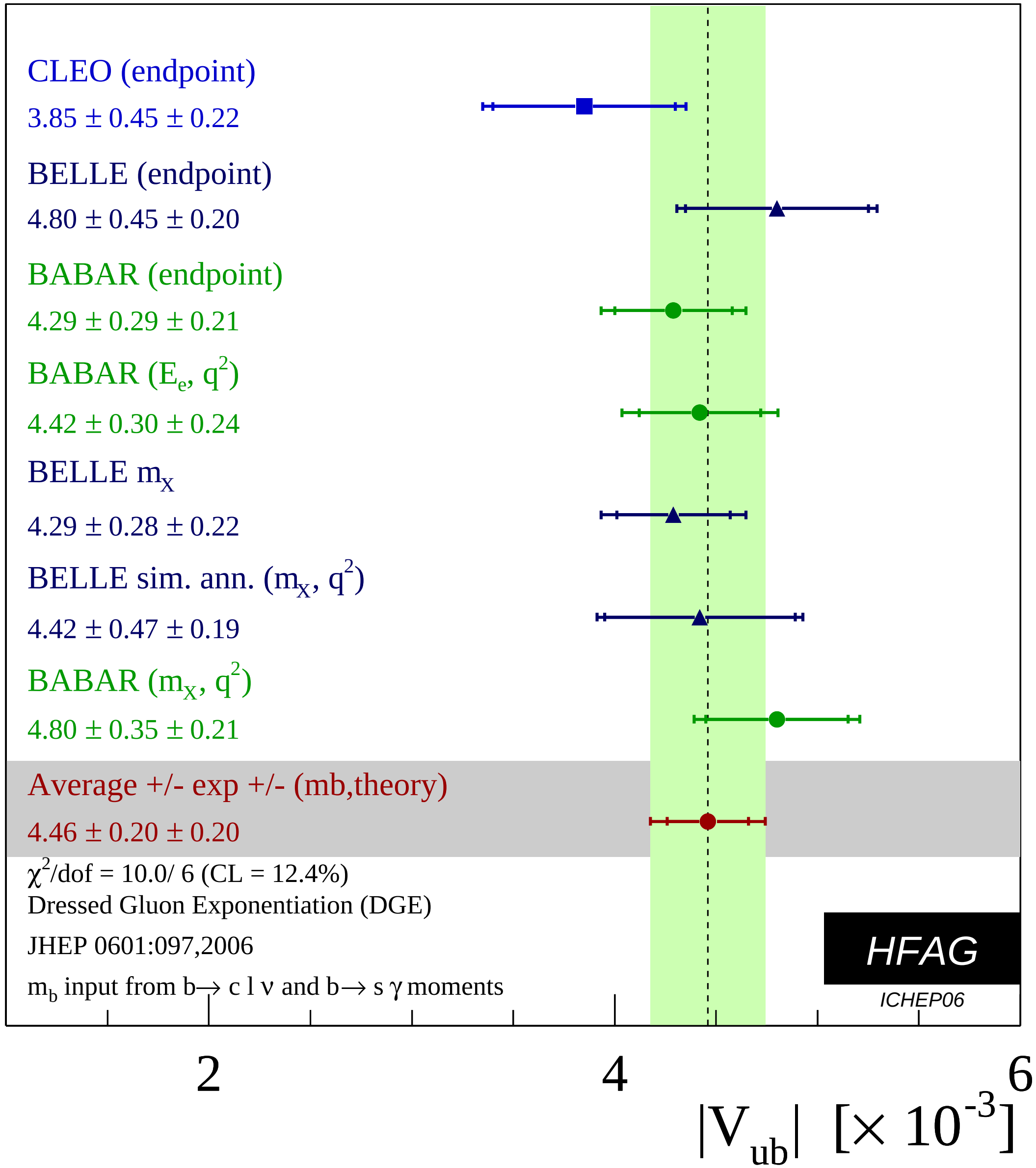
Einan Gardi (Cambridge)

- Inclusive $\bar{B} \to X_u l \bar{\nu}$ spectra
  - The essential rôle of resummation
  - Dressed Gluon Exponentiation (DGE) *vs.* shape function
- Recent progress and prospects for inclusive $\bar{B} \to X_u l \bar{\nu}$ spectra
  - Power corrections in the DGE approach
    implemented in the new release
  - NNLO corrections
    The $\mathcal{O}(\beta_0 \alpha_s^2)$ corrections to the fully differential width have been computed
    Gambino, Gardi & Ridolfi [hep-ph/0610140]
- Lessons from $\bar{B} \to X_s \gamma$ DGE@NNLO analysis
  Andersen & Gardi [hep-ph/0609250]

World Average $|V_{ub}|$ using DGE

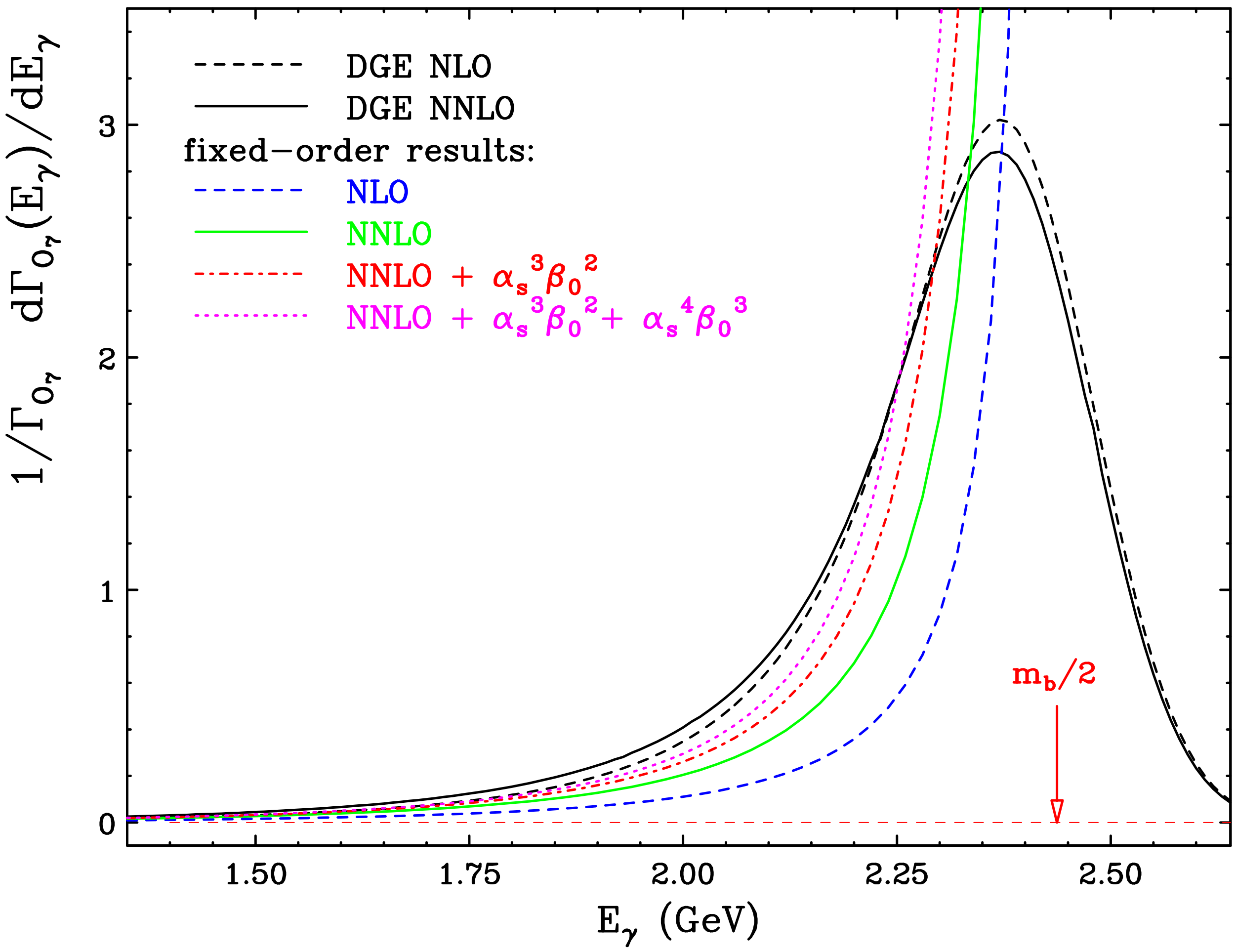
- CLEO, Belle & BaBar have performed several inclusive measurements of the partial $b \to u$ BF with different kinematic cuts on E_l, q^2, M_X, etc.
- March '06: each measurement was translated by *HFAG* into a value for $|V_{ub}|$ using DGE.

  HFAG update: July '06
- The results are all consistent.
- The uncertainty is the smallest obtained so far.



$\bar{B} \to X_s\gamma$: DGE *vs.* the fixed–order expansion

- Stability
- Support



Inclusive $\bar{B} \to X_u l \bar{\nu}$ spectra by Dressed Gluon Exponentiation

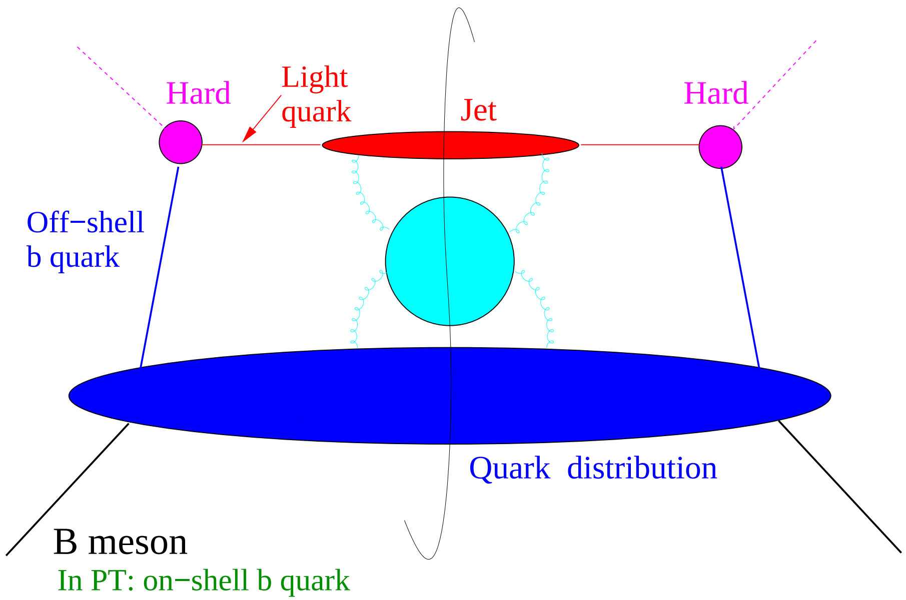
- Inclusive decays spectra are IR $\&$ Collinear safe:
  - Infrared cutoff and leading–power shape function are not necessary.
  - All non-perturbative effects can be taken into account as power corrections in moment space.
- Large perturbative corrections are associate with
  - Sudakov logarithms
  - running–coupling effects (renormalons)

  Both are resummed by DGE.
- General relation between the divergence of the perturbative expansion (renormalons) and infrared sensitivity (power corrections).
- In inclusive decays: resummation reveals that IR $\&$ Collinear safety extends beyond the logarithmic level!
  - Non-perturbative effects of the quark distribution first appear at $\mathcal{O}\left((N\Lambda/m_b)^3\right)$.
  - DGE spectrum without any power corrections is already a good approximation.
  - Given the known parametric dependence, power corrections can be effectively parametrized and quantified.

Factorization and DGE

Factorization in inclusive decays:

Korchemsky & Sterman '94



DGE:

Dressed Gluon Exponentiation: (**moment** space)	**Hard:**		**Jet:**		**Quark Distribution:**
	m_b	$\gg$	$m_b/\sqrt{N}$	$\gg$	m_b/N
	calculated		calculated		**calculated+p.c.****

**power corrections distinguishing $S(m_b/N;\mu)$ (the quark distribution in the meson) from $S_{\rm PT}(m_b/N;\mu)$ (the quark distribution in an on-shell quark) are parametrized.

DGE *vs.* shape function

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Cutoff–based "Shape Function" approach (BLNP):

"Shape Function" approach: (**momentum** space)	**Hard:**		**Jet:**		**Quark Distribution:**
	m_b	$\gg$	$\sqrt{m_b\Lambda}$	$\gg$	Λ
	calculated		calculated		**parametrized***

* the entire leading–power quark distribution in the meson is parameterized.

Power corrections in the DGE approach

- Resummed perturbation theory (on-shell heavy quark) yields:

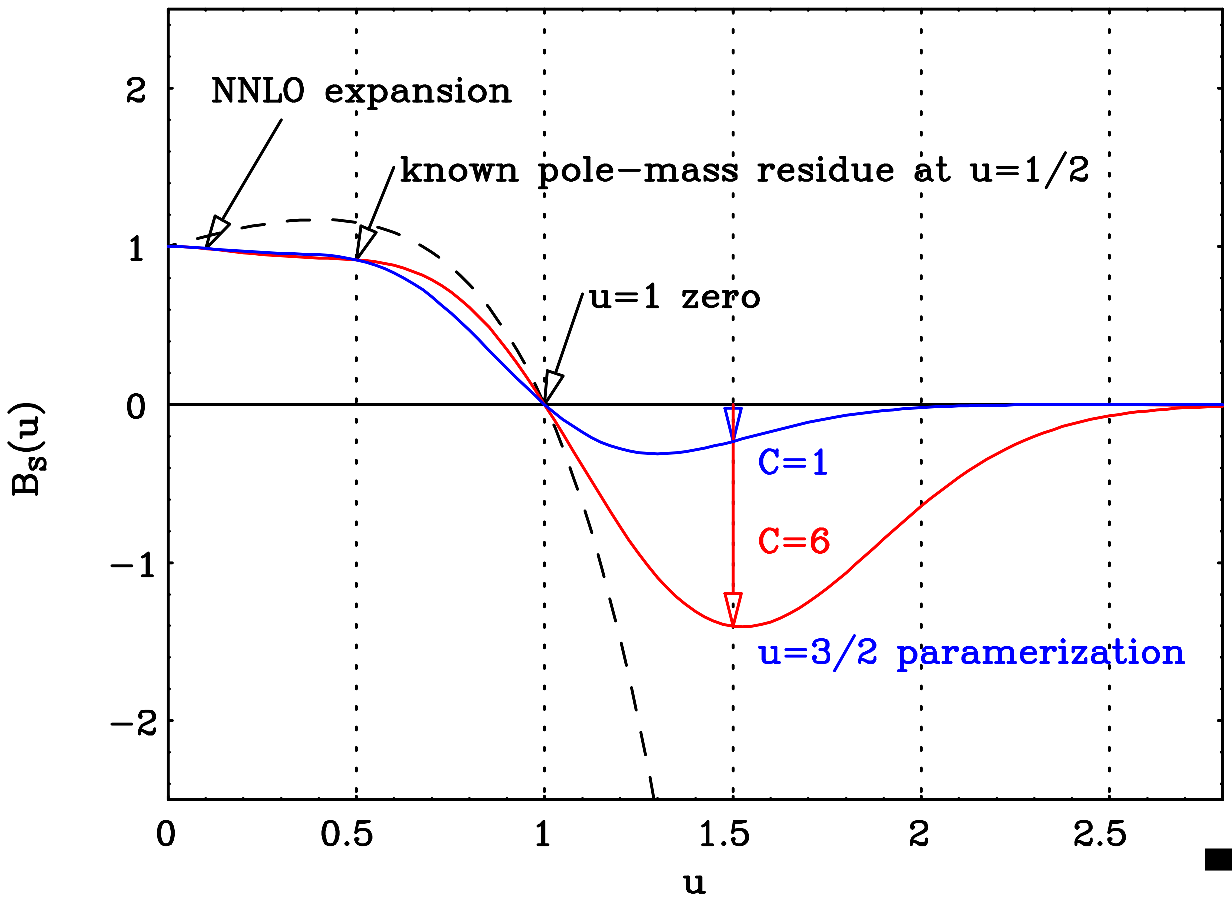
$$\mathrm{Sud}(p^-,N) = \exp\left\{\frac{C_F}{\beta_0}\int_0^\infty \frac{du}{u}\left(\frac{\Lambda}{p^-}\right)^{2u}\right.$$
$$\left.\Bigg[\underbrace{B_{\mathcal{S}}(u)\left(\frac{\Gamma(N)\Gamma(-2u)}{\Gamma(N-2u)}+\frac{1}{2u}\right)}_{\texttt{Quark Distribution: } N^{2u}} - \underbrace{B_{\mathcal{J}}(u)\left(\frac{\Gamma(N)\Gamma(-u)}{\Gamma(N-u)}+\frac{1}{u}\right)}_{\texttt{Jet: } N^{u}}\Bigg]\right\}$$

- Renormalon ambiguities indicate power corrections $(N\Lambda/p^-)^k$ in the exponent!
  - $u = 1/2$ renormalon ambiguity cancels with the pole mass renormalon.
  - $u = 1$ renormalon is missing ($B_{\mathcal{S}}(1) = 0$).
  - $u \geq 3/2$ ambiguities are present in the on-shell decay spectrum.
- To approximate the quark distribution in the meson: introduce one non-perturbative parameter f_{PV} to parametrize $(N\Lambda/p^-)^k$ corrections for $k \geq 3$:

$$\mathrm{Sud}(p^-,N) \longrightarrow \mathrm{Sud}(p^-,N)\big|_{\mathrm{PV}} \times$$
$$\exp\left\{\frac{C_F}{\beta_0}\,\pi\, f_{\mathrm{PV}} \sum_{k=3}^{\infty}\frac{(-1)^k}{k\,k!}\,B_{\mathcal{S}}(k/2)\left(\frac{\Lambda}{p^-}\right)^k \Pi_{j=1}^{k}(N-j)\right\},$$

The Sudakov factor (beyond fixed logarithmic accuracy)

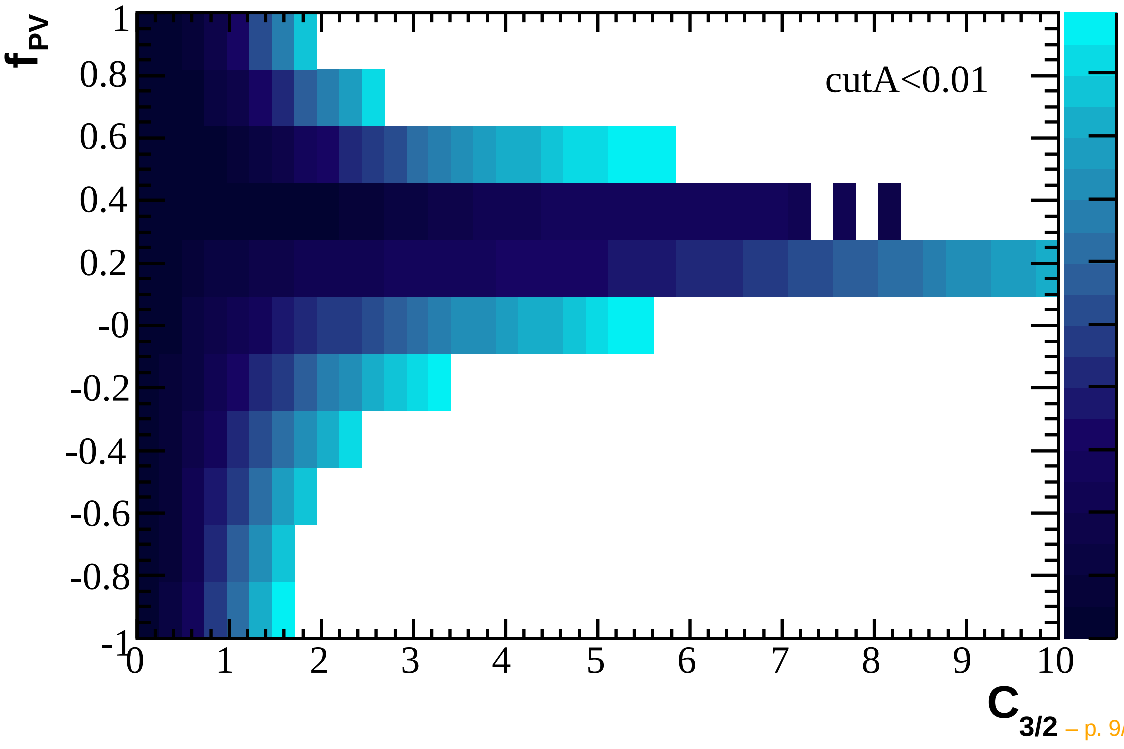
$$\mathrm{Sud}(p^-, N) \simeq \exp\left\{\frac{C_F}{\beta_0}\int_0^\infty \frac{du}{u}\left(\frac{\Lambda}{p^-}\right)^{2u}\right.$$
$$\left.\left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(N^{2u}-1\right) - B_{\mathcal{J}}(u)\Gamma(-u)\left(N^u-1\right)\right]\right\}$$



Power corrections in the DGE approach

- Simultaneous constraints on $B_{\mathcal{S}}(u)$ (the renormalon residue $C_{3/2}$) and $f_{\rm PV}$ based on
  - support (done)
  - data (fits to $\bar{B} \to X_s \gamma$ moments under way)
- $\bar{B} \to X_s \gamma$ values of $C_{3/2}$ and $f_{\rm PV}$ directly apply to $\bar{B} \to X_u l \bar{\nu}$.
- Variation of $R_{\rm cut} = {\rm BF}(M_X < 1.7\,{\rm GeV})/{\rm BF(total)}$ due to power corrections: $\pm 1\%$

Support–based exclusion map



Francesca Di-Lodovico (BaBar)

Andersen & Gardi [hep-ph/0609250]

NNLO corrections

- Lesson from $\bar{B} \to X_s\gamma$ DGE@NNLO analysis:
  - significant reduction of uncertainty along the tail upon including $\mathcal{O}(\alpha_s^2)$ corrections (was done for O_7).
  - renormalization scale in the matching coefficient in $\bar{B} \to X_s\gamma$: $\mu \sim m_b/2$ — stability NLO $\longrightarrow$ NNLO
- $\beta_0\alpha_s^2$ corrections dominate the $\mathcal{O}(\alpha_s^2)$ contribution to inclusive decays (both within the Sudakov region and out of it).
- The $\beta_0\alpha_s^2$ corrections (as well as $\beta_0^n\alpha_s^{n+1}$ for any n) are now available for the fully differential $\bar{B} \to X_u l\bar{\nu}$ width.
  Gambino, Gardi & Ridolfi [hep-ph/0610140].
- Matching to $\mathcal{O}(\beta_0\alpha_s^2)$ will soon be included in the DGE code.
- Estimated variation of $R_{\rm cut} = \mathrm{BF}(M_X < 1.7\,\mathrm{GeV})/\mathrm{BF}(\mathrm{total})$ due to NNLO corrections: $\pm 6\%$

Conclusions

DGE provides a systematic and effective framework to compute B-meson decay spectra.

Status of DGE–based experimental studies:

- $|V_{\rm ub}|$ has been determined from all available $\bar{B} \to X_u l \bar{\nu}$ BF measurements with a variety of cuts: consistent values; small error.
- Agrees well with $\bar{B} \to X_s \gamma$ moments as a function of the cut;
  Fits to $\bar{B} \to X_u l \bar{\nu}$ moments or spectrum have not yet been done (Weak Annihilation).

Recent Progress:

- New way to include and constrain(!) power corrections
  - New release of the DGE code is available for both $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_u l \bar{\nu}$.
  - Fits to data are necessary to determine m_b, $C_{3/2}$ and $f_{\rm PV}$.
- Available NNLO $\beta_0 \alpha_s^2$ corrections for the fully differential $\bar{B} \to X_u l \bar{\nu}$ width.

Prospects:

- Matching to $\mathcal{O}(\beta_0 \alpha_s^2)$: reduction of theoretical uncertainty on $|V_{\rm ub}|$ from any experimental cut.