Lattice QCD results for the $B \rightarrow D^{(*)} l \nu$ form factors: F(1) and G(1)

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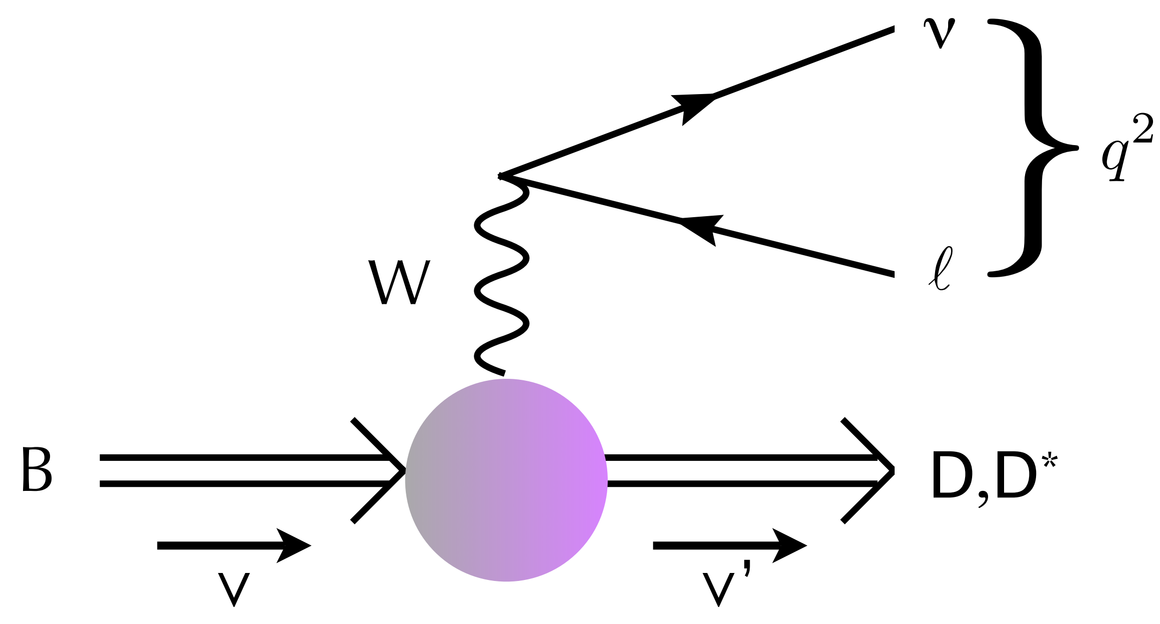
Why study semileptonic B→D$^{(*)}$ decays?



$$\frac{d\Gamma(B\to Dl\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B+m_D)^2 (w^2-1)^{3/2} |V_{cb}|^2 |\mathcal{F}_{B\to D}(w)|^2$$

$$\frac{d\Gamma(B\to D^* l\nu)}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B-m_{D^*})^2 \sqrt{w^2-1}\,\mathcal{G}(w) |V_{cb}|^2 |\mathcal{F}_{B\to D^*}(w)|^2$$

w ≡ v′ · v

w = 1 at zero recoil

- Experiments can only measure the product **(form factor) x |V$_{cb}$|**
- Lattice QCD calculations needed to determine **normalization** and extract the CKM matrix element | V$_{cb}$ |
- Only need one q^2 point from lattice -- choose **w=1** because easiest to calculate

Lattice calculations of F(1), G(1)

Quenched

- Calculated by Fermilab collaboration [Phys.Rev.D66:014503,2002; Phys.Rev.D61:014502,2000]
  - Wilson u,d,s quarks; Fermilab formulation for b,c quarks

Unquenched

- Calculations in progress by the Fermilab/MILC collaboration
- Use the publicly available "2+1 flavor" **MILC configurations** [Phys.Rev.D70:114501,2004] which have three flavors of improved staggered quarks:
  - Two degenerate light quarks and one heavy quark $(\approx m_s)$
  - Light quark mass ranges from $m_s/10 \leq m_l \leq m_s$
- Fermilab formulation for b,c quarks
- Preliminary B→D form factor, G(1), presented at Lattice '04 [Nucl.Phys.Proc.Suppl.140:461-463,2005]

Schematic of B→D form factor calculation

LATTICE → **QCD**

- G(1) depends upon **two form factors**, h_+ and h_-:

$$\mathcal{F}_{B\to D}(1) = h_+^{B\to D}(1) - \frac{m_B - m_D}{m_B + m_D}\, h_-^{B\to D}(1)$$

- Could calculate directly from lattice B→D correlation function, but errors would be large

- Instead construct **double ratios of correlation functions** to cancel out bulk of statistical fluctuations from Monte Carlo simulations as well as many systematic errors

$$\frac{\langle D|\bar{c}\gamma^4 b|\overline{B}\rangle\langle\overline{B}|\bar{b}\gamma^4 c|D\rangle}{\langle D|\bar{c}\gamma^4 c|D\rangle\langle\overline{B}|\bar{b}\gamma^4 b|\overline{B}\rangle} = |h_+(1)|^2 \qquad \frac{\langle D|\bar{c}\gamma_i b|\overline{B}\rangle\langle D|\bar{c}\gamma_0 c|D\rangle}{\langle D|\bar{c}\gamma_0 b|\overline{B}\rangle\langle D|\bar{c}\gamma_i c|D\rangle} = 1 - \frac{h_-(w)}{h_+(w)}$$

- Can calculate h_+ and h_- **directly at the tuned bottom & charm quark masses**

Schematic of $B \to D^*$ form factor calculation

LATTICE → **HQET** → **QCD**

- F(1) depends upon one form factor, h_{A_1}:

$$\mathcal{F}_{B\to D^*}(1) = h_{A_1}(1)$$

- h_{A_1}, however, cannot be determined from a single double ratio
- Instead **use heavy quark effective theory (HQET) as an intermediary ...**

Schematic of B→D* form factor calculation

LATTICE → **HQET** → **QCD**

- Can expand $h_{A_1}(1)$ in powers of the heavy quark mass using HQET
- To order $1/m_Q^2$, depends upon three different HQET matrix elements:

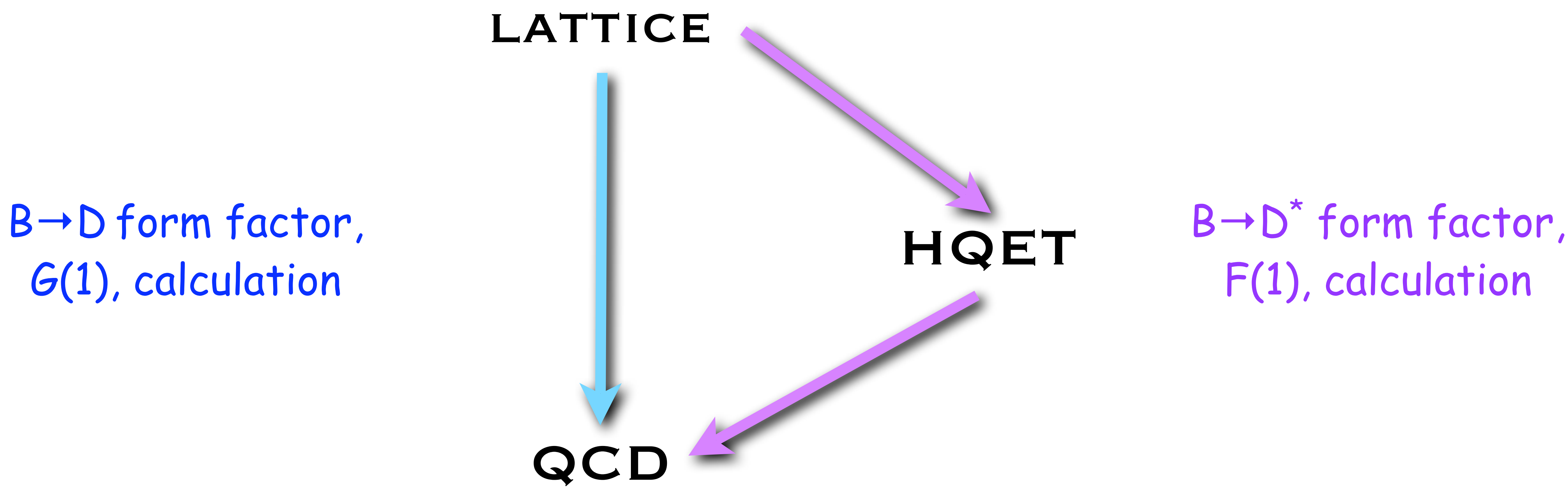
$$h_{A_1}(1) = \eta_A \left[1 - \frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{2m_c m_b} - \frac{\ell_P}{(2m_b)^2} \right]$$

- Can determine three HQET matrix elements from the heavy quark mass dependence of **three different double ratios**
- For example, use the heavy-quark mass dependence of R_+ to extract l_P:

$$\mathcal{R}_+ = \frac{\langle D|\bar{c}\gamma^4 b|\overline{B}\rangle\langle\overline{B}|\bar{b}\gamma^4 c|D\rangle}{\langle D|\bar{c}\gamma^4 c|D\rangle\langle\overline{B}|\bar{b}\gamma^4 b|\overline{B}\rangle} = |h_+(1)|^2 \qquad h_+(1) = \eta_V \left[1 - \ell_P \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \right]$$

- Then **recombine the HQET matrix elements get the form factor** $h_{A_1}(1)$

Comparison of F(1), G(1) lattice calculations



Similarities

- Essentially same code to calculate correlation functions
- Both calculations use double ratios to reduce errors

Differences

- G(1) comes directly from lattice data at tuned bottom & charm masses
- F(1) requires intermediate step of determining HQET matrix elements, and consequently lattice data at multiple heavy quark masses

Systematics in lattice calculations

- Lattice calculations typically quote the following sources of error:

  1. Monte carlo statistics & fitting
  2. Tuning lattice spacing, a, and quark masses
  3. **Matching lattice gauge theory to continuum QCD**
  4. Extrapolation to continuum
  5. **Chiral extrapolation to physical up, down quark masses**

- Some lattice simulations also neglect dynamical quark loops -- known as the "quenched approximation"

- Errors #3 and #5 are dominant sources of uncertainty in current heavy-light lattice calculations -- will discuss them in turn

Heavy quarks on the lattice

PROBLEM: Generic lattice quark action will have discretization errors $\propto (am_Q)^n$

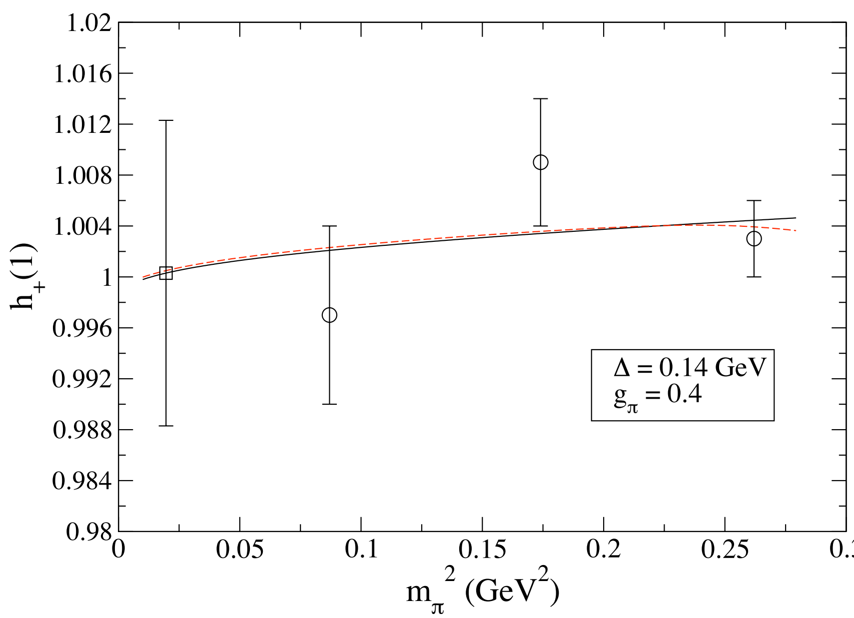
SOLUTION: **Fermilab method** uses knowledge of the heavy quark limit of QCD to systematically eliminate HQ discretization errors order-by-order [Phys.Rev.D55:3933-3957,1997; Phys.Rev.D62:014505,2000; Phys.Rev.D65:094514,2002]

Continuum QCD → (using HQET) → Lattice gauge theory

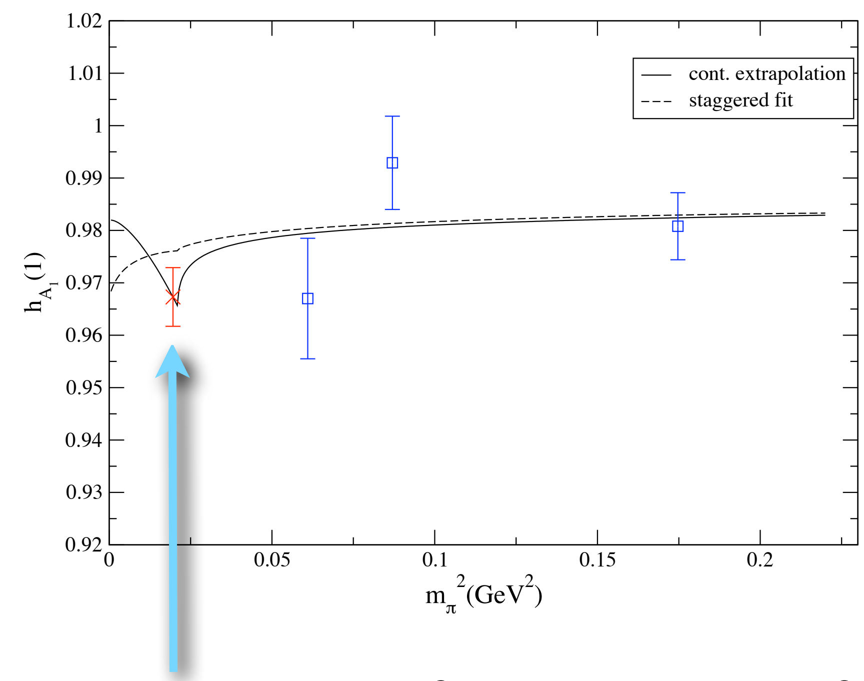
- Requires tuning parameters of lattice action and lattice currents to continuum
- Typically calculate matching coefficients in lattice perturbation theory [Phys.Rev.D48:2250-2264,1993]
- Combine all errors associated with discretizing action into "heavy quark discretization errors"
- Estimate errors using knowledge of short-distance coefficients and power-counting

Chiral extrapolation of lattice data

- Must extrapolate lattice results to physical values of up, down quark mass
- For MILC 2+1 flavor lattices, must use **staggered chiral perturbation theory** [Lee & Sharpe, Aubin & Bernard, Sharpe & RV]
  - Accounts for next-to-leading order light quark mass dependence and light quark discretization effects through $O(\alpha_S^2 a^2 \Lambda_{QCD}^2)$
- For B→D, **extrapolation under good control** [Phys.Rev.D73:054501,2006]
- For B→D*, extrapolation using **correct χPT is essential** [Laiho, Lattice '06]



black = finite lattice spacing
red = continuum extrapolation



cusp at $m_\pi^2 = (m_{D^*} - m_D)^2$

Quenched results for F(1), G(1)

Quenched results for $B\rightarrow D^{(*)}$ form factors

m_Q tuning & matching

chiral extrapolation

F(1):

$$\mathcal{F}_{B\rightarrow D^*}(1) = 0.9130 \pm^{+0.0238+0.0156+0.0032+0.0000}_{-0.0173-0.0157-0.0141-0.0163}$$

statistics

lattice spacing dependence

m_Q tuning

$$\mathcal{F}_{B\rightarrow D}(1) = 1.058 \pm 0.016 \pm 0.003^{+0.014}_{-0.005}$$

G(1): errors omit lattice spacing dependence

statistics & chiral extrapolation

perturbative matching

- **4%** determination of F(1) [Phys.Rev.D66:014503,2002]
- **2%** determination of G(1) [Phys.Rev.D61:014502,2000]
- **Proof-of-method**: use of double ratios allows precise calculation of $B\rightarrow D^{(*)}$ form factors at zero recoil

Potential sources of improvement

Necessary:

- **Unquenched calculation** with three light quark flavors (*in progress*)
- **Use chiral perturbation theory expressions** for extrapolation to physical up/down quark mass (Staggered chiral perturbation theory for F(1) and G(1) calculated by Laiho & RV)

Helpful:

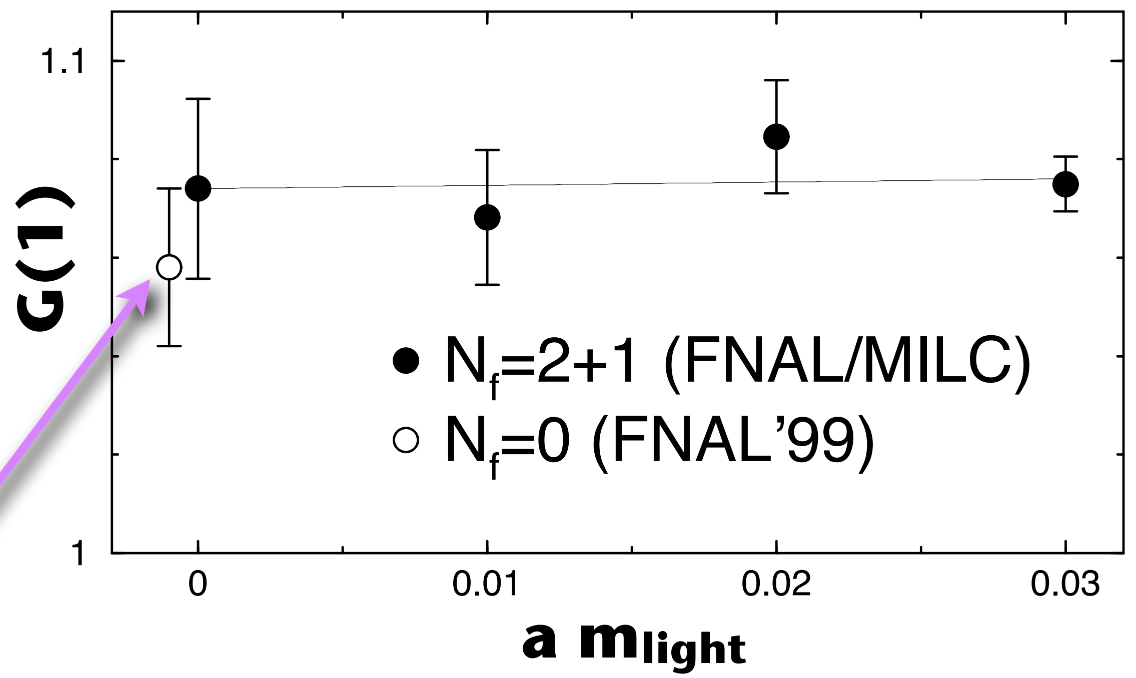
- Increased statistics (*in unquenched calculation*)
- Lighter quark masses (*in unquenched calculation*) and finer lattice spacings
- 2-loop perturbative (or nonperturbative) matching
- Improved heavy-quark action (*in progress* -- Kronfeld & Oktay)
- Heavy-light calculations with different light quark action, e.g domain-wall (RBC) or overlap fermions (JLQCD)

Progress in unquenched calculations of F(1) and G(1)

Unquenched result for B→D form factor

$$\mathcal{F}^{n_f=2+1}_{B\to D}(1) = 1.074\ (18)_{\rm sta}(16)_{\rm sys}$$
$$|V_{cb}| = 3.8\ (1)_{\rm sta}(6)_{\rm sys} \times 10^{-2}$$

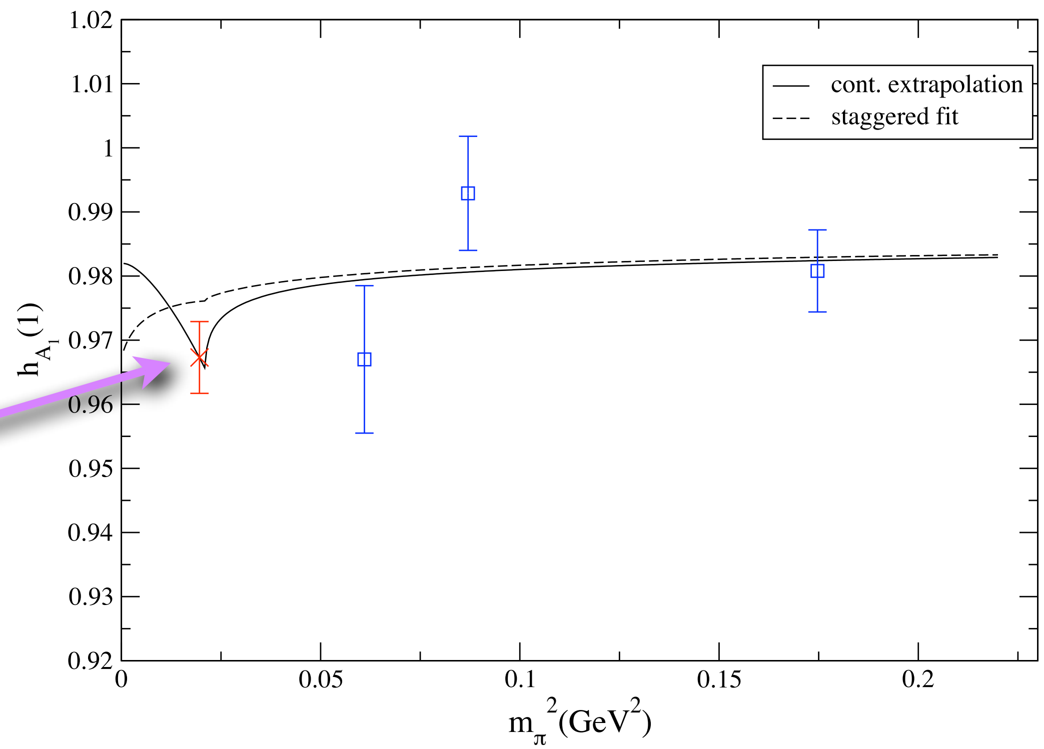
[Nucl.Phys.Proc.Suppl,140:461-463,2005]
(*preliminary*)



- Consistent with earlier quenched result -- still only 2% error
- Linear extrapolation in quark mass and single lattice spacing
- **G(1) calculation in progress with many improvements**:
  - 4 times statistics
  - Additional data point at even lighter quark mass
  - Extrapolation with staggered chiral perturbation theory expression
  - Additional (larger) lattice spacing to estimate lattice spacing dependence

"Prediction" for errors in B→D* form factor

- Total error in quenched F(1) is **4%**
- In addition to removing quenching uncertainty, unquenched calculation will make the following improvements:
  - Increased statistics
  - Extrapolation in light quark mass with staggered chiral perturbation theory
  - Because staggered χPT includes light quark discretization effects, will also reduce lattice spacing error
- Error in unquenched F(1) will **likely be 2-3%**
- To get below this, will help to reduce experimental uncertainty in the D-D*-π coupling, g_π
- Size of continuum cusp varies by ± 1% when g_π is varied within its experimental uncertainty



Summary and prospects for exclusive $|V_{cb}|$

- $|V_{cb}|$ currently known to 4% from exclusive decays [PDG]
- Limited by theoretical uncertainty in the $B \to D^{(*)}$ form factors at zero recoil
- Quenched calculations **demonstrate the capability of lattice calculations to determine F(1), G(1) to few percent accuracy**
- Unquenched calculations are in progress using the same methodology
- Although the errors will likely only go from 4% (+ ???%) → 2-3% between the quenched and unquenched calculations, **the results will be on a stronger theoretical footing, and hence more reliable**
- Reducing the errors to below 2-3% will require additional work, e.g.
  - Higher-order matching
  - Better determination of g_π
- Unquenched calculations are happening right now, so stay tuned...