$|V_{ub}|$ from QCD Sum Rules on the Light-Cone

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Based on Ball/Zwicky, hep-ph/0406232; Ball, hep-ph/0611108.

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Theory Input for Semileptonic Decays

Form factors:

$$\langle \pi(p)|\bar{u}\gamma_\mu(1-\gamma_5)b|B(p+q)\rangle$$
$$= \quad (q+2p)_\mu\, f_+(q^2) + \frac{m_B^2 - m_\pi^2}{q^2}\, q_\mu \left(f_0(q^2) - f_+(q^2)\right)$$

$$0 \le q^2 \le (m_B - m_\pi)^2 \qquad \longleftrightarrow \qquad m_\pi \le E_\pi \le \frac{1}{2m_B}\left(m_B^2 - m_\pi^2\right)$$
$$0 \le q^2 \le 26.4\,\mathrm{GeV}^2 \qquad \longleftrightarrow \qquad 0.14\,\mathrm{GeV} \le E_\pi \le 2.6\,\mathrm{GeV}$$

Theoretical methods:

- lattice $\rightarrow$ J. Flynn's talk
- dispersive constraints $\rightarrow$ I. Stewart's talk
- QCD sum rules on the light-cone $\rightarrow$ this talk!

QCD Sum Rules on the Light-Cone

Basic quantity: correlation function:

$$i\int d^4y e^{iqy}\langle\pi(p)|T[\bar{u}\gamma_\mu b](y)[m_b\bar{b}i\gamma_5 d](0)|0\rangle \stackrel{\text{LCE}}{=} \sum_n T_H^{(n)} \otimes \phi_\pi^{(n)}$$

- $\phi_\pi^{(n)}$: π distribution amplitudes (DAs)
- $T_H^{(n)}$: perturbative amplitudes
- n: twist
- LCE: light-cone expansion

$$= 2p_\mu\left(f_+(q^2)\,\frac{m_B^2 f_B}{m_B^2 - p_B^2} + \text{higher poles and cuts}\right) + \ldots$$

- B meson described by Euclidean current + plus analytical continuation

QCD Sum Rules on the Light-Cone

Features of LCSRs:

- terms in LCE ordered in powers of $1/m_b \to$ need to include higher-twist terms ($n > 2$)
- $\sum T_H^{(n)} \otimes \phi_\pi^{(n)}$ implies factorization – valid at higher twist?
  - calculate $O(\alpha_s)$, known for

    T2 (π (Khodjamirian et al. 97, Ball et al. 97), ρ (Ball/Braun 98))

    T3 (π (Ball/Zwicky 2001))

    $\to$ factorization OK
- use standard SR techniques: Borel-transformation, continuum model
  - introduce irreducible systematic uncertainty $\sim 10\%$

QCD Sum Rules on the Light-Cone

- Ball/Zwicky 04:

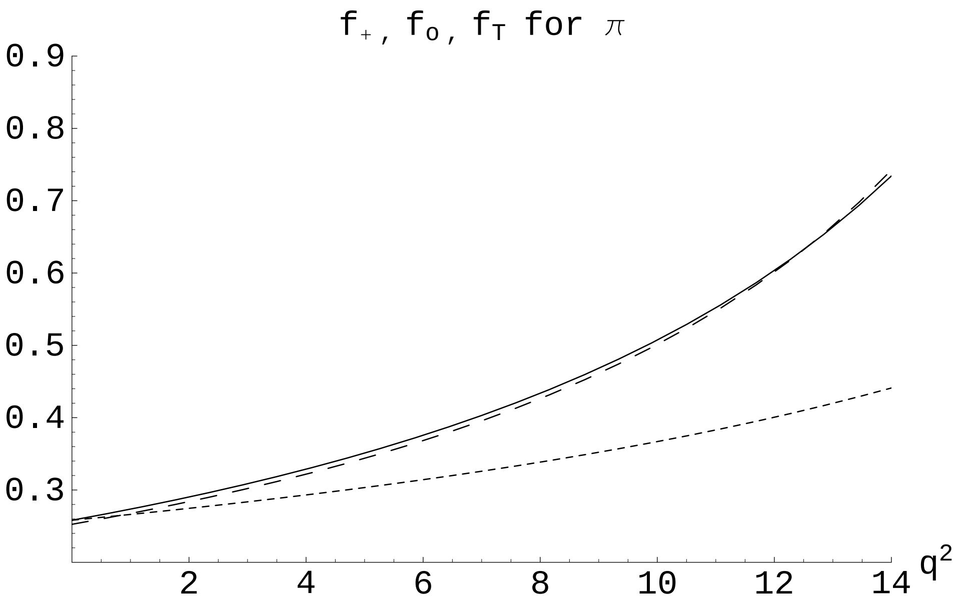
  $$f_+(0) = 0.258 \pm 0.031$$

  with theory input for leading-twist π distribution amplitude $\phi_{\pi;2}$

- Ball/Zwicky 05: constrain $\phi_{\pi;2}$ from experimental q^2 spectrum of $B \to \pi e \nu$:

  $f_+(0) \approx 0.27$ and

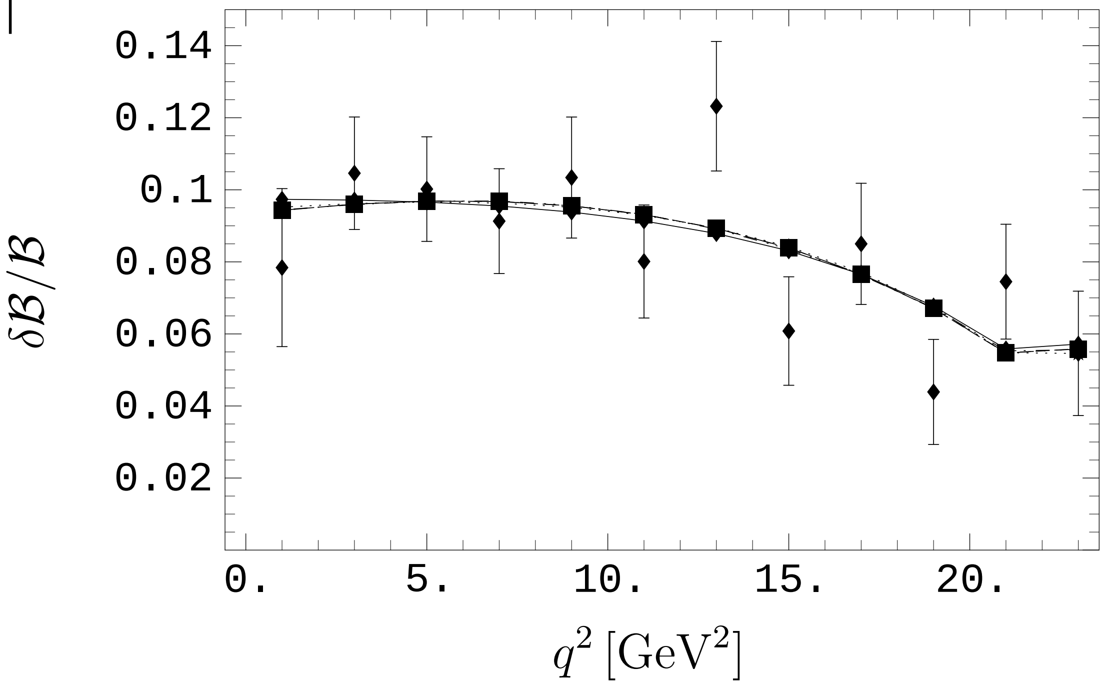
  $$|V_{ub}| = (3.2 \pm 0.4) \cdot 10^{-3}$$



BZ 04

Results for $B \to \rho e \nu$ also available — but less experimental information.

Theory Assisted by Experiment



2006 BaBar results for q^2 spectrum in $B \to \pi e \nu$ in 12 bins (up from 5 bins in 2005)

Strategy: Parametrise form factor, fit to data, extract $|V_{ub}| f_+(0)$.

Form Factor Parametrisations

- Becirevic/Kaidalov (BK) :

$$f_+(q^2) = \frac{f_+(0)}{\left(1 - q^2/m_{B^*}^2\right)\left(1 - \alpha_{\rm BK}\, q^2/m_B^2\right)}\,,$$

where $\alpha_{\rm BK}$ determines the shape of f_+ and $f_+(0)$ the normalisation;

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- Ball/Zwicky (BZ):

$$f_+(q^2) = f_+(0)\left(\frac{1}{1 - q^2/m_{B^*}^2} + \frac{r q^2/m_{B^*}^2}{\left(1 - q^2/m_{B^*}^2\right)\left(1 - \alpha_{\rm BZ}\, q^2/m_B^2\right)}\right),$$

with the two shape parameters $\alpha_{\rm BZ}$, r and the normalisation $f_+(0)$; BK is a variant of BZ with $\alpha_{\rm BK} := \alpha_{\rm BZ} = r$.

Form Factor Parametrisations

- the AFHNV parametrisation (Flynn et al.), based on an $(n+1)$-subtracted Omnes representation of f_+:

$$f_+(q^2) \overset{n \geqslant 1}{=} \frac{1}{s_{th} - q^2} \prod_{i=0}^{n} \left[f_+(q_i)^2 (s_{th} - q_i^2) \right]^{\alpha_i(q^2)} ,$$

with $$\alpha_i(s) = \prod_{j=0, j \neq i}^{n} \frac{s - s_j}{s_i - s_j} , \qquad s_{th} = (m_B + m_\pi)^2 ;$$

the shape parameters are $f_+(q_i^2)/f_+(q_0^2)$ with $q^2_{0,\ldots,n}$ the subtraction points; the normalisation is given by $f_+(0)$.

Form Factor Parametrisations

- the BGL parametrisation based on analyticity of f_+:

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, q_0^2)} \sum_{k=0}^{\infty} a_k(q_0^2)[z(q^2, q_0^2)]^k \,, \qquad \sum_k a_k^2 \leq 1 \,,$$

$$z(q^2, q_0^2) = \frac{\{(m_B + m_\pi)^2 - q^2\}^{1/2} - \{(m_B + m_\pi)^2 - q_0^2\}^{1/2}}{\{(m_B + m_\pi)^2 - q^2\}^{1/2} + \{(m_B + m_\pi)^2 - q_0^2\}^{1/2}}$$

  q_0^2: free parameter, determines maximum $|z|$; define

  - BGLa : $q_0^2 = 20.1\,\mathrm{GeV}^2$, $|z| < 0.28$
  - BGLb : $q_0^2 = 0$, $|z| < 0.52$

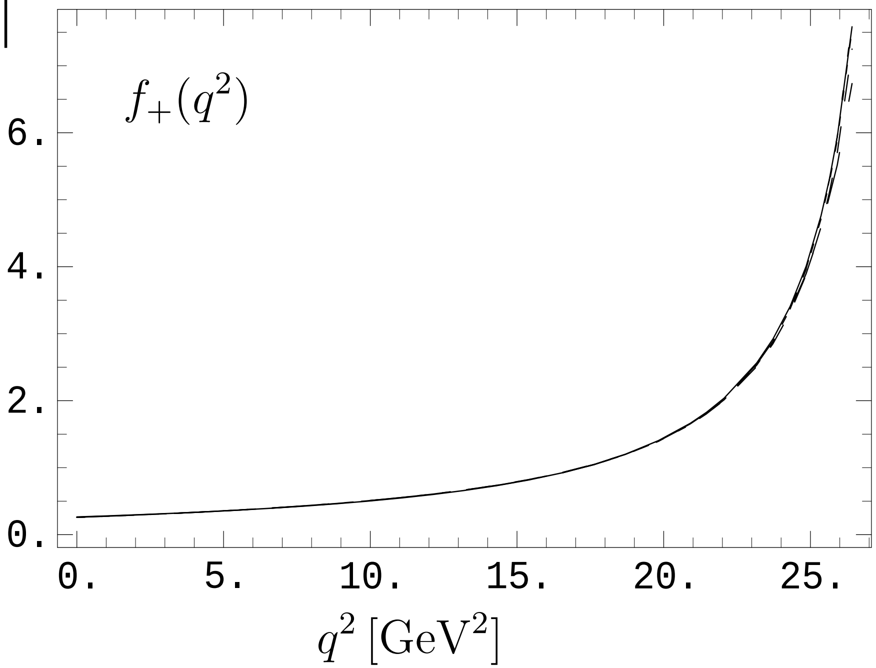
- systematic expansion in the small parameter z; truncate at $k_{\max}$; choose $k_{\max} = 2$ for BGLa and $k_{\max} = 3$ for BGLb .

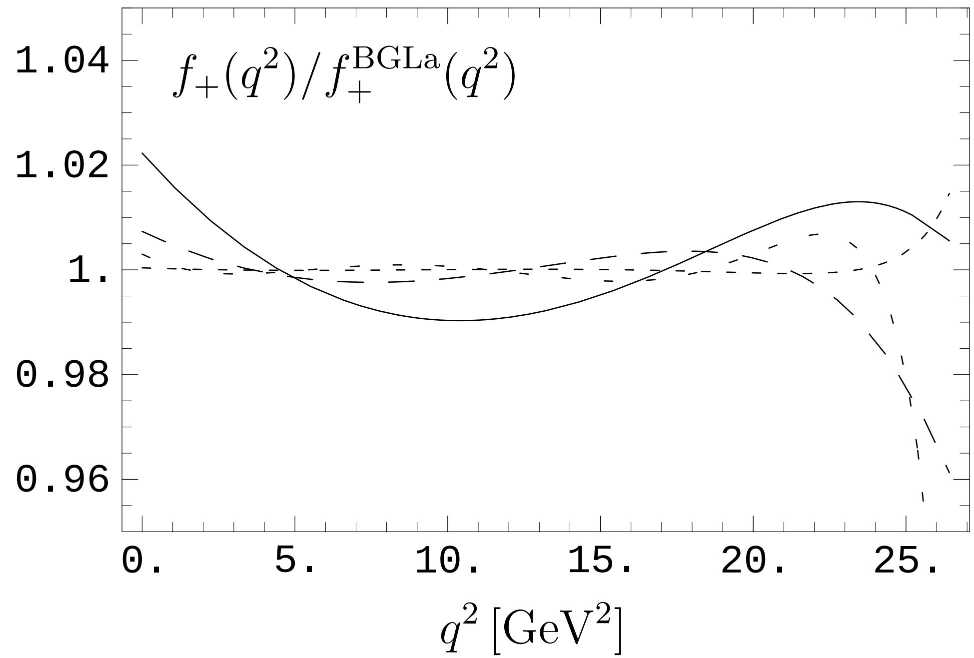
$|V_{ub}| f_+(0)$ from data

Param.	$\lvert V_{ub}\rvert f_+(0)$	Remarks
BK	$(9.3 \pm 0.3 \pm 0.3) \times 10^{-4}$	$\chi^2_{\min} = 8.74/11\,\mathrm{dof}$ $\alpha_{\mathrm{BK}} = 0.53 \pm 0.06$
BZ	$(9.1 \pm 0.5 \pm 0.3) \times 10^{-4}$	$\chi^2_{\min} = 8.66/10\,\mathrm{dof}$ $\alpha_{\mathrm{BZ}} = 0.40^{+0.15}_{-0.22},\ r = 0.64^{+0.14}_{-0.13}$
BGLa	$(9.1 \pm 0.6 \pm 0.3) \times 10^{-4}$	$\chi^2_{\min} = 8.64/10\,\mathrm{dof}$
BGLb	$(9.1 \pm 0.6 \pm 0.3) \times 10^{-4}$	$\chi^2_{\min} = 8.64/9\,\mathrm{dof}$
AFHNV	$(9.1 \pm 0.3 \pm 0.3) \times 10^{-4}$	$\chi^2_{\min} = 8.64/8\,\mathrm{dof}$
SCET	$(8.0 \pm 0.4) \times 10^{-4}$	from $B^- \to \pi^-\pi^0$ (Arnesen et al.) (tree-level, no $1/m_b$ corrections)

All parametrisations agree – model-independent result!

Fitted Form Factor





Left panel: best-fit form factors f_+ as a function of q^2. The line is an overlay of all five parametrisations.

Right panel: best-fit form factors normalised to BGLa.
Solid line: BK, long dashes: BZ, short dashes: BGLb, short dashes with long spaces: AFHNV.

Results for $|V_{ub}|$

Procedure 1: take FF from theory calculation, fit to BK and extract $|V_{ub}|$ from experimental partial branching ratio
($q^2 \leq 16\,\mathrm{GeV}^2$ for LCSR, $q^2 \geq 16\,\mathrm{GeV}^2$ for lattice)

LCSR	$f_+(0) = 0.26 \pm 0.03\,, \quad \alpha_{\mathrm{BK}} = 0.63^{+0.18}_{-0.21}$ $|V_{ub}| = (3.5 \pm 0.6(\mathrm{th}) \pm 0.1(\exp)) \times 10^{-3}$ $|V_{ub}| f_+(0) = (9.0^{+0.7}_{-0.6} \pm 0.4) \times 10^{-4}$
HPQCD	$f_+(0) = 0.21 \pm 0.03\,, \quad \alpha_{\mathrm{BK}} = 0.56^{+0.08}_{-0.11}$ $|V_{ub}| = (4.3 \pm 0.7 \pm 0.3) \times 10^{-3}$ $|V_{ub}| f_+(0) = (8.9^{+1.2}_{-0.9} \pm 0.4) \times 10^{-4}$
FNAL	$f_+(0) = 0.23 \pm 0.03\,, \quad \alpha_{\mathrm{BK}} = 0.63^{+0.07}_{-0.10}$ $|V_{ub}| = (3.6 \pm 0.6 \pm 0.2) \times 10^{-3}$ $|V_{ub}| f_+(0) = (8.2^{+1.0}_{-0.8} \pm 0.3) \times 10^{-4}$

Results for $|V_{ub}|$

Procedure 2: take FF from theory, fit to experimentally determined shape, BGLa , obtain $f_+(0)$, extract $|V_{ub}|$ from full branching ratio.

LCSR	$f_+(0) = 0.26 \pm 0.03$ $|V_{ub}| = (3.5 \pm 0.4(\text{shape}) \pm 0.1(\mathcal{B})) \times 10^{-3}$
HPQCD	$f_+(0) = 0.21 \pm 0.03$ $|V_{ub}| = (4.3 \pm 0.5 \pm 0.1) \times 10^{-3}$
FNAL	$f_+(0) = 0.25 \pm 0.03$ $|V_{ub}| = (3.7 \pm 0.4 \pm 0.1) \times 10^{-3}$

- reduced theoretical uncertainty as shape of FF is fixed by experimental data
- reduced experimental uncertainty as total $\mathcal{B}(B \to \pi e \nu)$ can be used

Summary

- form factor calculations from QCD sum rules on the light-cone in mature shape
- no scope for major improvement
- LCSR predictions for small and moderate $q^2 < 16\,\mathrm{GeV}^2$
  $\longleftrightarrow$ LQCD predictions for large $q^2 > 16\,\mathrm{GeV}^2$
- reduce error of $|V_{ub}|$ determination by fixing shape of form factor from experiment instead of theory data
- both LCSR and FNAL prefer small $|V_{ub}| \sim 3.6 \times 10^{-3}$
- HPQCD points at larger $|V_{ub}| \sim 4.3 \times 10^{-3}$
- UTangles gives $|V_{ub}| = (3.50 \pm 0.18) \times 10^{-3}$

How sure are we about the inclusive result? (both th. and exp.)