V_{us} from kaon decays

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4th Int. Workshop on the CKM Unitarity Triangle

Nagoya, Japan - 15 December 2006

http://www.lnf.infn.it/wg/vus

Working Group on Precise SM Tests in K Decays

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Acknowledgements

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Vladimir Obraztsov (Protvino)

KLOE:
Matthew Moulson (Frascati) web coordinator
Patrizia De Simone (Frascati)

KTeV:
Sasha Glazov (DESY)

NA48:
Rainer Wanke (Uni. Mainz)
Michele Veltri (Uni. Urbino)
Mauro Piccini (CERN)

Theory:
Johan Bijnens (Lund)
Vincenzo Cirigliano (Los Alamos)
Juerg Gasser (Bern)
Claudio Gatti (Frascati)
Richard Hill (FNAL)
Federico Mescia (Frascati)
Jan Stern (Orsay)

Honorary chair: **Paolo Franzini (LNF)** Coordinators: **Mario Antonelli (LNF)** and **Gino Isidori (LNF)**

K_{l3} decays, V_{us}, and CKM unitarity

At present, most precise test of CKM unitarity is from 1st row:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \equiv 1 - \Delta$$

$0^+ \to 0^+ \beta$ decays:	$2|V_{ud}|\delta V_{ud} = 0.0005$
$Kl3$ decays:	$2|V_{us}|\delta V_{us} < 0.0010$

→ **2002** (2004 PDG)	**Old $Kl3$ data give $\Delta = 0.0035(15)$** A 2.3σ hint of unitarity violation?
2003	**BNL 865 measures BR($K^+ \to \pi^0 e^+ \nu$) = 5.13(10)%** Value for V_{us} consistent with unitarity
2004-2006	**Many new measurements from KTeV, KLOE, ISTRA+, NA48** • **BRs, lifetimes, form-factor slopes** • **Much higher statistics** than older measurements • Importance of **radiative corrections** • Proper reporting of **correlations** between measurements
2005 CKM WG1 report	**$|V_{us}|f_+(0)$ = 0.2173(8): unitarity to better than 1σ** Includes many (but not all) of these important developments
This talk	**Update with all recent measurements (even if preliminary)**

Determination of V_{us} using K_{l3} rates

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{Kl}(\{\lambda\}_{Kl}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{Kl}^{EM})$$

with $K \in \{K^+, K^0\}$; $l \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

S_{EW} Universal SD EW correction (1.0232)

Inputs from theory:

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta^K_{SU(2)}$ Form-factor correction for $SU(2)$ breaking

Δ^{Kl}_{EM} Form-factor correction for long-distance EM effects

Inputs from experiment:

$\Gamma(K_{l3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$I^{Kl}(\{\lambda\}_{Kl})$ Integral of form-factor over phase space: λs parameterize evolution in t

- K_{e3}: Only λ_+ (or λ_+', λ_+'')
- $K_{\mu3}$: Need λ_+ and λ_0

K_L branching ratios

KTeV
PRD 70 (2004)

5 ratios of main BRs from independent samples of 10^5-10^6 events collected with a single trigger

2-track ratios

BR($K_{\mu3}/K_{e3}$) = 0.6640(26)

BR($\pi^+\pi^-\pi^0/K_{e3}$) = 0.3078(18)

BR($\pi^+\pi^-/K_{e3}$) = 0.004856(28)

Neutral ratio

BR($2\pi^0/3\pi^0$) = 0.004446(25)

Mixed ratio

BR($3\pi^0/K_{e3}$) = 0.4782(55)

6 decays = 99.93% of K_L width
KTeV combines ratios to extract BRs

Our fit uses these BR ratios
Correlations available

NA48
PLB 602 (2004)

K_L beam only, 2-track sample, 80M events (6M signal)

$$\frac{\mathrm{BR}(K_{e3})}{\mathrm{BR}(2\ \mathrm{track})} = 0.4978(35) \approx \frac{\mathrm{BR}(K_{e3})}{1 - \mathrm{BR}(3\pi^0)}$$

NA48
preliminary

From BR($K_L \to 3\pi^0$)/BR($K_S \to 2\pi^0$)

BR($3\pi^0$) = 0.1966(34)

K_L branching ratios and lifetime

KLOE
PLB 632 (2006)

Absolute BRs: K_L decays tagged by $K_S \to \pi^+\pi^-$
Errors on absolute BRs dominated by error on τ_L
Dependence on τ_L of geometrical efficiency known

For KLOE results: Set Σ_x BR($K_L \to x$) = 1 and solve for τ_L
For our fit: Use unconstrained BRs with dependence on τ_L

BR$^{(0)}$($Ke3$) = 0.4049(21)
BR$^{(0)}$($K\mu3$) = 0.2726(16)
BR$^{(0)}$($3\pi^0$) = 0.2018(24)
BR$^{(0)}$($\pi^+\pi^-\pi^0$) = 0.1276(15)

at $\tau_L^{(0)}$ = 51.54 ns, with
d BR/BR = $0.67 d\tau_L/\tau_L$

Correlations available

KLOE
PLB 626 (2005)

Lifetime: Direct measurement with $K_L \to 3\pi^0$ events
High, uniform reconstruction efficiency over $0.4\lambda_L$
Independent of BR measurement

τ_L = 50.92(30) ns

cf. Vosburgh '72: τ_L = 51.54(44) ns

$K_L \to \pi^+\pi^-$

New measurements of $K_L \to \pi^+\pi^-(\gamma)$ also useful in global fit

KTeV
PRD 70 (2004)

BR($\pi^+\pi^-/Ke3$) = 4.856(29) × 10^{-3}
1 of 5 ratios in K_L BR analysis
Contribution from direct emission (DE) negligible

KLOE
PLB 638 (2006)

BR($\pi^+\pi^-/K\mu3$) = 7.275(68) × 10^{-3}
Fully inclusive of DE component

NA48
hep-ex/0611052

BR($\pi^+\pi^-/Ke3$) = 4.826(27) × 10^{-3}
Residual DE contribution of 0.19% subtracted

For consistency and to better satisfy Σ BR = 1 in global fit,
DE contribution of **1.52(7)%*** added to **KTeV** and **NA48** results

* From E731 '93, KTeV '01 and KTeV '06 $K_L \to \pi^+\pi^-\gamma$ results

Fit to K_L BR and lifetime measurements

Availability of **comprehensive new K_L data set** with proper radiative corrections has **radically changed the PDG fit**

- **2004 fit used 50 measurements** - all but a handful pre-1990
- **2006 fit uses 17 measurements** - all but 2 post-2003

Compared to PDG 2006, our fit:

- Uses the KTeV BR ratios and KLOE BRs quoted before application of constraints

  In each case, PDG uses the constrained results and removes the $3\pi^0$ from the fit

- Uses the NA48 preliminary BR($3\pi^0$) and new BR($\pi^+\pi^-$) measurements
- Implements consistent treatment of DE for $K_L \to \pi^+\pi^-$

Results of fit to K_L BRs, τ

18 input measurements:

- **5 KTeV** ratios
- **NA48** BR($Ke3$/2 track)
- **NA48** BR($3\pi^0$) [prelim.]
- **4 KLOE** BRs
  with dependence on τ_L
- **KLOE**, **NA48** BR($\pi^+\pi^-/Kl3$)
- **KLOE**, **NA48** BR($\gamma\gamma/3\pi^0$)
- **PDG** ETAFIT BR($2\pi^0/\pi^+\pi^-$)
- **KLOE** τ_L from $3\pi^0$
- **Vosburgh ’72** τ_L

1 constraint: Σ BR = 1

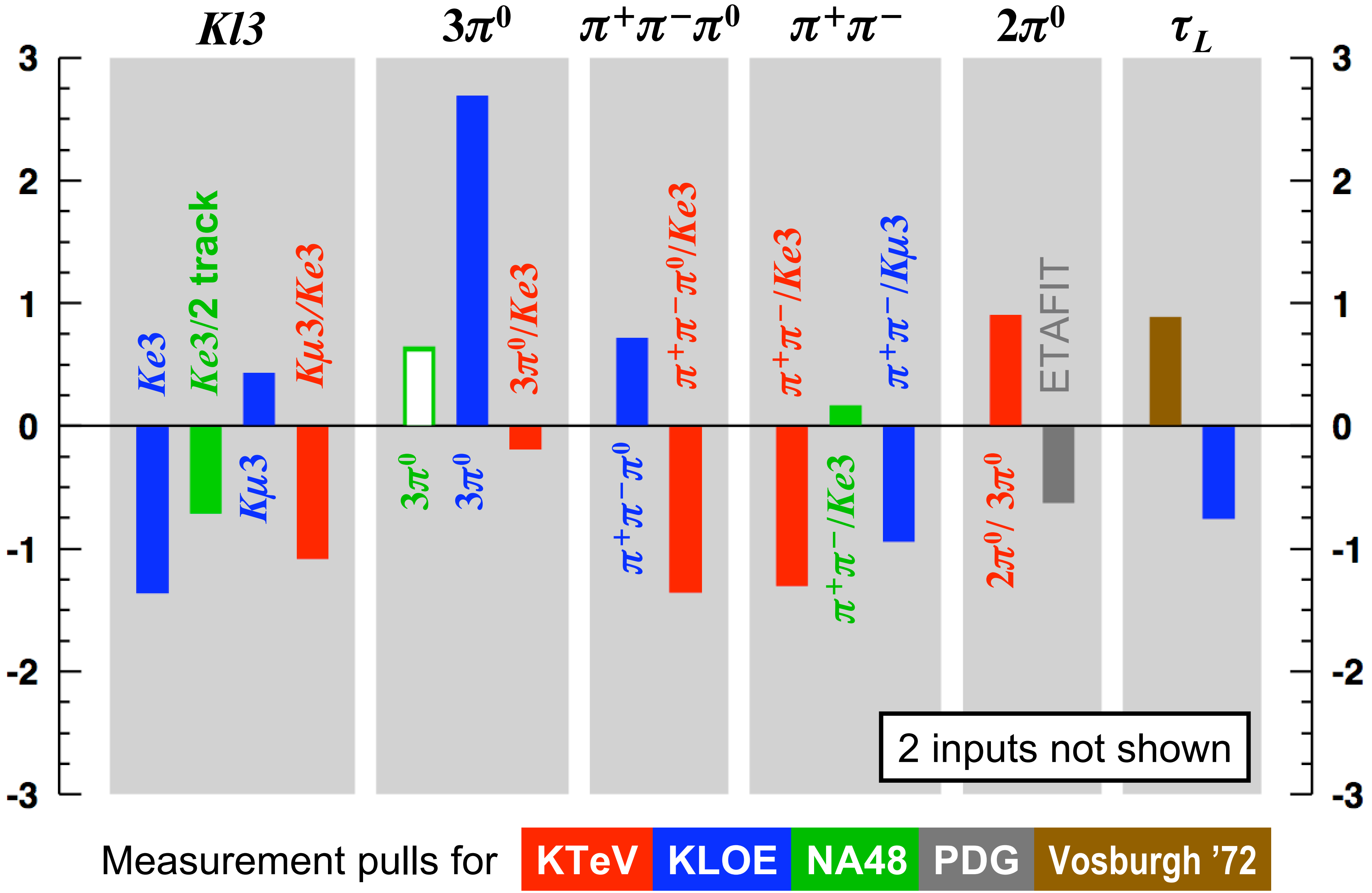
Parameter	Value	S
BR($Ke3$)	0.40571(89)	1.4
BR($K\mu3$)	0.27055(81)	1.3
BR($3\pi^0$)	0.19447(103)	1.4
BR($\pi^+\pi^-\pi^0$)	0.12588(78)	1.5
BR($\pi^+\pi^-$)	$1.9860(74)\times10^{-3}$	1.2
BR($2\pi^0$)	$8.603(54)\times10^{-4}$	1.7
BR($\gamma\gamma$)	$5.453(43)\times10^{-4}$	1.1
τ_L	51.148(211) ns	1.1

χ^2/ndf = 20.2/11 (Prob = 4.3%)

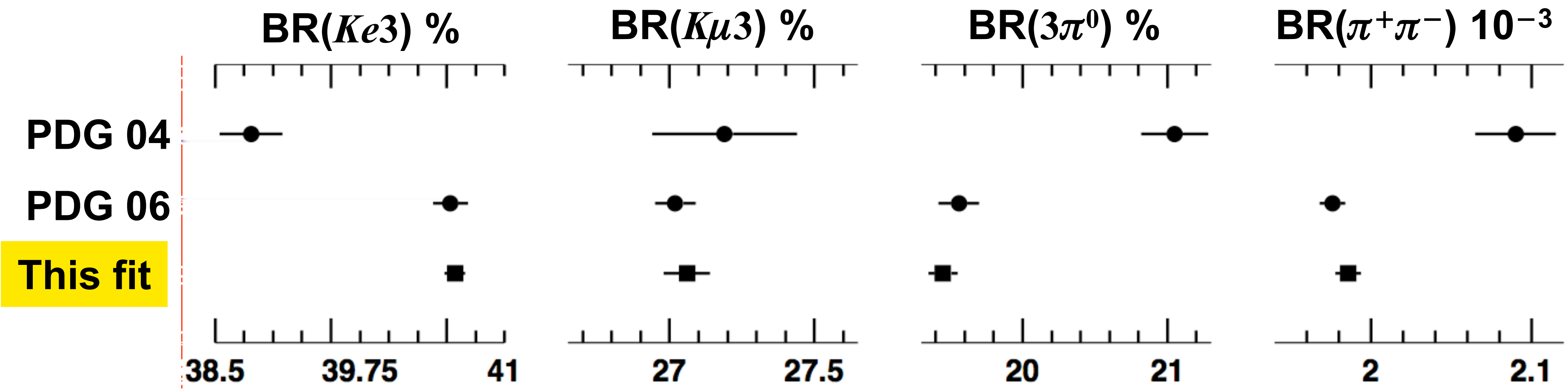
compare PDG ’06: 14.8/10 (14%)

- Without preliminary results & DE correction, our fit reproduces PDG
- Including ETAFIT pulls down $2\pi^0$ & $3\pi^0$ BRs, clashes with KLOE $3\pi^0$
- Even with scale factor, our fit reduces error on $Ke3$

Comparison: K_L BR fit vs. data



Evolution of K_L BRs



Differences between our fit and PDG '06 are minor

Since PDG '04 fit:

- BR($Ke3$) significantly increased
- BR($K\mu3$) changed little
- BR($3\pi^0$) significantly decreased

Proper radiative corrections
Especially for $Ke3$
Exclusion of NA31 measurements
Significantly reduces errors

$R_{\mu e}$ decreases from 0.701(9) to 0.6668(26)
Better agreement with lepton universality

BR($K_S \to \pi e \nu$) and K_S lifetime

KLOE
PLB 636 (2006)

Using tagged K_S beam

BR($K_S \to \pi e \nu$)/BR($K_S \to \pi^+\pi^-$) = 10.19(13) × 10^{-4}

KLOE
EPJC 48 (2006)

410 pb^{-1}, averaged with KLOE ’02 result (17 pb^{-1})

BR($K_S \to \pi^+\pi^-$)/BR($K_S \to \pi^0\pi^0$) = 2.2459(54)

These two measurements completely determine main K_S BRs

BR($K_S \to \pi e \nu$) = 7.046(91) × 10^{-4}

PDG

τ_S = 0.08958(5) ns

From fit to CP parameters, does not assume CPT

Dominated by **NA48 ’02** and **KTeV ’03** τ_S values

Preliminary results on $K^{\pm}_{l3}$ BRs

NA48/2
preliminary

New results on BR($K^{\pm}e3$)/BR($\pi\pi^0$) and BR($K^{\pm}\mu3$)/BR($\pi\pi^0$)!

Ratio for $K^{\pm}e3$ higher than previous preliminary value

Statistics dominated; main systematic from FF for acceptance

BR($K^{\pm}e3$)/BR($\pi\pi^0$) = 0.2496(9)(4)

BR($K^{\pm}\mu3$)/BR($\pi\pi^0$) = 0.1637(6)(3)

Correlation provided

ISTRA+
preliminary

Updated preliminary value

BR($K^{\pm}e3$)/BR($\pi\pi^0$) = 0.2449(4)(15)

KLOE
preliminary

Absolute BR($K^{\pm}e3$) and BR($K^{\pm}\mu3$) measurements

Separate measurements for each charge

Tagged by $K \to \mu\nu$ and $K \to \pi\pi^0$: 8 measurements total

BR$^{(0)}$($K^{\pm}e3$) = 5.047(92)%

BR$^{(0)}$($K^{\pm}\mu3$) = 3.310(81)%

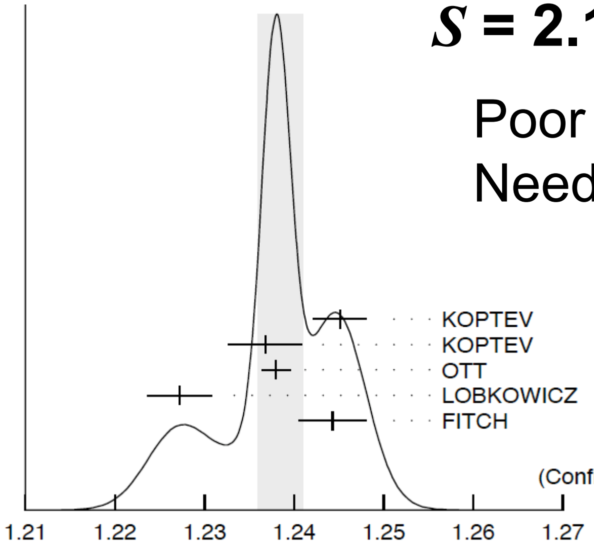
at $\tau_{\pm}^{(0)}$ = 12.36 ns, with d BR/BR = −0.50 $d\tau_{\pm}/\tau_{\pm}$

$K^{\pm}$ lifetime

PDG
average

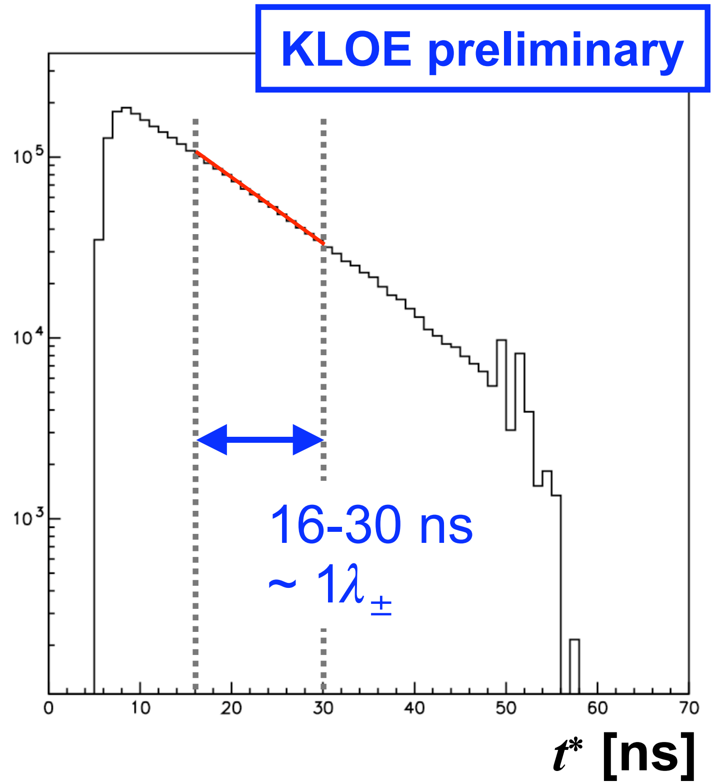
12.385(25) ns
S = 2.1

Poor consistency
Needs confirmation



			χ^2
KOPTEV	95	CNTR	4.9
KOPTEV	95	CNTR	0.2
OTT	71	CNTR	0.1
LOBKOWICZ	69	CNTR	9.8
FITCH	65B	CNTR	2.4
			17.3

(Confidence Level = 0.002)



KLOE
preliminary

Fit to t^* distribution from decay length
Use all $K \to \mu\nu$-tagged vertices in drift chamber

$\tau_{\pm}$= 12.367(78) ns

Fit to $K^{\pm}$ BR and lifetime measurements

Not possible to fit to only new $K^{\pm}$ data (unlike for K_L)

Only $Kl3$ and $Kl3/\pi\pi^0$ have been measured recently

- $Kl3$ and $\pi\pi^0$ highly correlated in fit (-0.64, -0.79 for $Ke3$, $K\mu3$)
- New measurement of $\pi\pi^0$ is crucial

For channels like $\pi\pi^0$ and $\pi^+\pi^+\pi^-$, fit rests heavily on Chiang '72

- No radiative corrections
- 6 BRs constrained by Σ BR = 1, correlations unavailable

Compared to PDG 2006, our fit:

- Uses preliminary results from KLOE, ISTRA+, and NA48/2
- Does not use BR($\pi^0\pi^0 e\nu$) as a free parameter

Results of fit to $K^{\pm}$ BRs, τ

30 input measurements:

- **5 older τ values in PDG**
- **KLOE τ**
- **KLOE BR($\mu\nu$)**
- **KLOE $Ke3$, $K\mu3$ BRs**
  with dependence on τ
- **ISTRA+ BR($Ke3/\pi\pi^0$)**
- **NA48/2 $Ke3/\pi\pi^0$, $K\mu3/\pi\pi^0$**
- **E865 BR($Ke3/K$Dal)**
- **6 Chiang '72 BRs**
- **3 old BR($\pi\pi^0/\mu\nu$)**
- **2 old BR($Ke3$/2 body)**
- **3 $K\mu3/Ke3$ (2 old)**
- **2 old + 1 KLOE results on 3π**

1 constraint: Σ BR = 1

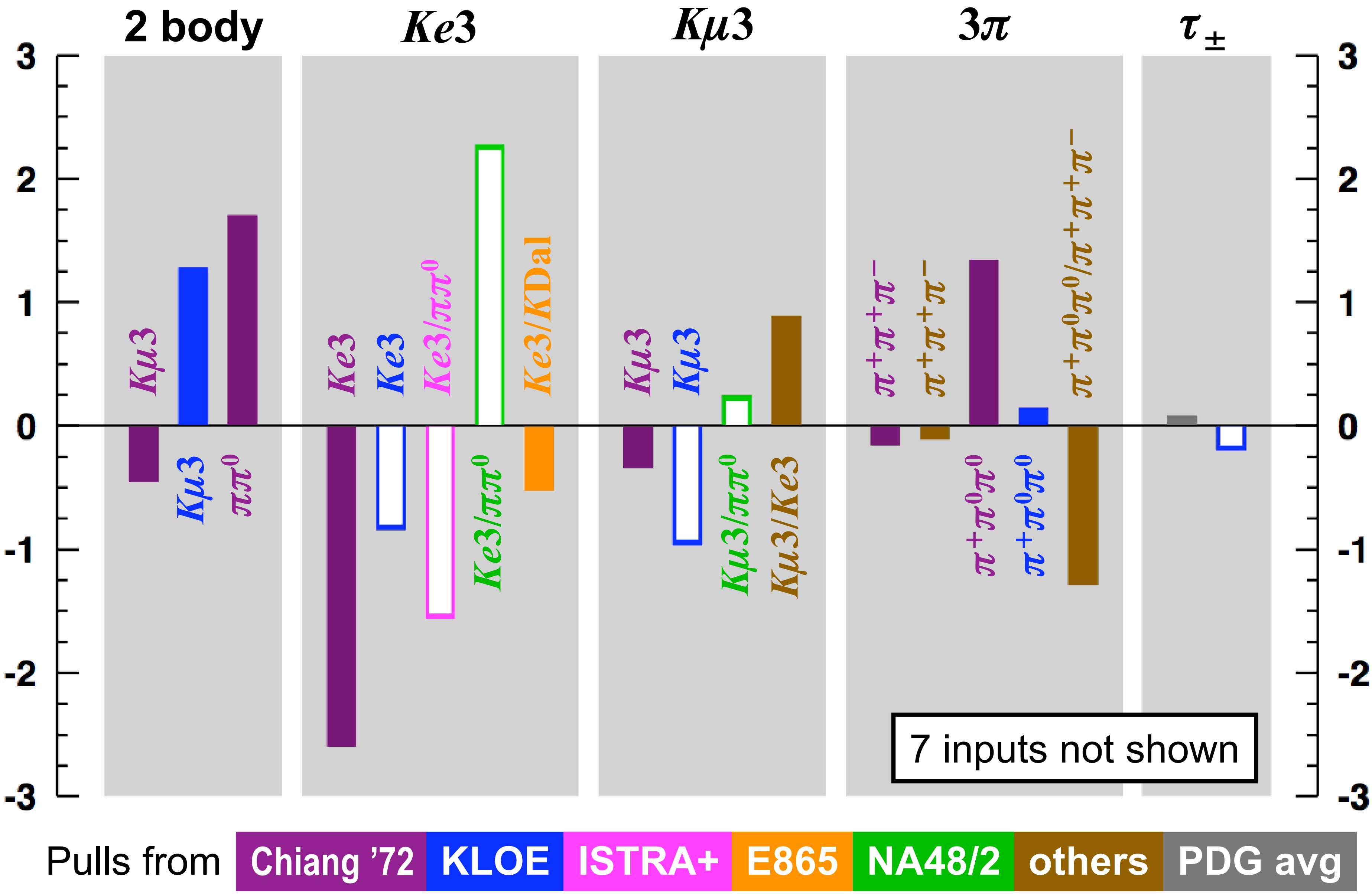
Parameter	Value	S
BR($\mu\nu$)	63.442(145)%	1.3
BR($\pi\pi^0$)	20.702(109)%	1.3
BR($\pi\pi\pi$)	5.5921(305)%	
BR($Ke3$)	5.120(38)%	1.6
BR($K\mu3$)	3.3853(205)%	1.2
BR($\pi\pi^0\pi^0$)	1.7592(234)%	1.1
$\tau_{\pm}$	12.3840(213) ns	1.8

χ^2/ndf = 48.5/24 (Prob = 0.22%)

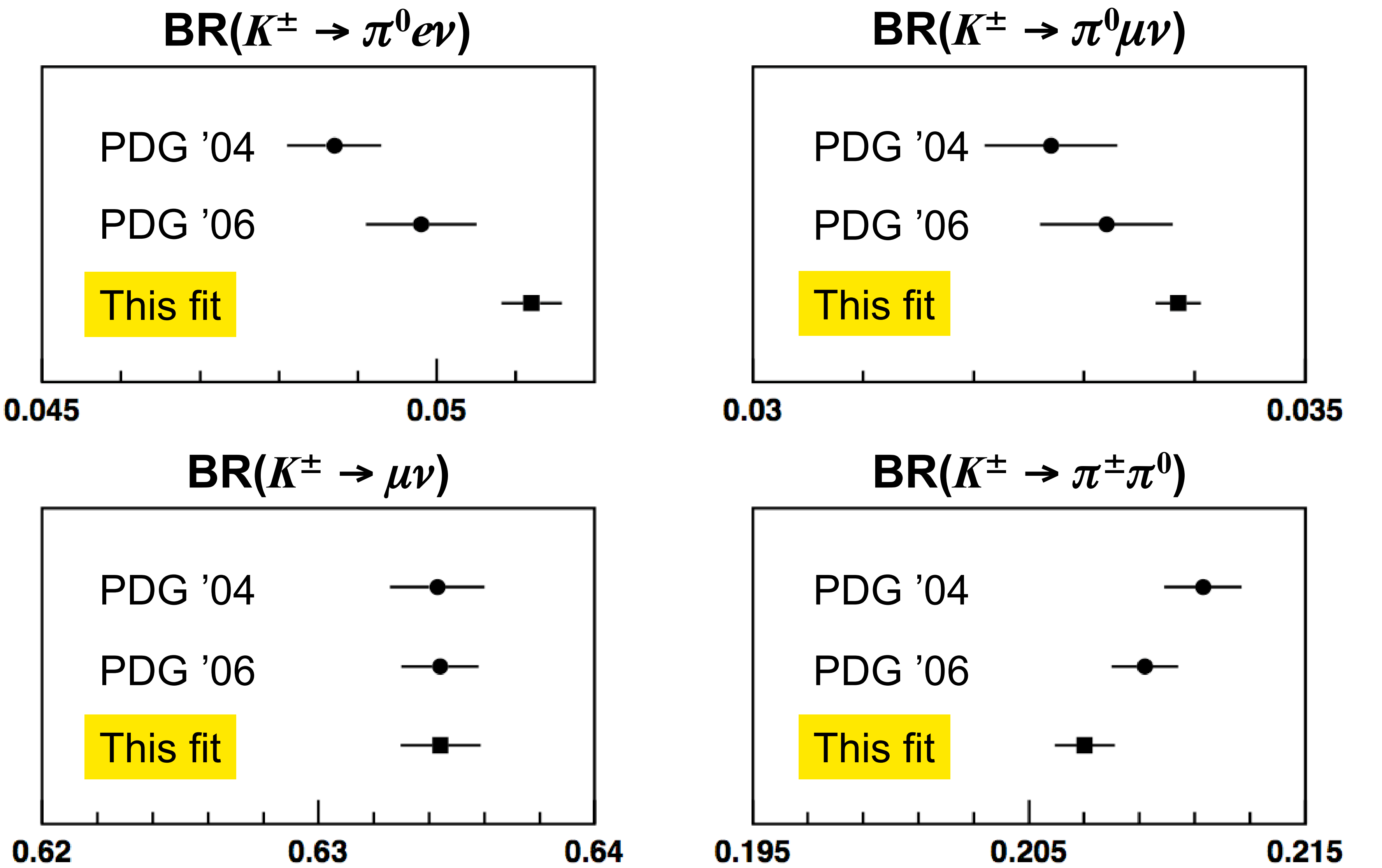
compare PDG '06: 30.0/19 (5.2%)

Improves to χ^2/ndf = 31.2/20 (5.3%) with no changes to central values or errors, if 5 older τ_{+} measurements replaced by PDG avg (with S = 2.1)

Comparison: $K^{\pm}$ BR fit vs. input data



Evolution of $K^{\pm}$ BRs

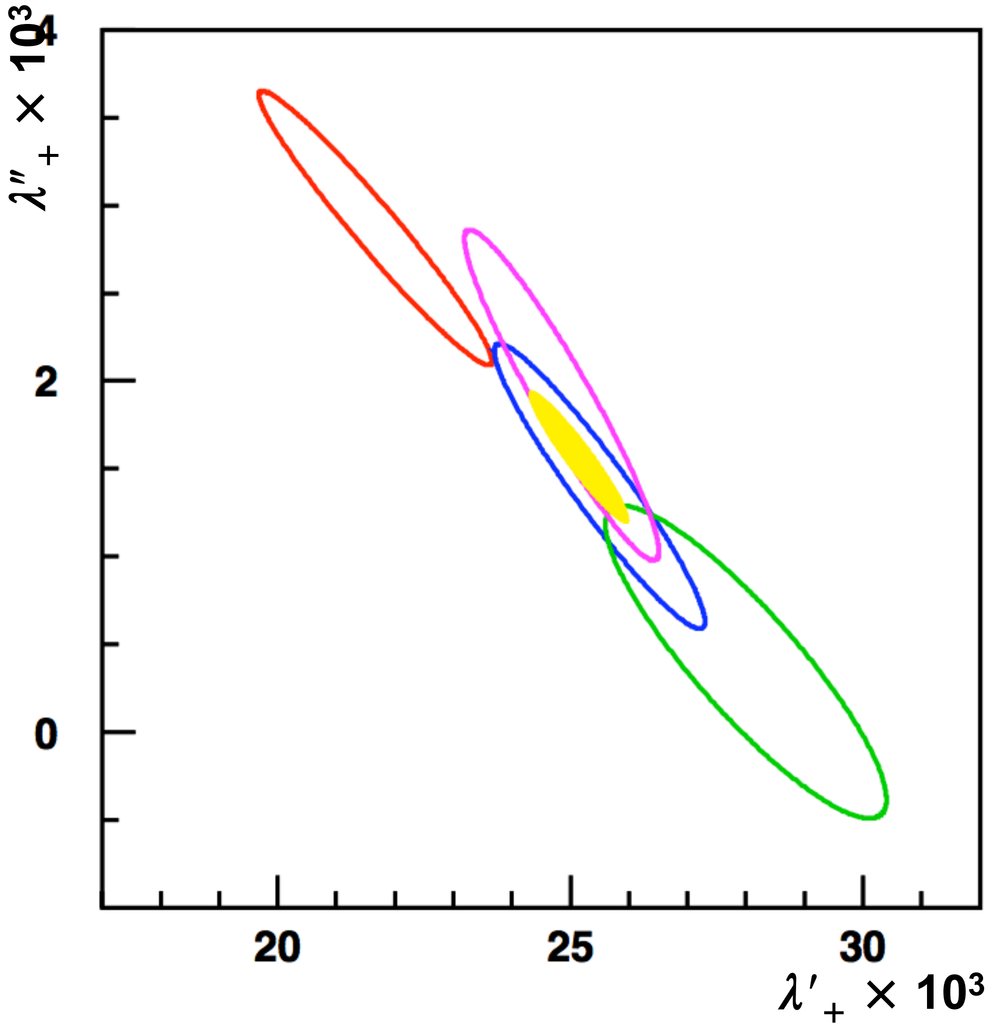


Current data on K_{l3} form-factor slopes

	Note	$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	$\lambda_0 \times 10^3$
KTeV PRD 70 (2004)	$K_L e3$	21.7 ± 2.0	2.9 ± 0.8	
	$K_L \mu3$	17.0 ± 3.7	4.4 ± 1.5	12.8 ± 1.8
KLOE PLB 636 (2006)	$K_L e3$	25.5 ± 1.8	1.4 ± 0.8	
NA48 PLB 604 (2004)	$K_L e3$	28.0 ± 2.4	0.4 ± 0.9	
NA48 **preliminary**	$K_L \mu3$	16.8 ± 3.3	4.0 ± 1.4	9.1 ± 1.4
ISTRA+ PLB 581 (2004)	$K^- e3$	24.9 ± 1.7	1.9 ± 0.9	
ISTRA+ PLB 589 (2004)	$K^- \mu3$	23.0 ± 6.4	2.3 ± 2.3	17.1 ± 2.2

K_{e3} slopes: Quadratic fits

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **This fit**



Slope parameters × 10³

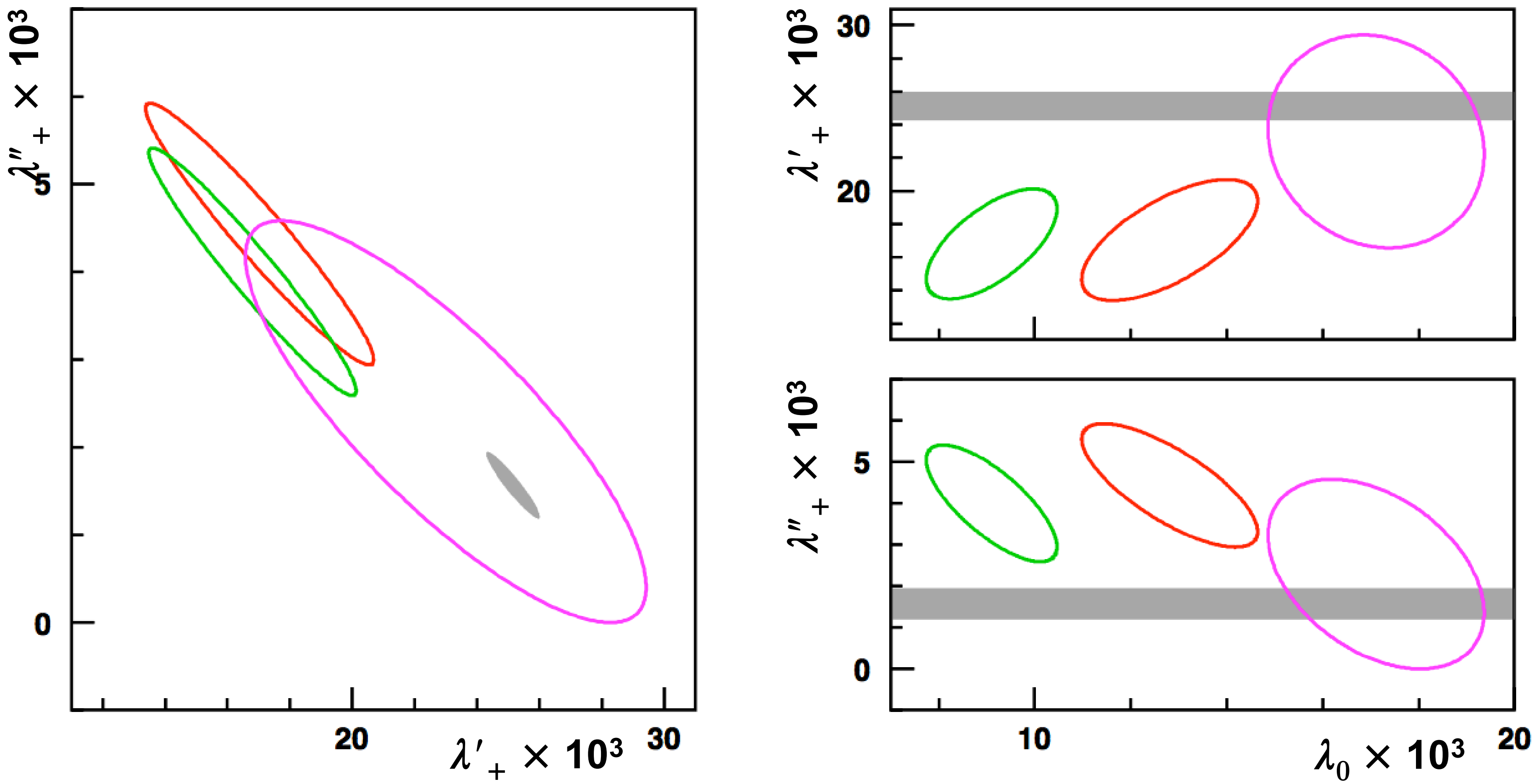
$$\lambda'_+ = 25.15 \pm 0.87$$
$$\lambda''_+ = 1.57 \pm 0.38$$
$$\rho(\lambda'_+, \lambda''_+) = -0.941$$
$$\chi^2/\text{ndf} = 5.3/6\ (51\%)$$

Excellent compatibility
Significance of $\lambda''_+ > 4\sigma$

$I(K^0e3)$ = **0.15465(21)**
$I(K^+e3)$ = **0.15901(22)**

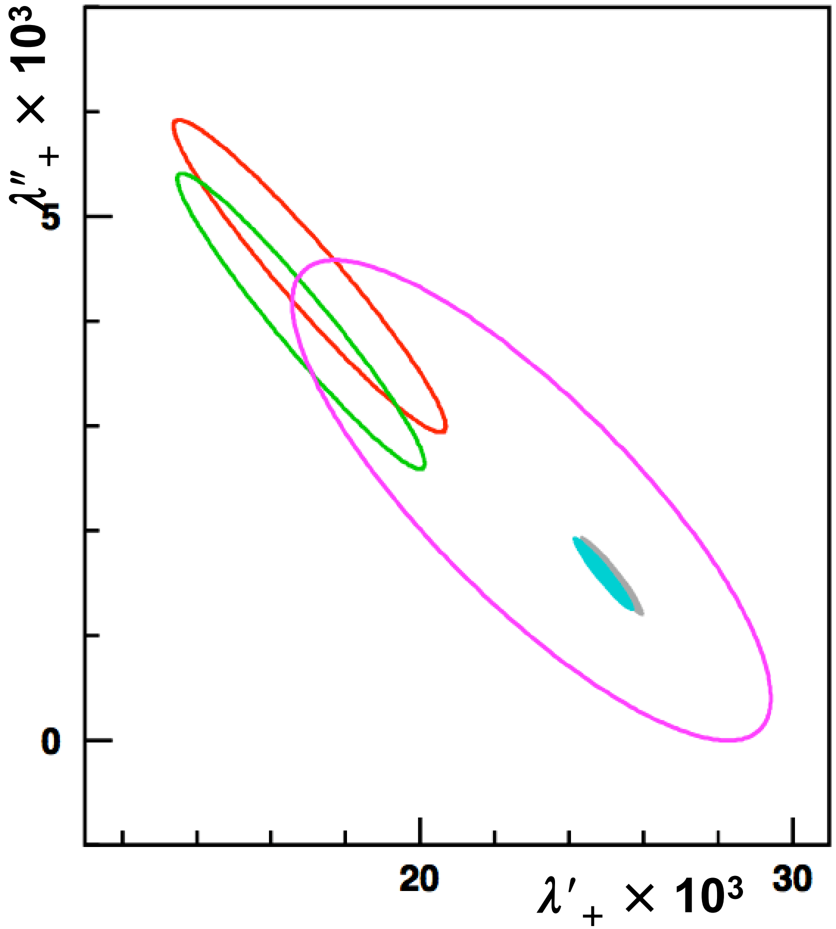
$K_{\mu3}$ form-factor slopes

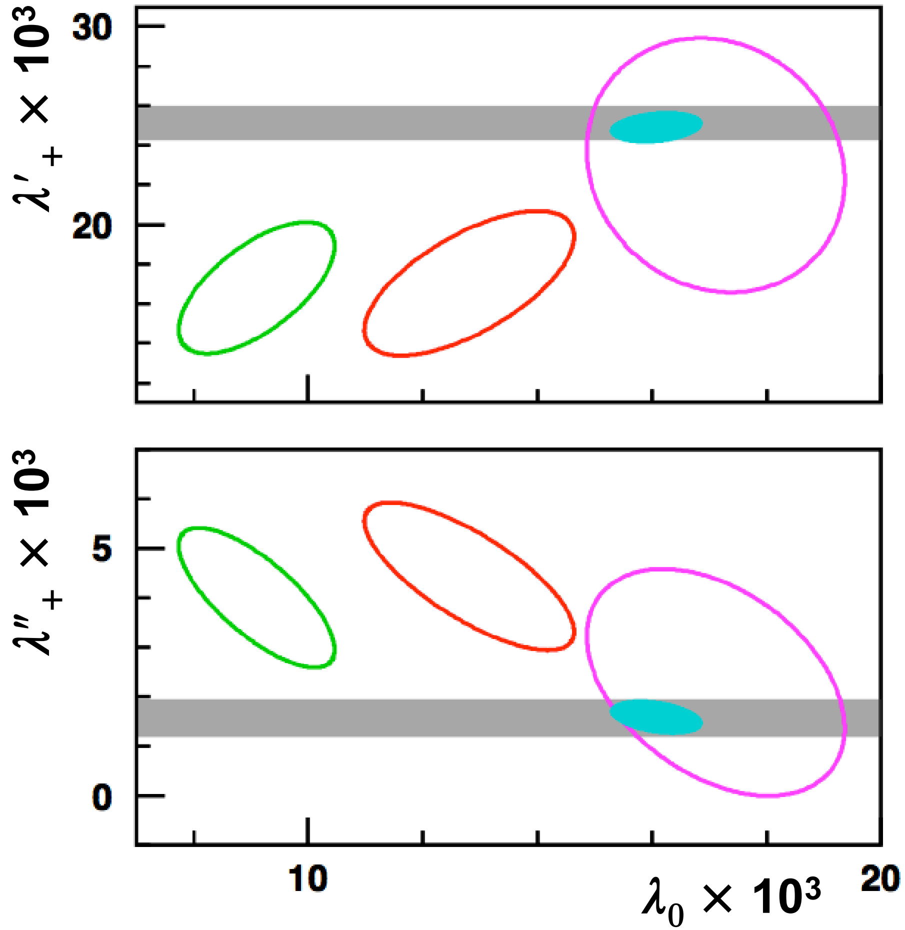
Slopes from **KTeV $K\mu3$** **ISTRA+ $K\mu3$** **NA48 $K\mu3$** **Fit to $Ke3$ data**



Fit to K_{l3} form-factor slopes

Slopes from | KTeV $K\mu3$ | ISTRA+ $K\mu3$ | NA48 $K\mu3$ | Fit to $Ke3$ data

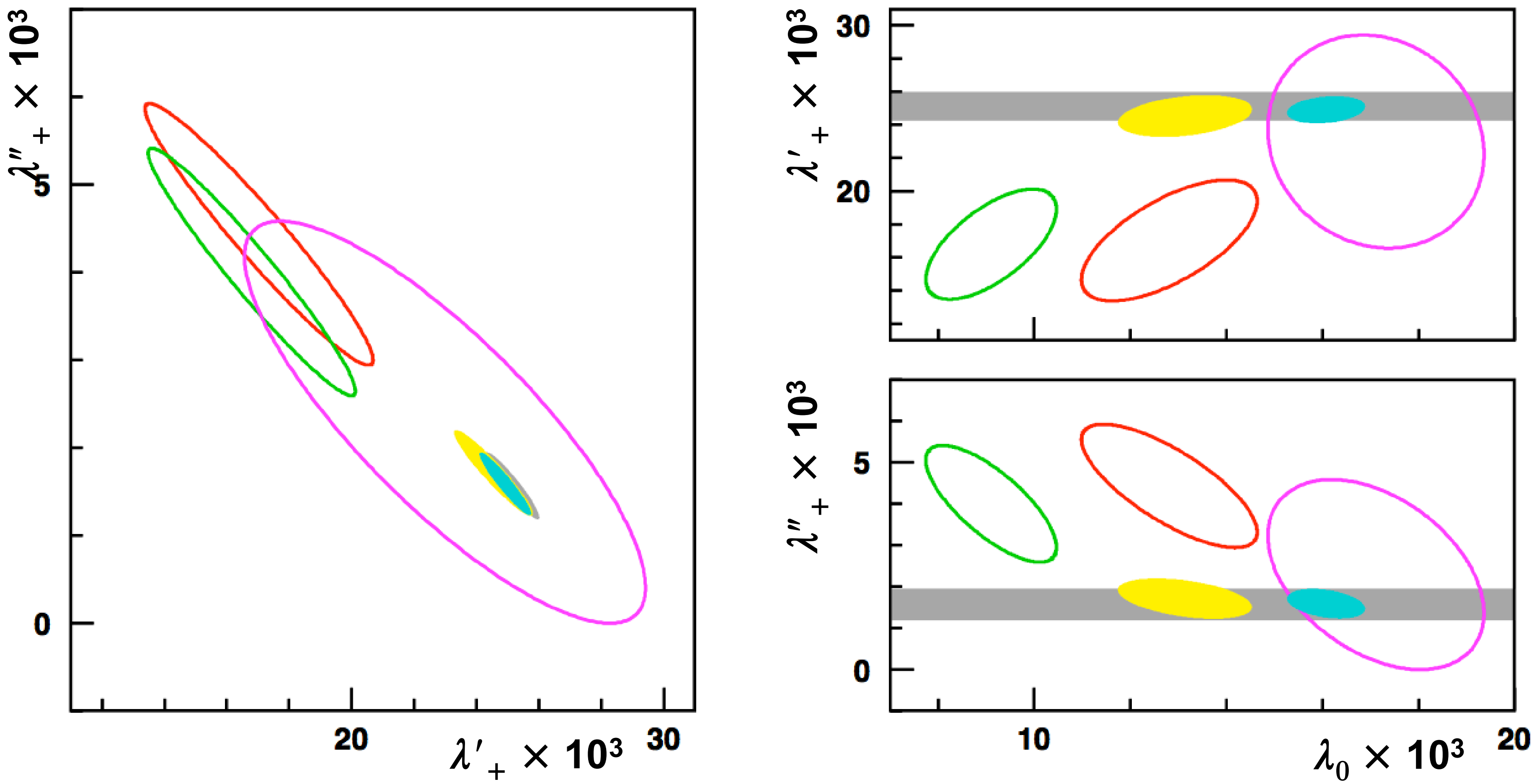




$Kl3$ fit, no NA48 $K\mu3$: χ^2=11.9/9 (21.7%)

Fit to K_{l3} form-factor slopes

Slopes from | KTeV $K\mu3$ | ISTRA+ $K\mu3$ | NA48 $K\mu3$ | Fit to $Ke3$ data



$Kl3$ fit, no NA48 $K\mu3$: χ^2=11.9/9 (21.7%) | $Kl3$ fit, all data, χ^2=58/12 (10^{-6})

K_{l3} form-factor slopes: Fit results

Although compatibility poor, no *a priori* reason to exclude NA48 $K\mu3$ data
Inconsistency parameterized by scale factors for fit results

Slope parameters × 10^3:

$\lambda'_+ = 24.54 \pm 1.26$ $\quad S = 1.6$

$\lambda''_+ = 1.71 \pm 0.49$ $\quad S = 1.4$

$\lambda_0 = 13.14 \pm 1.40$ $\quad S = 2.2$

χ^2/ndf = 58/12 (10^{-6})

Correlation coefficients:

	λ'_+	λ_0
λ'_+	−0.95	+0.31
λ''_+		−0.42

Integrals	
$I(K^0e3)$	0.15444(35)
$I(K^+e3)$	0.15879(35)
$I(K^0\mu3)$	0.10199(37)
$I(K^+\mu3)$	0.10493(38)

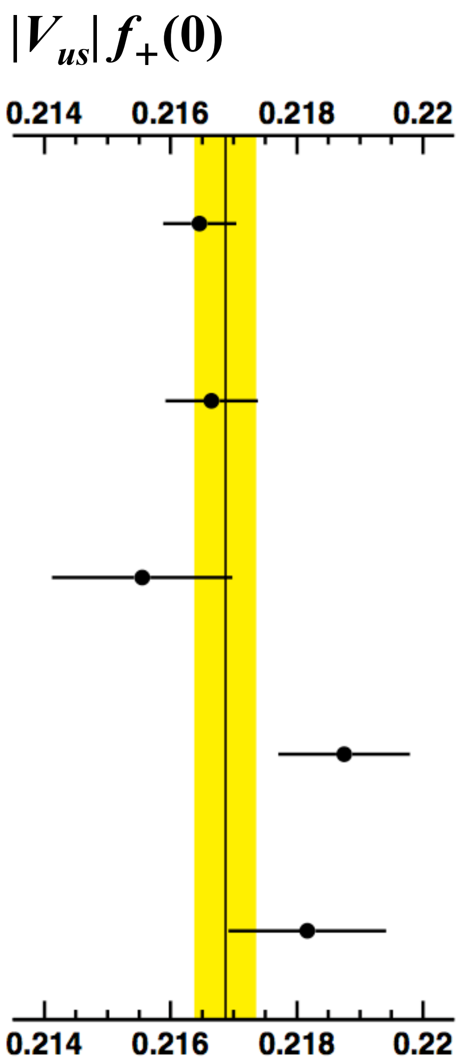
These results used to evaluate $|V_{us}|\, f_+(0)$ for all modes

$SU(2)$ and EM corrections

	$\Delta^{SU(2)}$	Δ^{EM}
K^0e3	0	+0.55(10)%
$K^0\mu3$	0	+0.95(15)%
K^+e3	+2.31(22)%	−0.10(16)%
$K^+\mu3$	+2.31(22)%	+0.20(40)%

- Δ^{EM} for full phase space - all measurements assumed fully inclusive
- $\Delta^{SU(2)}$ and $\Delta^{\text{EM}}(Ke3)$ are ChPT estimates from Cirigliano et al.
- $\Delta^{\text{EM}}(K^0\mu3)$ is hadronic-model estimate from Andre
  $\Delta^{\text{EM}}(K^0e3)$ = +0.65(15) from Andre agrees with value in table
- $\Delta^{\text{EM}}(K^+\mu3)$ a rough guess, with conservative error
  Currently the dominant contribution to error on $|V_{us}|f_+(0)$ from this mode

$|V_{us}|\,f_+(0)$ from K_{l3} data



Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
K_Le3	**0.21646(59)**	**0.27**	0.09	0.19	0.10	0.11
$K_L\mu3$	**0.21665(71)**	**0.33**	0.12	0.19	0.15	0.18
K_Se3	**0.21555(143)**	**0.66**	0.65	0.02	0.10	0.11
$K^\pm e3$	**0.21875(104)**	**0.47**	0.37	0.07	0.27	0.11
$K^\pm\mu3$	**0.21817(125)**	**0.57**	0.30	0.07	0.45	0.18

Average: $|V_{us}|\,f_+(0)$ = 0.21686(49) **χ^2/ndf = 5.0/4 (29.0%)**

$|V_{us}|\,f_+(0)$: $K^\pm$ vs. $K_{L,S}$

Fit 5 modes with separate values of $|V_{us}|\,f_+(0)$ for $K^\pm$ and $K_{L,S}$ modes

- Using results of overall fit to form-factor slopes
- With $SU(2)$ corrections for $K^\pm$ modes [$\Delta^{SU(2)}{}_{\text{th}}$ = 2.31(22)%]

$K^\pm$ modes
$|V_{us}|\,f_+(0)$ = 0.21854(95)

↔

$K_{L,S}$ modes
$|V_{us}|\,f_+(0)$ = 0.21644(53)

2.05σ difference

χ^2/ndf = 0.77/3 (86%) ρ = 0.13

When fit performed without $SU(2)$ corrections for $K^\pm$ modes, obtain an experimental value for $\Delta^{SU(2)}$

$K^\pm$ modes, no $SU(2)$
$|V_{us}|\,f_+(0)$ = 0.22359(85)

$\Delta^{SU(2)}{}_{\text{exp}}$ = 3.30(43)%

K_{l3} data and lepton universality

For each state of kaon charge, we evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\text{obs}}}{(R_{\mu e})_{\text{SM}}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \cdot \frac{I_{e3}\,(1+\delta_{e3})}{I_{\mu 3}\,(1+\delta_{\mu 3})} = \frac{[|V_{us}|\,f_+(0)]^2_{\mu 3,\text{ obs}}}{[|V_{us}|\,f_+(0)]^2_{e3,\text{ obs}}} = \frac{(G_F^{\mu})^2}{(G_F^{e})^2}$$

$K^{\pm}$ modes

$r_{\mu e}$ = 0.9947(111)

Using 2004 BRs*

$r_{\mu e}$ = 1.010(20)

$K_{L,S}$ modes

$r_{\mu e}$ = 1.0022(62)

Using 2004 BRs*

$r_{\mu e}$ = 1.052(15)

Average

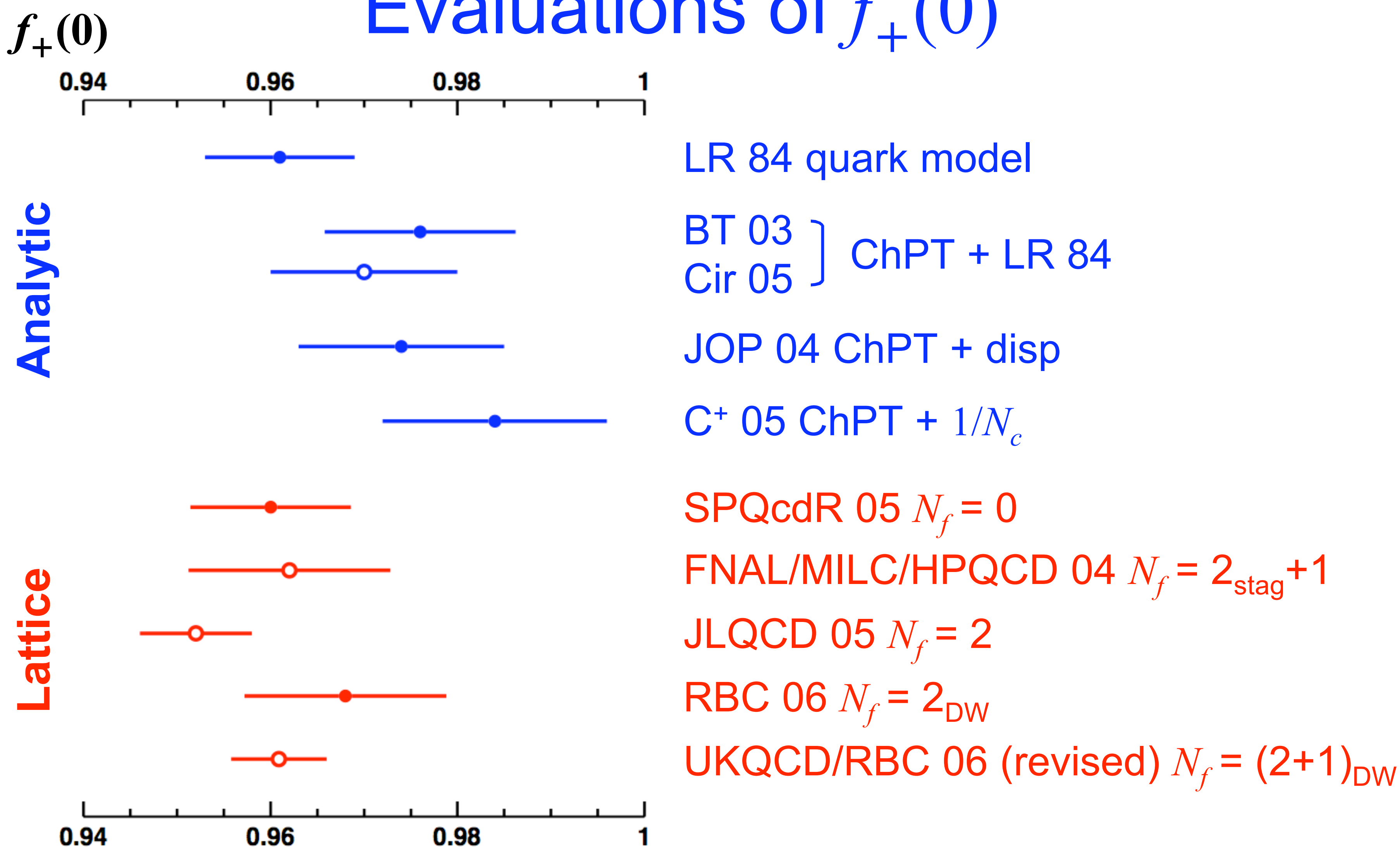
(incl. ρ = 0.11)

$r_{\mu e}$ = 1.0007(56)

Compare sensitivity from $\pi \rightarrow l\nu$ decays: **$(r_{e\mu})_{\pi l2}$ = 0.9966(30)**
see Erler, Ramsey-Musolf '06

*Assuming current values for form-factor slopes

Evaluations of $f_+(0)$



Leutwyler & Roos estimate (LR 84) still widely used: **$f_+(0)$ = 0.961(8)**

Lattice evaluations generally agree well with this value

$|V_{us}|$ from K_{l3} data and CKM unitarity

K_{l3} average: $|V_{us}| f_+(0)$ = 0.21686(49)

Leutwyler & Roos '84
$f_+(0)$ = 0.961(8)

Conventional choice for value of $f_+(0)$ until a definitive evaluation becomes available

K_{l3} average: $|V_{us}|$ = 0.2257(20)

Marciano & Sirlin '06
$|V_{ud}|$ = 0.97377(27)

Average from $0^+ \rightarrow 0^+$ β decays with recent evaluation of EW radiative corrections

$V_{ud}^2 + V_{us}^2 - 1 = -0.0008(10)$
Compatibility with unitarity -0.8σ

V_{ud}, V_{us} & BR($K^{\pm} \to \mu^{\pm}\nu$)

Marciano '04

$$\frac{\Gamma(K^{\pm} \to \mu^{\pm}\nu(\gamma))}{\Gamma(\pi^{\pm} \to \mu^{\pm}\nu(\gamma))} = \frac{|V_{us}|^2 f_K^2 m_K (1 - m_\mu^2/m_K^2)^2}{|V_{ud}|^2 f_\pi^2 m_\pi (1 - m_\mu^2/m_\pi^2)^2} \times 0.9930(35)$$

Uncertainty from SD virtual corrections

MILC '06 preliminary

f_K/f_π = **1.208(2)($^{+7}_{-14}$)**
$N_f = (2+1)_{\text{stag}}$
Cancellation of lattice-scale uncertainties

KLOE
PLB 636 (2006)

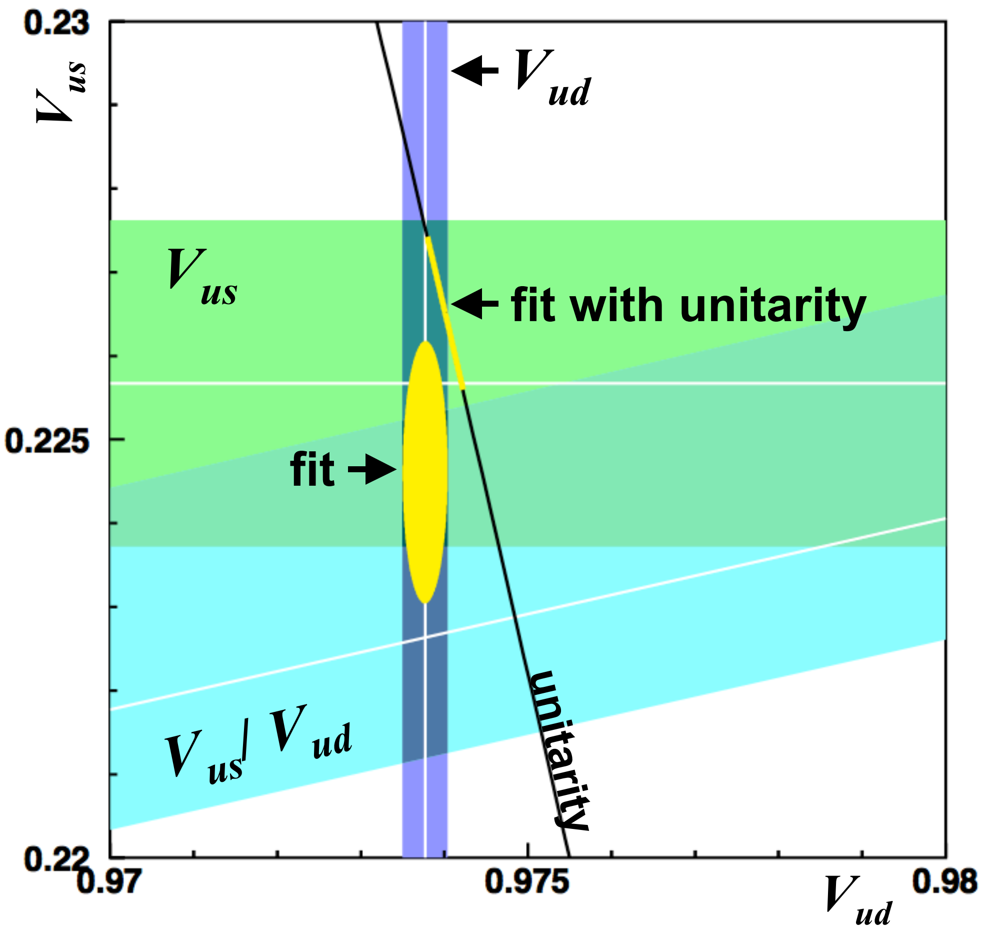
BR($K^+ \to \mu^+\nu(\gamma)$) = 0.6366(17)
Uses $K^- \to \mu^-\nu$ to tag 2-body K decays
Counts $K^+ \to \mu^+\nu$ from decay-momentum spectrum

Use KLOE BR($K^+ \to \mu^+\nu(\gamma)$) instead of value from BR/lifetime fit:
Error slightly larger, but radiative contribution under better control

$$V_{us}/V_{ud} = 0.2286(^{+27}_{-15})$$

V_{ud}, V_{us} & BR($K^{\pm} \to \mu^{\pm}\nu$)

$f_+(0)$ from LR 84
$|V_{us}|$ = 0.2257(20) from $Kl3$



Fit results, no constraint:

V_{ud} = 0.97377(27)
V_{us} = 0.2246(16)
χ^2/ndf = 0.85/1 (36%)

Fit results, unitarity constraint:

V_{ud} = 0.97402(22)
V_{us} = 0.2265(9)
χ^2/ndf = 3.04/2 (22%)

Agreement with unitarity at 1.2σ

Summary and outlook

- Several new $K^{\pm}$ measurements! None yet published
- BR/lifetime fits for both K_L and $K^{\pm}$ have χ^2 probability ~ 5%
  For $K^{\pm}$, new measurements in normalization channels could help
- New NA48 $K\mu 3$ form-factor slopes disagree with other data
- Accuracy of $\Delta^{SU(2)}$, Δ^{EM} a significant issue for charged modes
  Some evidence that $\Delta^{SU(2)}$ may be underestimated
- Experimental uncertainty on $|V_{us}|\,f_+(0)$ at 0.2% level
- Dominant contribution to uncertainty on $|V_{us}|$ still from $f_+(0)$
- With $f_+(0)$ = 0.961(8), first-row unitarity test satisfied at ~1σ level

K_{l3} average: $|V_{us}|\,f_+(0)$ = 0.21686(49)

Additional information

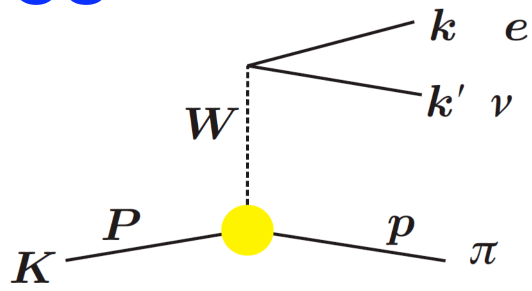
Fit to K_L BRs, τ: Comparison to PDG

Parameter	This fit Value	This fit S	PDG 2006 Value	PDG 2006 S
$Ke3$	0.40571(89)	1.4	0.4053(15)	2.1
$K\mu3$	0.27055(81)	1.3	0.2702(7)	
$3\pi^0$	0.19447(103)	1.4	0.1956(14)	1.9
$\pi^+\pi^-\pi^0$	0.12588(78)	1.5	0.1256(5)	
$\pi^+\pi^-$	$1.9860(74) \times 10^{-3}$	1.2	$1.976(8) \times 10^{-3}$	
$2\pi^0$	$8.603(54) \times 10^{-4}$	1.7	$8.69(4) \times 10^{-4}$	1.1
$\gamma\gamma$	$5.453(43) \times 10^{-4}$	1.1	$5.48(5) \times 10^{-4}$	1.2
τ	51.148(211) ns	1.1	51.14(21) ns	
	18 measurements		17 measurements	
	χ^2/ndf = 20.2/11 (4.3%)		χ^2/ndf = 14.8/10 (14.0%)	

Fit to $K^{\pm}$ BRs, τ: Comparison to PDG

Parameter	This fit Value	This fit S	PDG 2006 Value	PDG 2006 S
$\mu\nu$	63.442(145)%	1.3	63.44(14)%	1.2
$\pi\pi^0$	20.702(109)%	1.3	20.92(12)%	1.1
$\pi\pi\pi$	5.5921(305)%		5.590(31)%	1.1
$Ke3$	5.120(38)%	1.6	4.98(7)%	1.3
$K\mu3$	3.3853(205)%	1.2	3.32(6)%	1.2
$\pi\pi^0\pi^0$	1.7592(234)%	1.1	1.757(24)%	1.1
$\pi^0\pi^0e\nu$	Not in fit		$2.2(4) \times 10^{-5}$	
τ	12.3840(213) ns	1.8	12.385(24) ns	2.1
	30 measurements		26 measurements	
	χ^2/ndf = 48.5/24 (0.22%)		χ^2/ndf = 30.0/19 (5.2%)	

K_{l3} form-factor slopes



Hadronic matrix element:

$$\langle\pi|\,J_\alpha\,|K\rangle = f(0)\times[\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha]$$

$f_-(t)$ term only important for $K_{\mu 3}$.
For $K_{\mu 3}$, use $f_+(t)$ and $f_0(t) = f_+(t) + \dfrac{t}{m_K^2 - m_{\pi+}^2} f_-(t)$

For V_{us}, need integral over phase space of squared matrix element

Expand form factor:

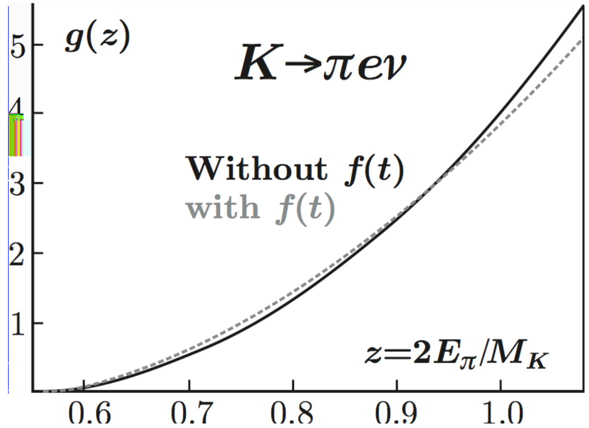
Linear: $\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0}\,[t/m_{\pi+}^2]$

Quadratic: $\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0}\,[t/m_{\pi+}^2] + 1/2\,\lambda''_{+,0}\,[t/m_{\pi+}^2]^2$

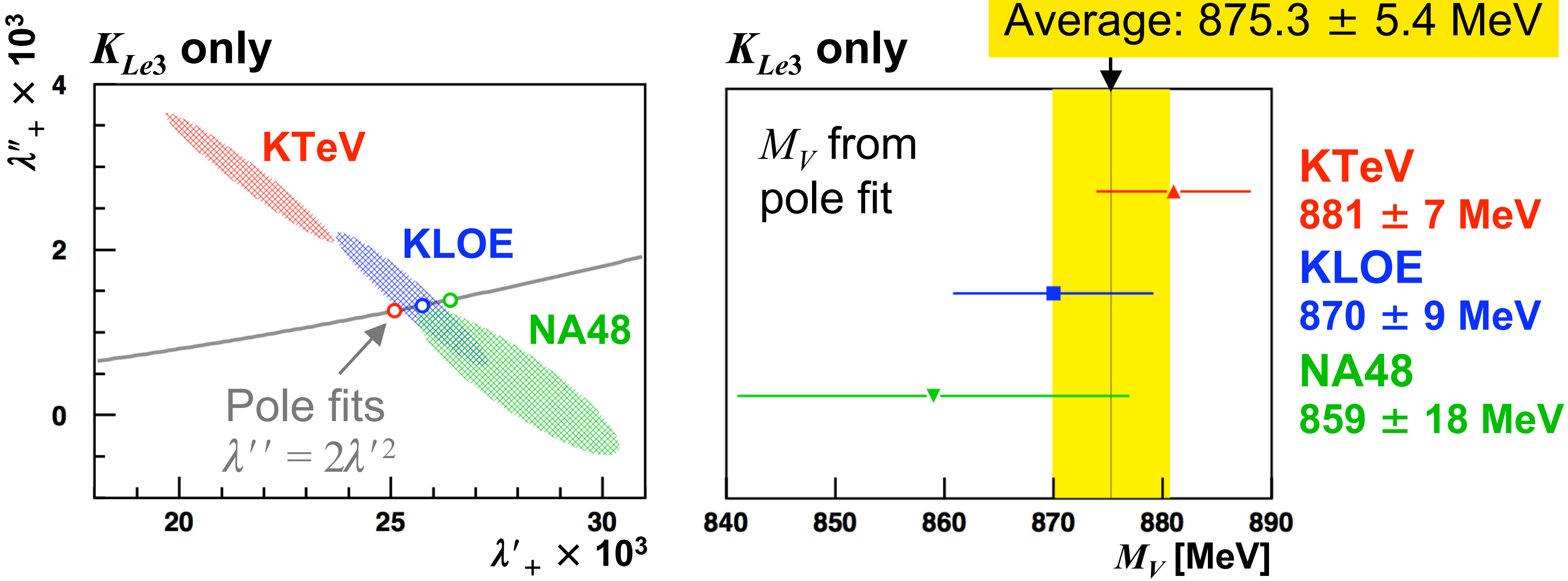
Fits to t-distribution give poor sensitivity to quadratic terms

Polar fit:

$$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t} \qquad \lambda' = (m_{\pi+}/M)^2 \qquad \lambda'' = 2\lambda'^2$$



K_{e3} slopes: Quadratic vs. pole fits



K_{Le3} data	$P(\chi^2)$ fit Quad	K_{Le3} integral Quad	$P(\chi^2)$ fit Pole	K_{Le3} integral Pole	Difference
KTeV	54%	0.15378(51)	43%	0.15449(25)	**+0.46%**
KLOE	92%	0.15472(42)	92%	0.15489(33)	**+0.11%**
K_{Le3} avg		0.15456(29)		0.15469(19)	**+0.09%**

Beyond quadratic and pole fits

Hill, PRD 74 (2006):

Power series expansion based on analyticity of form factors
Constraints from crossing symmetry, e.g., bounds on $f_+(t)$ from $\tau \rightarrow K\pi\nu$ data
Rigorous estimate of error from truncation of series expansion

KTeV
PRD 70 (2004)

Refit $Ke3$ data using Hill parameterization
$I(K^0e3)$ = 0.15392(48)$_{\text{exp}}$(6)$_{\text{th}}$

Bernard et al., PLB 638 (2006):

Dispersion relation for ln $f_0(t)$ subtracted at t = 0 and $t = m_K{}^2 - m_\pi{}^2$, giving:

$$f_0(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2}\left(\ln C - G(t)\right)\right]$$

$G(t)$ evaluated using $K\pi$ scattering data and given as a polynomial

1 fit parameter: ln C = ln $f_0(m_K{}^2 - m_\pi{}^2)$
Value of ln C a test for right-handed quark currents

NA48
preliminary

From new analysis of $K\mu3$ form-factor slopes
ln C = 0.1533(138)

$|V_{us}| f_+(0)$: $K_{L,S}$ vs. $K^\pm$

Using separate fit results for form-factor slopes:

$K^\pm$ only, $e3$ and $\mu3$:

ISTRA+ $e3$, $\mu3$

λ'_+	=	24.80 ± 1.54
λ''_+	=	1.94 ± 0.86
λ_0	=	16.76 ± 1.20
χ^2/ndf	=	0.100/2 (95%)
$I(Ke3)$	=	0.159097(319)
$I(K\mu3)$	=	0.105949(298)

K_L only, $e3$ and $\mu3$:

KTeV avg, KLOE $e3$, NA48 $e3$, $\mu3$

λ'_+	=	25.26 ± 1.96 (S=1.9)
λ''_+	=	2.20± 0.72 (S=1.7)
λ_0	=	11.14 ± 1.61 (S=2.1)
χ^2/ndf	=	31.2/7 (0.01%)
$I(Ke3)$	=	0.15413(55)
$I(K\mu3)$	=	0.10140(49)

With $SU(2)$ corrections for $K^\pm$ modes:

$|V_{us}| f_+(0)$ = **0.21804(94)**
χ^2/ndf = 1.39/1 (23.9%)

$|V_{us}| f_+(0)$ = **0.21676(61)**
χ^2/ndf = 1.26/2 (53%)

With NO $SU(2)$ corrections for $K^\pm$ modes:

$|V_{us}| f_+(0)$ = **0.22308(84)**

$\Delta^{SU(2)}_{\text{exp}}$ = **2.92(48)%**

Results without NA48 $K\mu3$ slopes

Slope parameters × 10^3
λ'_+ = **24.92 ± 0.83**
λ''_+ = **1.59 ± 0.36**
λ_0 = **16.07 ± 0.82**
χ^2/ndf = 11.9/9 (21.7%)

Integrals	
$I(K^0e3)$	0.15455(21)
$I(K^+e3)$	0.15890(21)
$I(K^0\mu3)$	0.10268(21)
$I(K^+\mu3)$	0.10565(21)

$\lvert V_{us}\rvert f_+(0)$	
K_Le3	**0.21638(56)**
$K_L\mu3$	**0.21592(62)**
K_Se3	**0.21547(142)**
K^+e3	**0.21868(102)**
$K^-\mu3$	**0.21743(120)**

$Kl3$ average
$\lvert V_{us}\rvert f_+(0)$ = 0.21655(43)
χ^2/ndf = 6.1/4 (19.0%)

$K^+ - K^0$ diff.: 2.1σ
$\Delta^{SU(2)}_{\text{exp}}$ = 3.31(43)%

$r_{\mu e}$ = 0.9946(51)

$\lvert V_{us}\rvert$ = 0.2253(19)
Unitarity test: -1.0σ