Dispersion Analyses for Vub and form factor shape parameters

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Dispersion Relations

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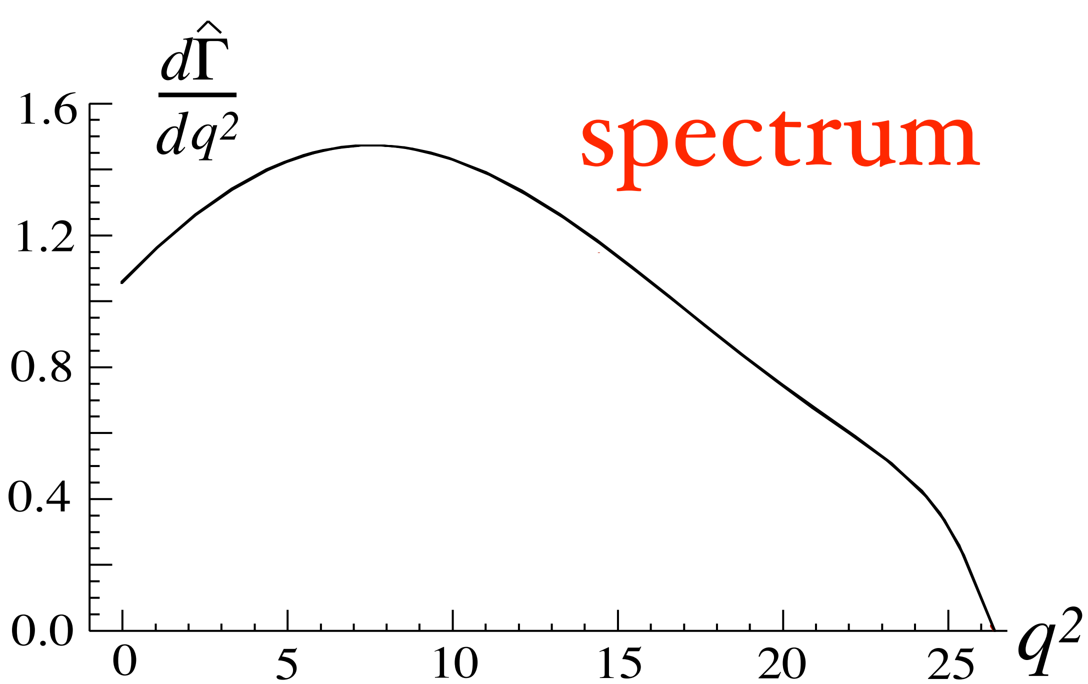
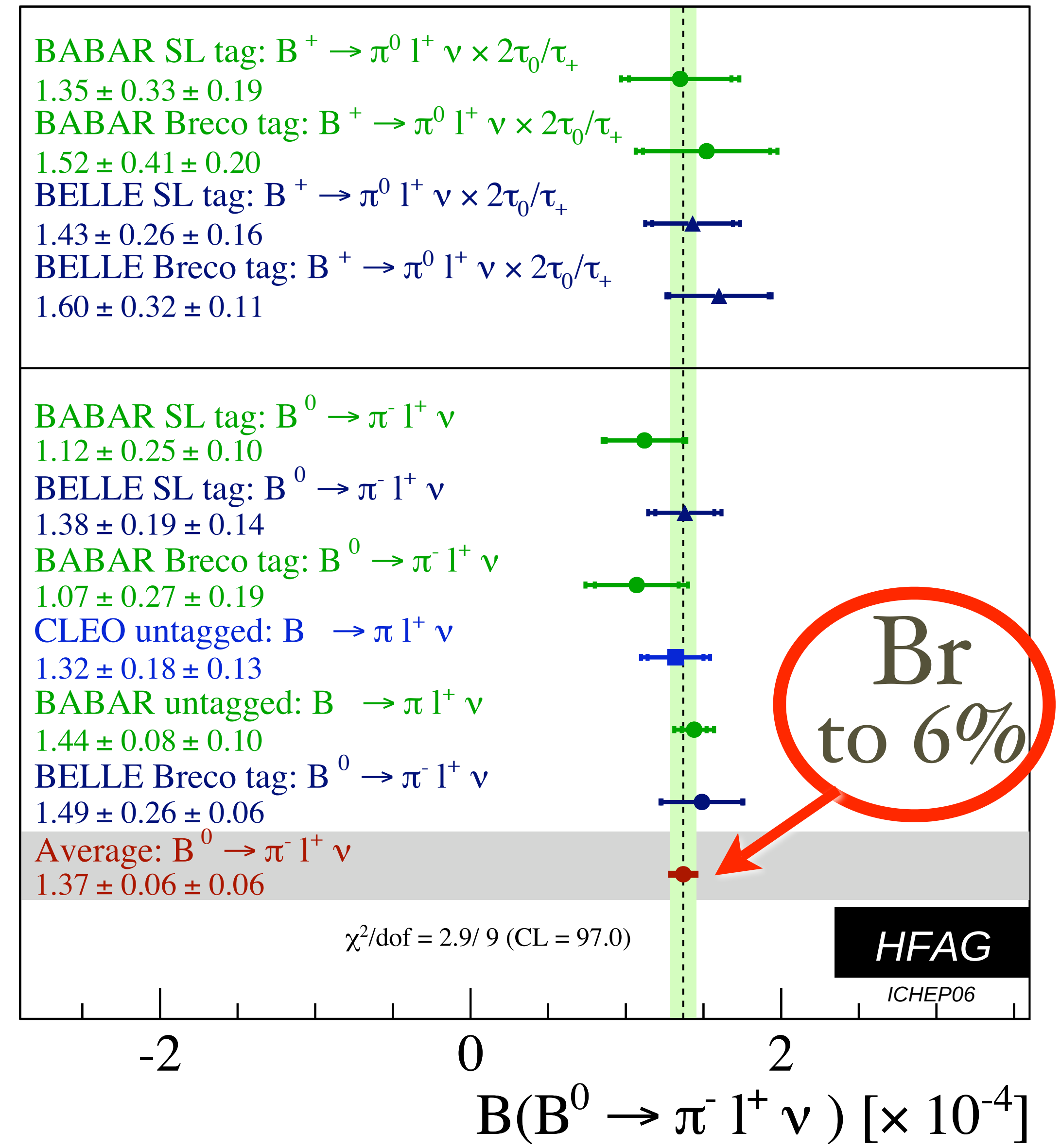
S. Okubo, I. Fushih, '71
V. Singh, A. K. Raina, '79
C. Bourrely, B. Machet, E de Rafael, '81
E. de Rafael, J. Taron, '92 & '94
B.Grinstein, P. Mende, '93
C.G.Boyd, B.Grinstein, R. Lebed, '95,'96,'97
L. Lellouch, '96
C.G.Boyd, M. Savage, '97
I. Caprini, L.Lellouch, M. Neubert '97
M. Fukunaga, T. Onogi, '05
C. Arnesen, B. Grinstein, I. Rothstein, I.S., '05
T. Becher, R. Hill, '06
J. Flynn, J. Nieves, '06

V_{ub} $\quad B \to \pi \ell \bar{\nu}$

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+ \ell \bar{\nu})}{dq^2} = \frac{G_F^2 |\vec{p}_\pi|^3}{24\pi^3} |V_{ub}|^2 \left|f_+(q^2)\right|^2$$





$\Rightarrow$ $|V_{ub}|$ to 3% !?!

Uncertainty from theory dominates.

Sources of Information

Experimental q^2 spectra (bins)

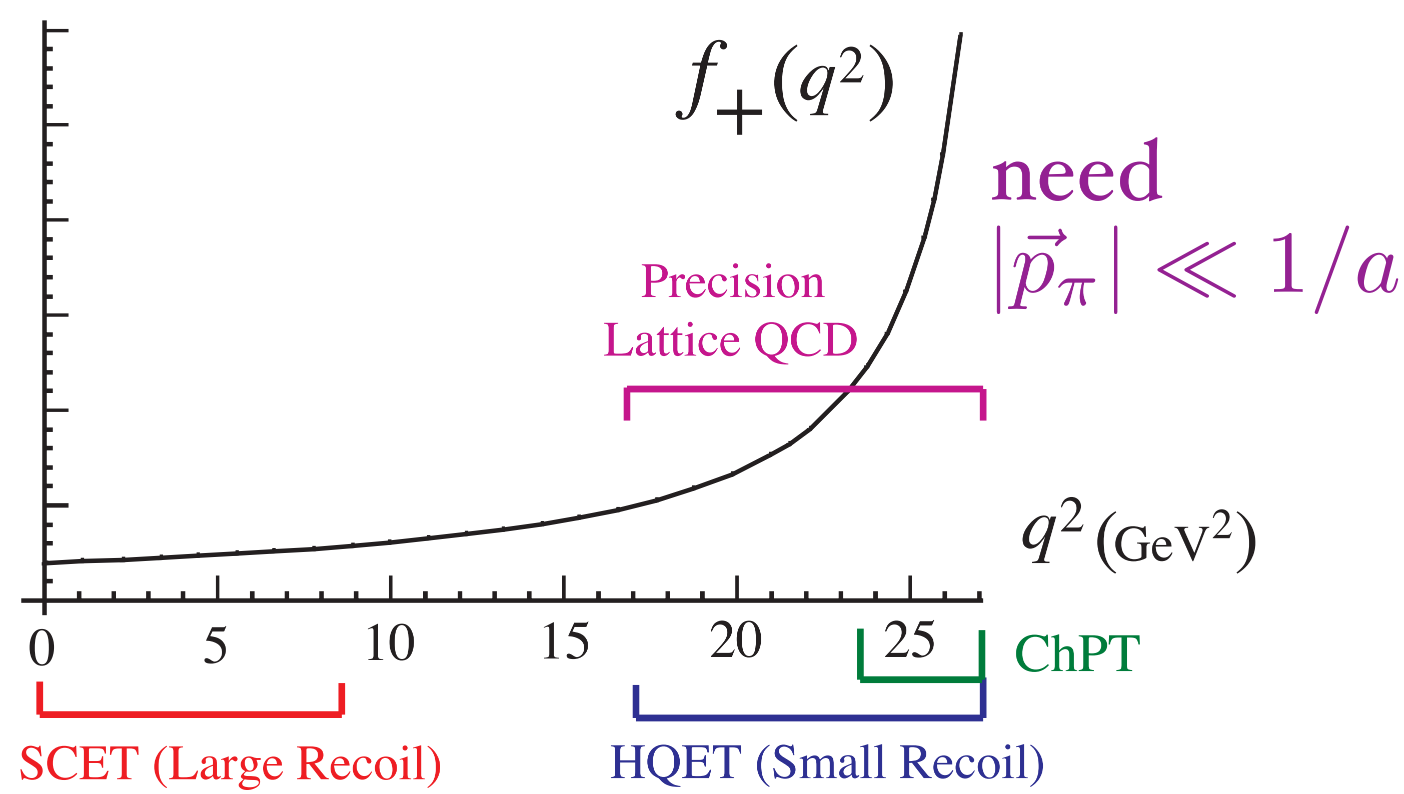
Dispersion Relations

Unquenched Lattice QCD

Chiral Perturbation Theory

SCET, $f_+(0)$

$\dots$



Dispersion Relations

Define

$$\Pi_J^{\mu\nu}(q) = \frac{1}{q^2}(q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_J^T(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi_J^L(q^2) \equiv i\int d^4x\, e^{iqx}\langle 0|\mathrm{T} J^\mu(x) J^{\dagger\nu}(0)|0\rangle$$

Dispersion relations

$$\chi^{(0)} = \frac{1}{2}\frac{\partial^2 \Pi_J^T}{\partial (q^2)^2}\bigg|_{q^2=0} = \frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\,\Pi_J^T(t)}{t^3}$$

Inequality

$$\mathrm{Im}\,\Pi_J^{T,L} = \frac{1}{2}\sum_X (2\pi)^4 \delta^4(q-p_X)|\langle 0|J|X\rangle|^2 \geq \int [p.s.]\pi(2\pi)^3\delta^4(q-p_X)|\langle 0|J|B\pi\rangle|^2$$

Perturbative QCD
(OPE)

Related by crossing to decay form factor

Bound on Form factor

$$1 \geq \int_{t^+}^\infty dt\, \frac{W(t)|f(t)|^2}{t^3}$$

$B\pi$ production threshold

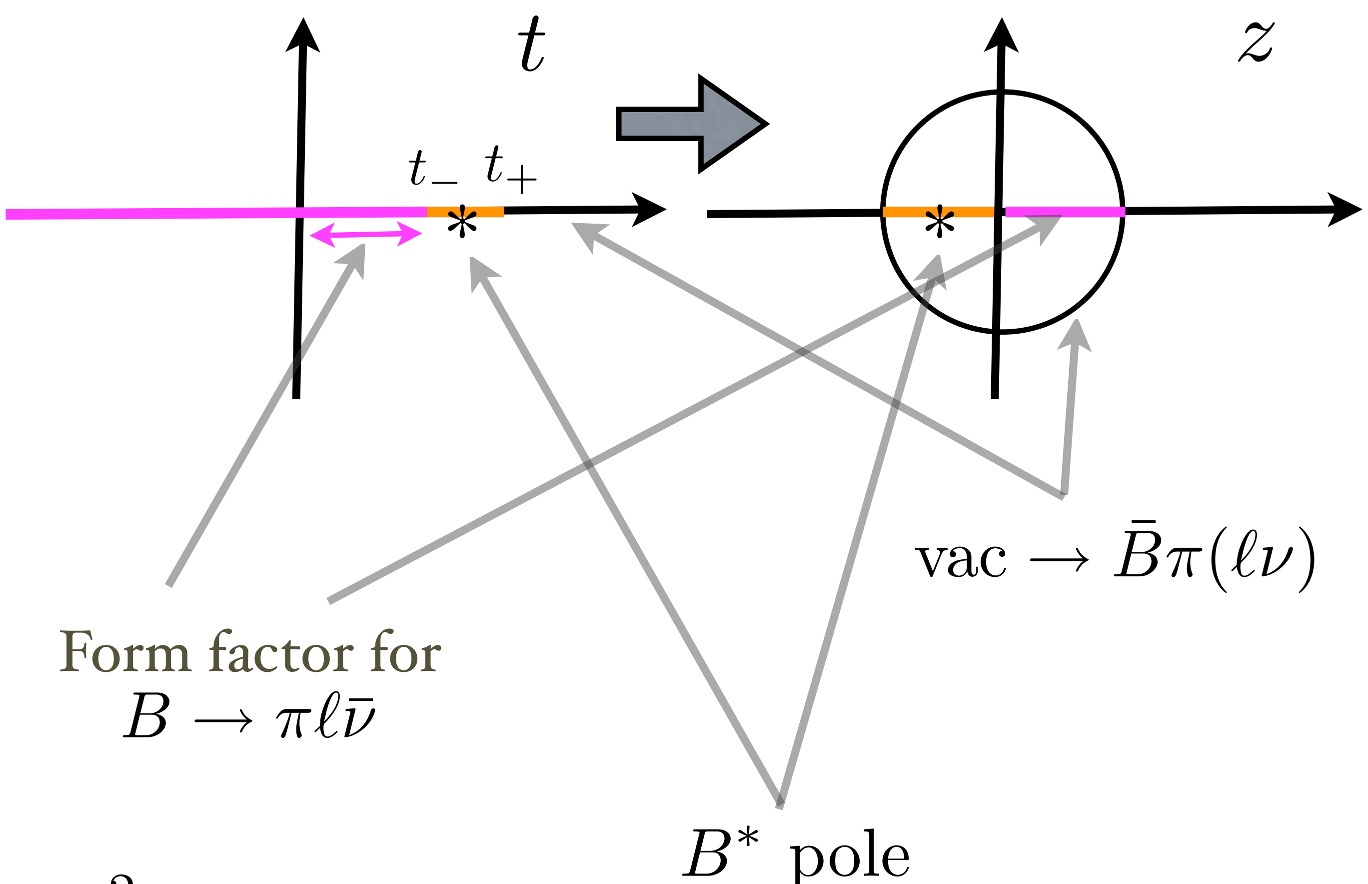
$$t_+ = (m_B + m_\pi)^2$$

Complex Magic

$$1 \geq \int_{t+}^{\infty} dt \, \frac{W(t)|f(t)|^2}{t^3}$$

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$



$$P(t)\phi(t)f(t) = \sum_{n=0}^{\infty} a_n \, z^n$$

Blaschke Factor: remove pole at $t = m_{B^*}^2$

Outer function: phase space, Jacobian, $\chi^{(0)}$ in QCD

eg. $t_0 = 0.65 \, t_-$ then

$-0.34 \leq z \leq 0.22$ kinematic expansion parameter

$t = q^2$

series form

$$\boxed{f_+(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n \, z^n}$$

parameters

$$\boxed{\sum_n a_n^2 \leq 1}$$

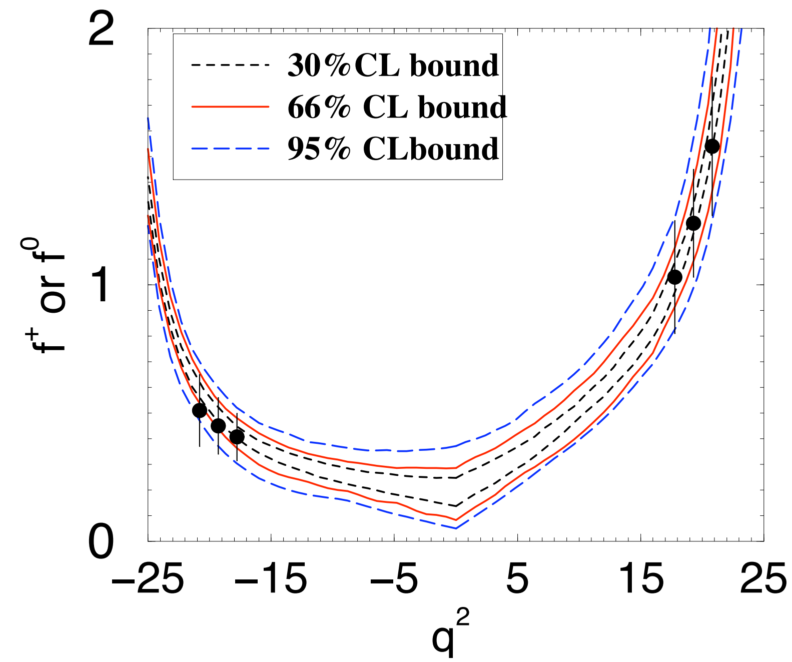
More Dispersion Relations

- Bound Form $$F_{\text{low}}(t, f_i) \le f(t) \le F_{\text{high}}(t, f_i)$$

  direct, no a's to deal with

  first combined analysis of lattice and experimental data (CLEO)

M. Fukunaga, T. Onogi



- Series Form $$f_+(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n\, z^n$$

χ^2 fit to include lattice and expt. points

C. Arnesen, B. Grinstein, I. Rothstein, I.S.

$$\chi^2 = \sum_{i,j} E_{ij}^{expt} [\mathrm{Br}_i^{\exp} - \mathrm{Br}_i(V_{\mathrm{ub}}, a_n, b_n)][\mathrm{Br}_j^{\exp} - \mathrm{Br}_j(V_{ub}, a_n, b_n)]$$
$$+ \sum_{ij} E_{ij}^{\mathrm{lat}} [f_{\mathrm{in}}^i - f^i(a_n, b_n)][f_{\mathrm{in}}^j - f^j(a_n, b_n)] + \dots$$

to go beyond $|V_{ub}|$ to get shape parameters, requires tighter constraints

T. Becher, R. Hill

$$\sum_{n=0}^{\infty} a_n^2 \le A \sim \left(\frac{\Lambda}{m_b}\right)^3$$

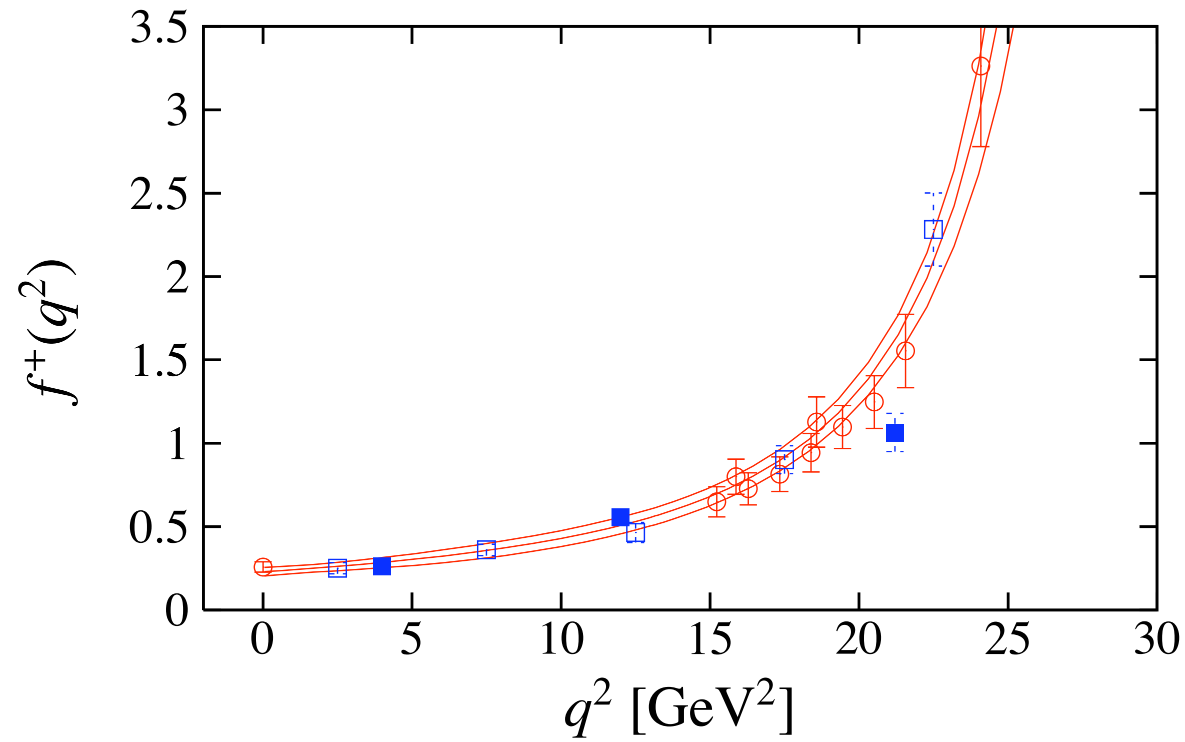
- Multiply-Subtracted Omnes Dispersion Relation

J. Flynn, J. Nieves

$$f_+(t) = \frac{1}{t_+ - t} \prod_{i=0}^{n} \left[f_i\,(t_+ - t_i) \right]^{\prod_{j=0, j\neq i}^{n} \left(\frac{t - t_j}{t_i - t_j} \right)} \qquad n \gg 1$$

χ^2 fit to include lattice, expt. points

completely compatible with series results for same inputs

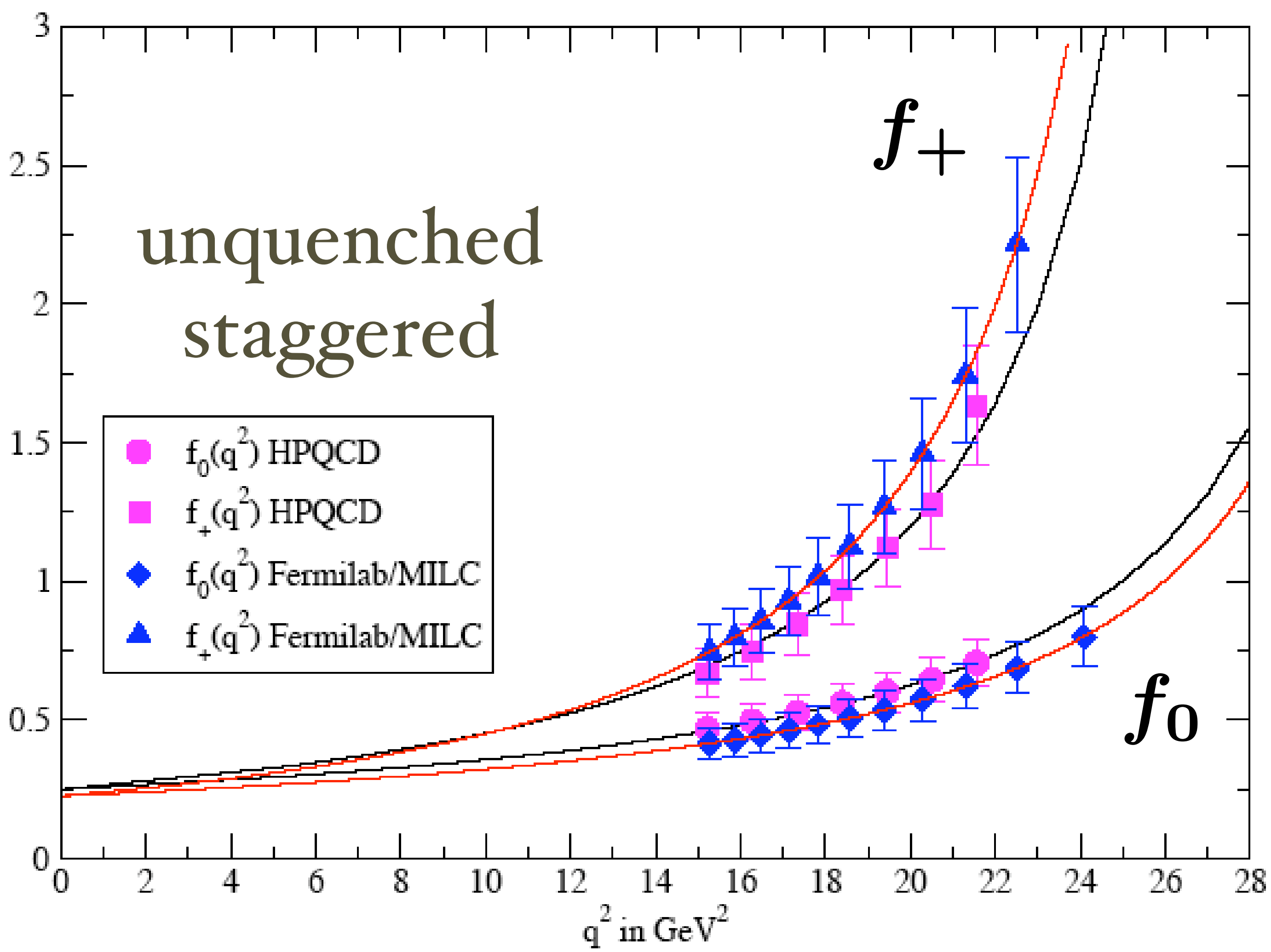


Unquenched Lattice QCD

- a list of systematic uncertainties for ALL known sources is crucial.

need
$|\vec{p}_\pi| \ll 1/a$

$q^2 \gtrsim 15\,\mathrm{GeV}^2$

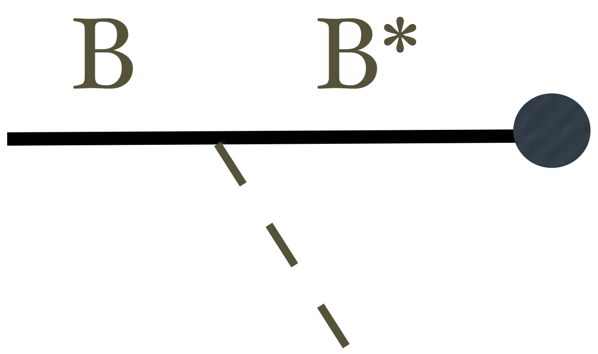


Errors	HPQCD errors
perturbative matching	9%
chiral extrap. & statistics	8%
action discretization	3%
relativistic	1%

Systematics	Fermilab/ MILC errors
matching	1%
chiral extrapolation	4%
q^2 interp.	4%
finite a	9%
statistics	8%

Chiral Perturbation Theory

- B* pole



$$f_+\left(q^2(E_\pi)\right) = \frac{g f_B m_B}{2 f_\pi (E_\pi + m_{B^*} - m_B)}\left[1 + \mathcal{O}\left(\frac{E_\pi}{\Delta}\right)\right] \qquad \Delta \sim 600\,\mathrm{MeV}$$

$$g f_B = ?$$

- important for lattice chiral extrapolation in m_π

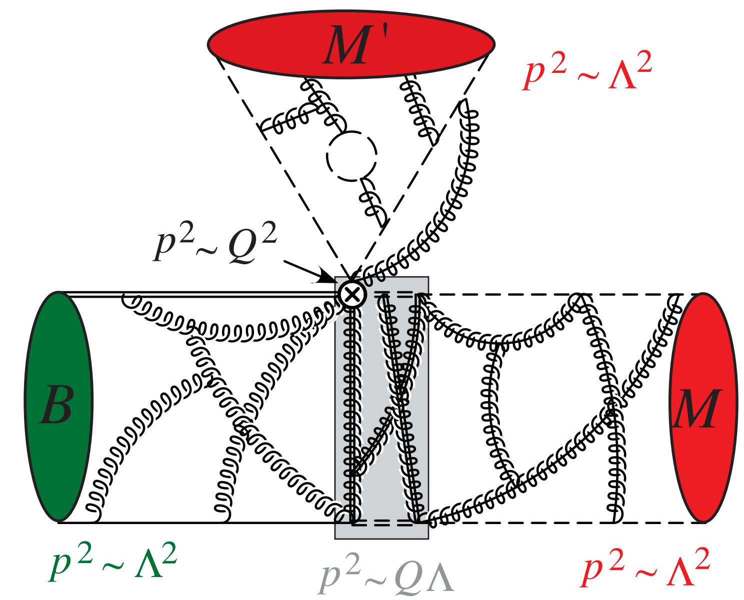
Shape Parameters, SCET, etc.

- gf_B
- $|V_{ub}|f_+(0)$
- $$\frac{1}{\beta} = \frac{(m_B^2 - m_\pi^2)}{f_+(0)} \left.\frac{df_0(q^2)}{dq^2}\right|_{q^2=0}$$
- $$\delta = 1 - \frac{(m_B^2 - m_\pi^2)}{f_+(0)} \left(\left.\frac{df_+}{dq^2}\right|_{q^2=0} - \left.\frac{df_0}{dq^2}\right|_{q^2=0} \right)$$

R.Hill

Factorization in SCET

BBNS; Chay and Kim;
Bauer, Pirjol, Rothstein, I.S.



Factorization at m_b

Nonleptonic $B \to M_1 M_2$

$$A(B \to M_1 M_2) = A^{c\bar{c}} + N\Big\{ f_{M_2} \zeta^{BM_1} \int du\, T_{2\zeta}(u) \phi^{M_2}(u) + f_{M_2} \int du dz\, T_{2J}(u,z) \zeta_J^{BM_1}(z) \phi^{M_2}(u) + (1 \leftrightarrow 2) \Big\}$$

Form Factors

$B \to$ pseudoscalar: $f_+,\ f_0,\ f_T$
$B \to$ vector: $V,\ A_0,\ A_1,\ A_2,\ T_1,\ T_2,\ T_3$

$$f(E) = \int dz\, T(z,E)\, \zeta_J^{BM}(z,E)$$

} "hard spectator", "factorizable"

$$+\ C(E)\, \zeta^{BM}(E)$$

} "soft form factor", "non-factorizable"

universality at $E\Lambda$

$$f_+(0) = \zeta^{B\pi} + \zeta_J^{B\pi}$$

$|V_{ub}| f_+(0)$

From nonleptonic data: $B \to \pi\pi$

- Uses data to remove complex penguin amplitude, and color suppressed amplitude. ie. to eliminate LO hadronic parameters

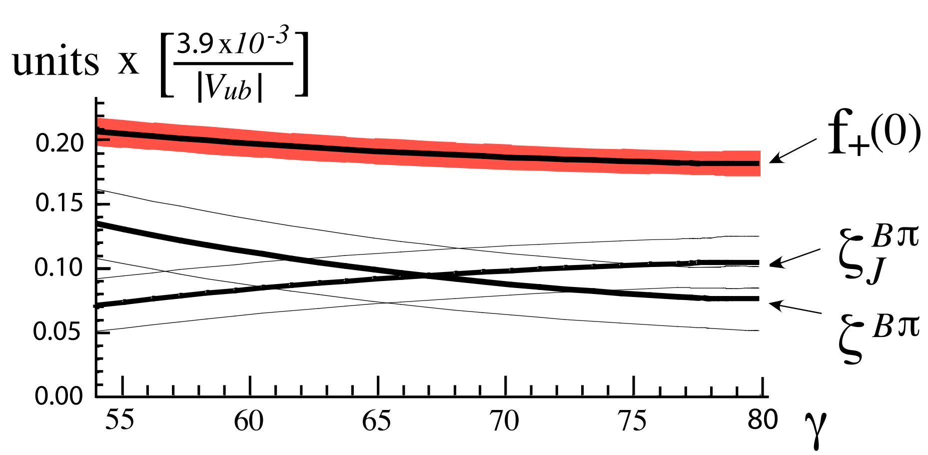
$$\boxed{|V_{ub}| f_+(0) = \left[\frac{64\pi}{m B^3 f_\pi^2} \frac{Br(B^- \to \pi^0\pi^-)}{\tau_{B^-}} |V_{ud}|^2 G_F^2 \right]^{1/2} \left[\frac{(C_1 + C_2) t_c - C_2}{C_1^2 - C_2^2} \right] \left[1 + \mathcal{O}\Big(\alpha_s(m_b), \frac{\Lambda_{\rm QCD}}{E} \Big) \right]}$$

$$t_c = \frac{|T_{\pi\pi}|}{|T_{\pi\pi} + C_{\pi\pi}|} = \sqrt{ \frac{Br(B^0 \to pi^+\pi^-)\tau_{B^-}}{Br(B^- \to \pi^0\pi^-)\tau_{B^0}} \, \frac{(1 + (1 - C^2_{\pi^+\pi^-} - S^2_{\pi^+\pi^-})^{1/2} \cos(2\beta) + S_{\pi^+\pi^-} \sin(2\beta)}{4 \sin^2 \gamma} }$$

see Arnesen et al.

$$|V_{ub}| f_+(0) = (7.6 \pm 1.9) \times 10^{-4}$$ (agrees at 1-sigma with fits)

flat with γ



Light Cone Sum rules give

$$f_+(0) = 0.258 \pm 0.031$$

(also agrees)

talk by P. Ball

Shape Parameters, SCET, etc.

- gf_B
- $|V_{ub}|f_+(0)$
- $$\frac{1}{\beta} = \frac{(m_B^2 - m_\pi^2)}{f_+(0)} \left.\frac{df_0(q^2)}{dq^2}\right|_{q^2=0}$$
- $$\delta = 1 - \frac{(m_B^2 - m_\pi^2)}{f_+(0)} \left(\left.\frac{df_+}{dq^2}\right|_{q^2=0} - \left.\frac{df_0}{dq^2}\right|_{q^2=0} \right)$$

E scaling of SCET f.f.

$$\begin{aligned} -\frac{1}{\beta} - 1 &= \left.\frac{d \ln(\zeta^{\mathrm{B}\pi} + \zeta_J^{\mathrm{B}\pi})}{d \ln E}\right|_{E=\frac{m_B}{2}} \\ &\simeq -2 \\ &\quad \text{if } \zeta^{\mathrm{B}\pi} + \zeta_J^{\mathrm{B}\pi} \sim \frac{1}{E^2} \end{aligned}$$

$$\delta = \left.\frac{2\zeta_J^{\mathrm{B}\pi}}{\zeta^{\mathrm{B}\pi} + \zeta_J^{\mathrm{B}\pi}}\right|_{E=\frac{m_B}{2}}$$

size of "hard scattering"
relative to "soft f.f."

Inputs to code

i) Expt. data with correlation matrix when available turn on/off datasets

ii) Lattice FNAL /MILC and/or HPQCD

take systematic error to be 100% correlated

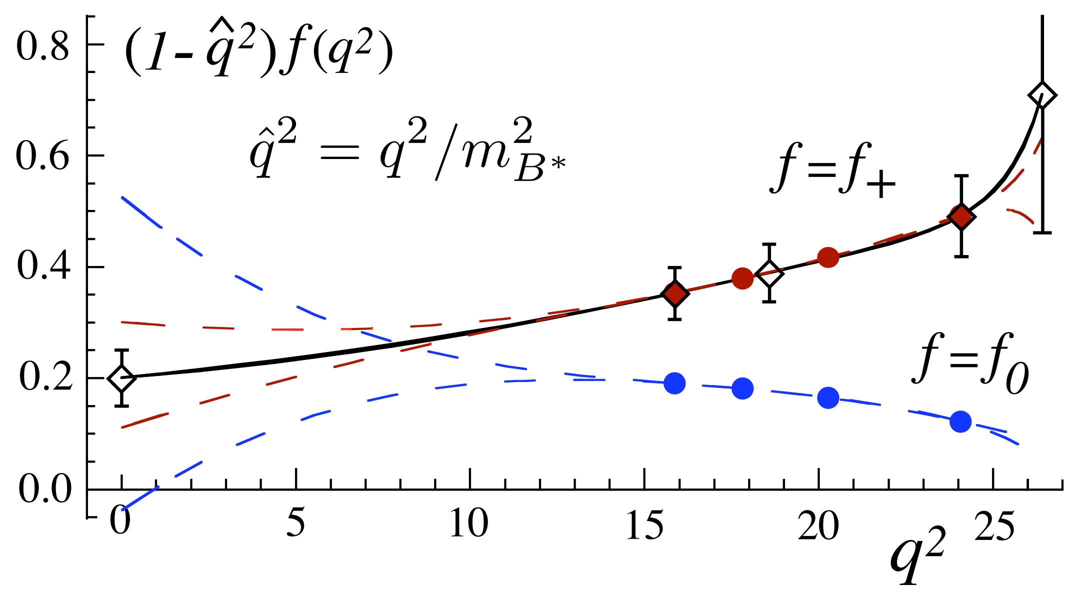
$E_{ij} = \sigma_i^2 \delta_{ij} + y^2 f_{\rm in}^i f_{\rm in}^j$ (this increases uncertainty)

play with:
- number of f+ and f0 points;
- error correlation analysis
  ala Bob Kowalewski

iii) $f_+(0) = f_0(0)$ simult. dispersion fit to both turn on/off

iv) number of $a_n^{(f_+)}$ and $a_n^{(f_0)}$ dispersion parameters vary the choice

v) bounds $\sum_n a_n^2 \leq 1$ or $\sum_n a_n^2 \leq A$, treatment of truncation error



The Default Vub Fit

- 3 lattice points for each form factor (100% syst. correlation)
- Do FNAL and HPQCD separately
- All experimental data sets included
- NO scet or chpt points
- 3 a_n's for f_+ and 2 b_n's for f_0 , $\sum_n a_n^2 \leq 1$
- $f_+(0) = f_0(0)$

$$V_{\mathrm{ub}}^{\mathrm{HPQCD}} = (4.21 \pm 0.45) \times 10^{-3}$$

errors dominated by lattice

$$V_{\mathrm{ub}}^{\mathrm{FNAL}} = (3.82 \pm 0.44) \times 10^{-3}$$

experimental errors ~ 3%

Vub

$$V_{\rm ub}^{\rm HPQCD} = (4.21 \pm 0.45) \times 10^{-3}$$
$$V_{\rm ub}^{\rm FNAL} = (3.82 \pm 0.44) \times 10^{-3}$$

- insensitive to: dispersion bound $\sum_n a_n^2 \leq A$, chpt/scet input
- also insensitive to: lattice corr. matrix ala Bob K. ~ 0.01
- mild: number of lattice points and which ones, num(a's) ~ 0.1
- depends on: FNAL vs. HPQCD, lattice norm (obviously) ~ 0.3

$f_+(0)$

- effects from: FNAL vs. HPQCD, num(a's)

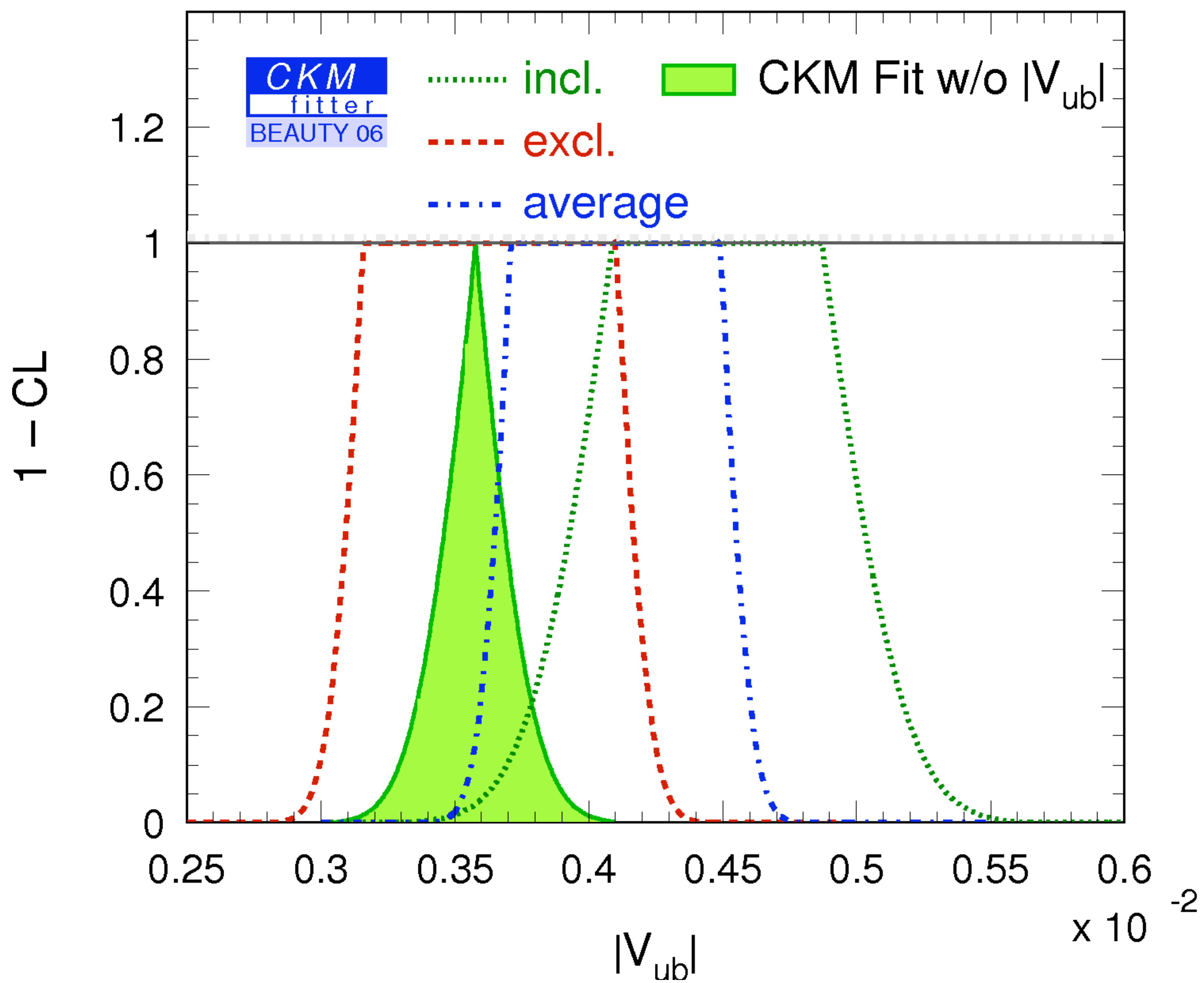
$$f_+^{\rm HPQCD}(0) = 0.20 \pm 0.02 \qquad f_+^{\rm HPQCD}(0) = 0.22 \pm 0.03$$

Other Shape parameters

$$\delta \equiv 1 - \frac{m_B^2 - m_\pi^2}{F_+(0)} \left(\frac{dF_+}{dq^2}\bigg|_{q^2=0} - \frac{dF_0}{dq^2}\bigg|_{q^2=0} \right) = 0.4 \pm 0.6 \pm 0.1 \pm 0.4 \,,$$

from Becher & Hill

The End


CKM fitter
BEAUTY 06
incl.
excl.
average
CKM Fit w/o $|V_{ub}|$
1 – CL
1.2
1
0.8
0.6
0.4
0.2
0
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
x 10 -2
$|V_{ub}|$