V_{us} FROM HADRONIC τ DECAY

K. Maltman + C. Wolfe, CKM 2006, Nagoya

[See also PLB639 (2006) 283 and in preparation]

OUTLINE

- *background/notation/terminology*

- *convergence/error estimate issues (non-spectral vs. spectral weights)*

- *results (with current data) and prospects*

BACKGROUND/NOTATION/TERMINOLOGY

- V,A $ij = ud,\ us,\ (J) = (0+1),\ (0)$ spectral functions from experimental differential decay distribution

$$R_{V/A;ij} = 12\pi^2 |V_{ij}|^2 S_{EW} \int_0^1 dy_\tau \left[w_T(y_\tau)\, \rho^{(0+1)}_{V/A;ij}(s) + w_L(y_\tau)\, \rho^{(0)}_{V/A;ij}(s) \right]$$

with $R_{V/A;ij} \equiv \frac{\Gamma[\tau \to \nu_\tau \,\text{hadrons}_{V/A;ij}\,(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]}$, $y_\tau = s/m_\tau^2$

$w_T(y) = (1-y)^2(1+2y)$, $w_L(y) = -2y(1-y)^2$,

- "longitudinal" : pure $(J) = (0)$ term

- "(k, m) spectral weights" : expt'l distribution multiplied by $(1 - y_\tau)^k\, y_\tau^m$ before integration

- $(0, 0)$ spectral weight: kinematic weight case (no additional factors) $\Rightarrow R_{V/A;ij}^{(0,0)}$ from sum over branching fractions

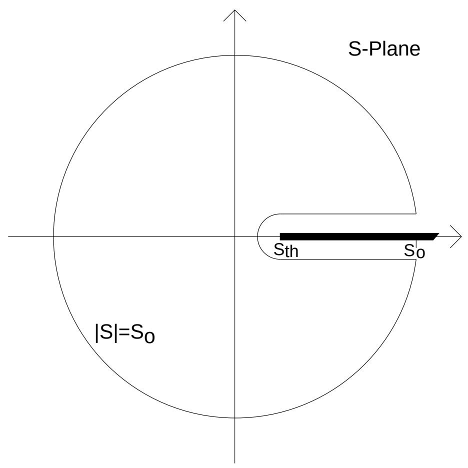
- chiral constraints on longitudinal spectral contributions

  - no chiral suppression of π, K pole terms
  - $O(m_q^2)$ chiral suppression of non-pole ("continuum") terms ($\Rightarrow$ negligible for $ij = ud$, suppressed for us)

EXTRACTING V_{us} (and m_s)

- Basic FESR relation

$$\int_0^{s_0} w(s)\,\rho(s)\,ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} w(s)\,\Pi(s)\,ds$$



(LHS: experimental spectral data, RHS: OPE)

- $R^w_{ij}(s_0)$: generic $(J) = (0+1)$ or (0); V or A; $w(s)$-reweighted $0 < s \leq s_0 \leq m_\tau^2$ spectral integral

- V_{us} (and m_s) from flavor-breaking combinations

$$\delta R^w(s_0) = \left[R^w_{ud}(s_0)/|V_{ud}^2|\right] - \left[R^w_{us}(s_0)/|V_{us}^2|\right]$$

  - $[\delta R^w(s_0)]^{OPE}_{D=0} = 0$ *for physical V_{us}*

  - incorrect $V_{us} \Rightarrow$ residual (large) $D = 0$ OPE contribution, hence leverage for V_{us} [Gamiz et al., JHEP 0301: 060 (2003)]

  - $([\delta R^w(s_0)]^{OPE}_{D=2} \propto m_s^2 \Rightarrow$ joint fit for V_{us}, m_s also possible)

- Given m_s from other sources, V_{us} via

$$|V_{us}| = \sqrt{R^w_{us}(s_0) / \left[\frac{R^w_{ud}(s_0)}{|V^2_{ud}|} - \delta R^w_{OPE}(s_0) \right]}$$

  - $R^w_{ud}(s_0)$ typically $>> \delta R^w_{OPE}(s_0) \Rightarrow$
    - fractional OPE-induced error on $V_{us} \sim \frac{\delta R^w_{OPE}(s_0)}{2R^w_{ud}(s_0)}$ $<<$ that on $\delta R^w_{OPE}(s_0)$ itself
    - $\Rightarrow$ good precision on V_{us} requires only modest OPE precision
  - current data errors (3$-$4% errors on $R^w_{us} \Rightarrow$ 1.5$-$2% on $|V_{us}|$) soon to be significantly reduced

COMPLICATIONS/CONVERGENCE ISSUES

THE MAIN ISSUES

- *severe* problems with longitudinal OPE representation, hence need for "longitudinal subtraction"

- ameliorating problems associated with slow convergence of integrated $(0+1)$ $D=2$ OPE series

- (controlling possible higher D OPE contributions)

PROBLEMS WITH THE LONGITUDINAL OPE

- integrated longitudinal OPE series badly non-convergent for all kinematically-allowed scales

- (even worse) *ALL* truncation schemes employed in the literature *BADLY* violate longitudinal continuum spectral positivity [KM, J. Kambor, PRD64: 093014]

$\Rightarrow$ *NO sensible option other than to subtract longitudinal spectral contributions from experimental distribution and work with* $(0+1)$ *sum rules*

THE LONGITUDINAL SUBTRACTION

- pole terms very accurately known and dominant for chiral+kinematic reasons

- residual us continuum subtraction
  - Jamin, Oller, Pich dispersive analysis for us scalar
  - KM, J. Kambor sumrule analysis for us PS
  - *both determinations strongly constrained by implications for m_s from scalar, PS sum rules*
  - even allowing $\sim$ 50% uncertainty, impact small on present analyses

SLOW $D = 2$ (0 + 1) OPE SERIES CONVERGENCE

- $\delta R^{w,(0+1)}(s_0)$, OPE thereof involve

$$\Delta\Pi(Q^2) = \Pi^{(0+1)}_{ud;V+A} - \Pi^{(0+1)}_{us;V+A}$$

and corresponding spectral function $\Delta\rho(s)$

- $D = 2$ OPE series, $\bar{m}_s = m_s(Q^2)$, $\bar{a} = \alpha_s(Q^2)/\pi$, $\overline{MS}$ scheme [Baikov, Chetyrkin, Kuhn PRL95:012003]

$$\left[\Delta\Pi(Q^2)\right]_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s}{Q^2}\Big[1 + 2.333\bar{a} + 19.933\bar{a}^2 + 208.746\bar{a}^3 + (2378 \pm 200)\bar{a}^4 + \cdots\Big]$$

- $a(m_\tau^2) \sim 0.10 - 0.11$ hence series *very* slowly converging at spacelike point on $|s| = s_0 = m_\tau^2$

- running of $\alpha_s(Q^2) \Rightarrow$ convergence improved away from spacelike point

- $s = s_0 e^{i\phi}$, $y = s/s_0 \Rightarrow |1-y| = 2\,sin(\phi/2) \Rightarrow$ higher $(k,0)$ spectral weights ($\propto (1-y)^{k+2}$) strongly peaked in spacelike (slowest convergence) direction

- can improve convergence with non-spectral weights [w_{20}, $\hat{w}_{10}$, w_{10} (KM, Kambor PRD62(2000)093020), w_8]

- s_0-instability of physical output wrt s_0 as sign of premature truncation of slowly converging series

- (s_0-stability tests also important re neglect of $D > 6$ OPE terms [Supplementary pages])

CONVERGENCE/STABILITY STUDIES

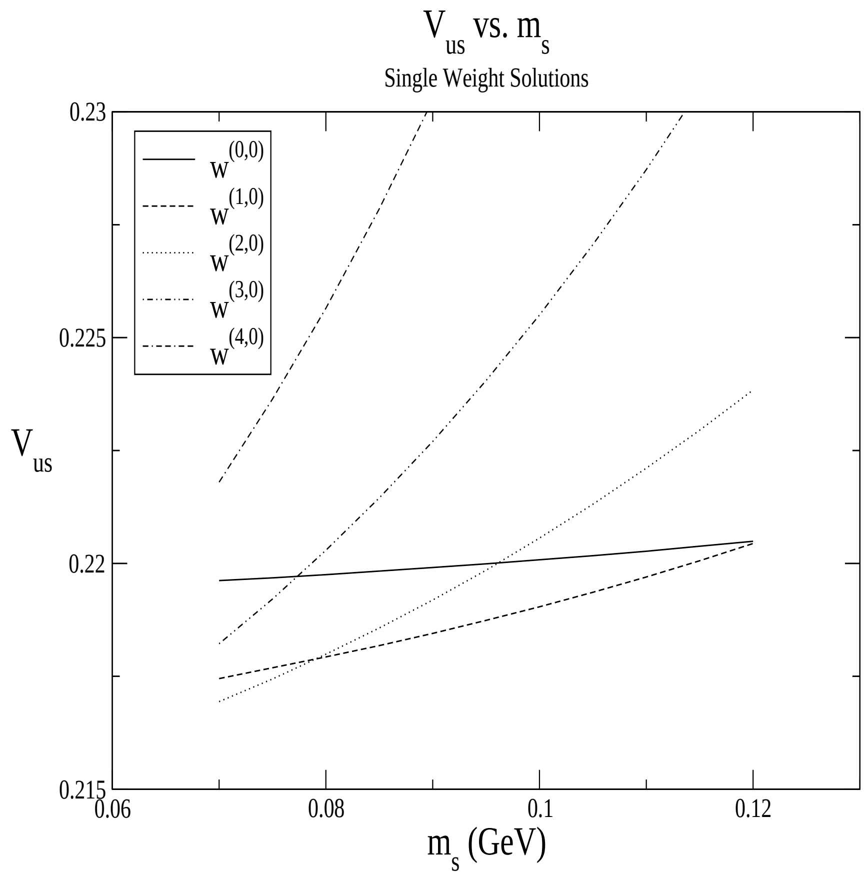
- ALEPH us data, covariance matrix, with mode-by-mode rescaling for changed BR's [Davier et al Tau'00 and EPJC22 (2001) 31 strategy]

- largest (and large) rescaling for $K^{\pm}\pi^{+}\pi^{-}$
  - OPAL, CLEO consistent, disagree with ALEPH, but use 3-fold average (consistent with preliminary high-precision BABAR result [I. Nugent, Tau'06])
  - CAUTION: difference of CLEO-OPAL and 3-fold average (0.07%) appears small, but is $\sim$ 2% of total us branching fraction $\leftrightarrow \sim$ 1% ($\sim$ 0.0020) impact on V_{us}

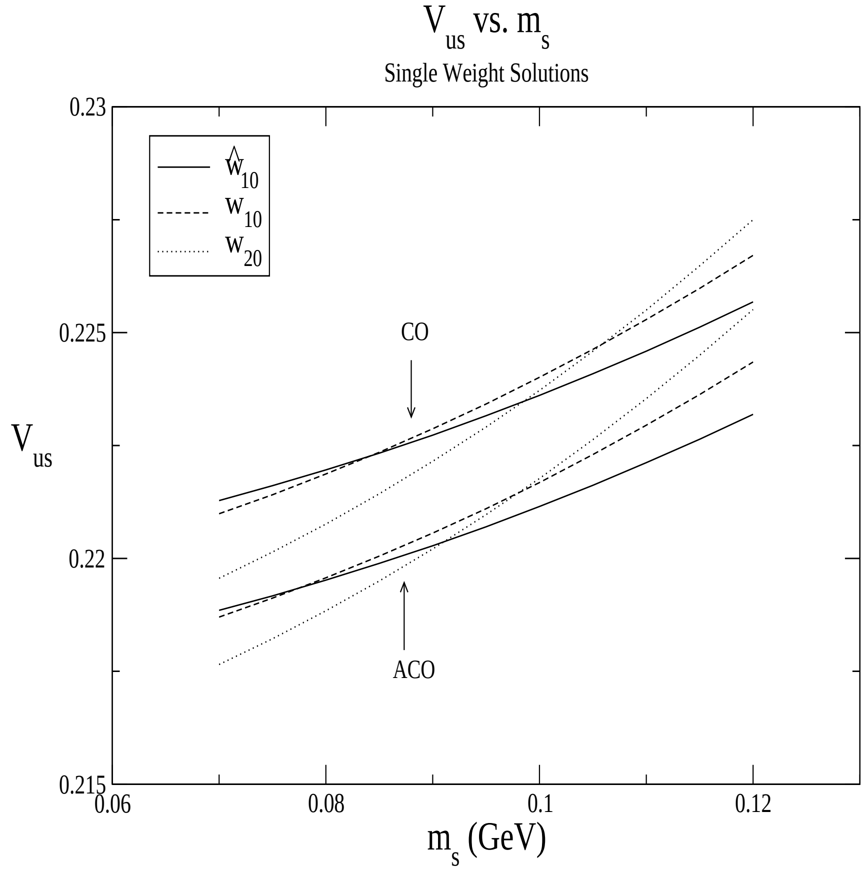
- for each $w(y)$, can fix m_s, $s_0 = m_\tau^2$, solve for V_{us}, then
  - look for m_s (if any) giving consistent V_{us} for different weights
  - check OPE/spectral integral match vs. s_0 for each weight separately with given m_s, resulting V_{us}, to see if any m_s yields s_0-stability

- results [FIGURES] show
  - poor consistency, no m_s with s_0-stability for $(k, 0)$ spectral weights
  - much improved consistency, s_0-stability for non-spectral weights

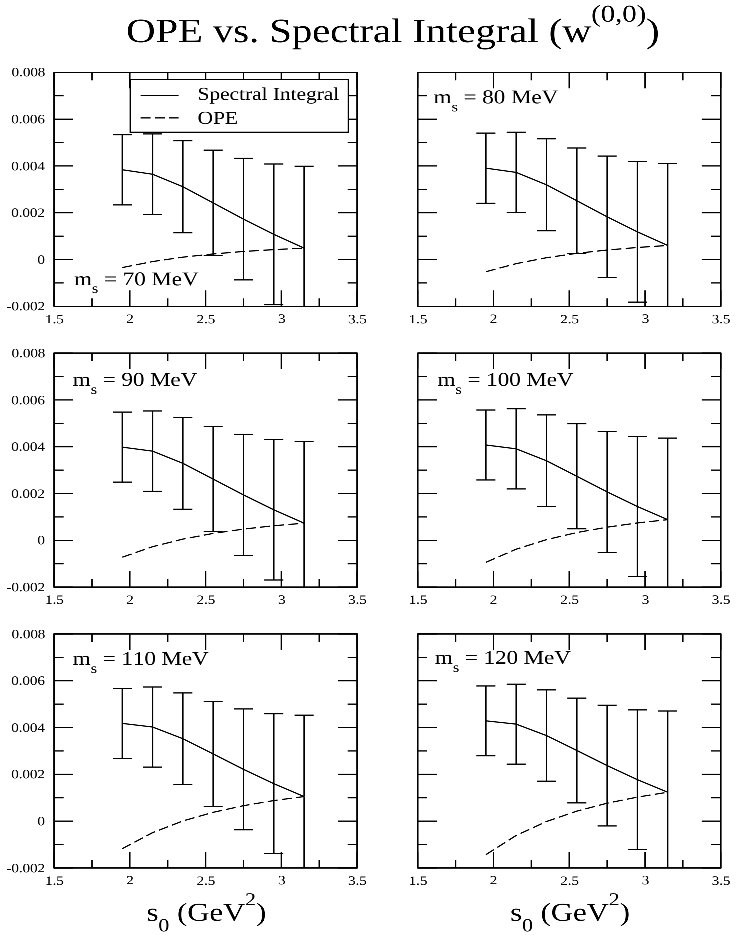
$w^{(k,0)}$ CONSISTENCY CHECKS



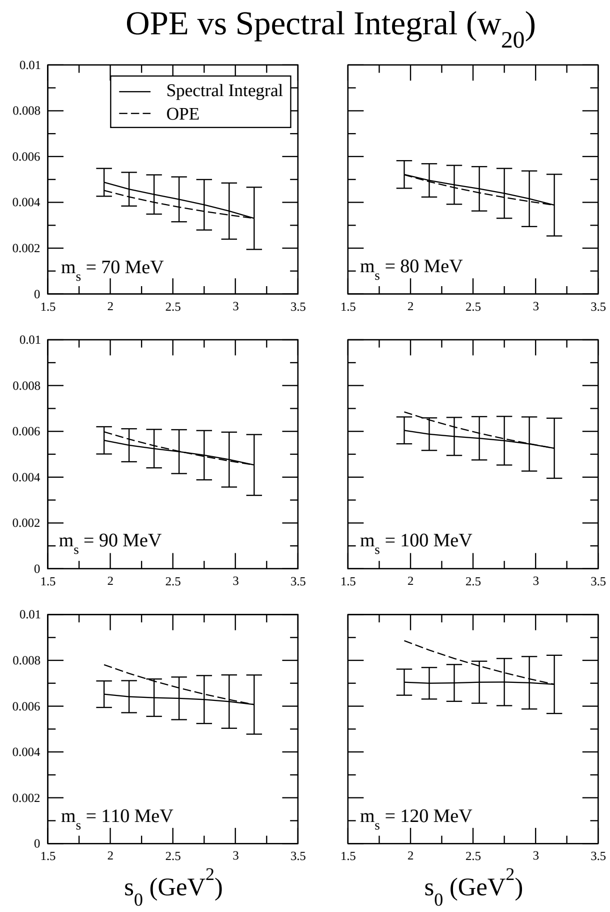
NON-SPECTRAL WEIGHT CONSISTENCY

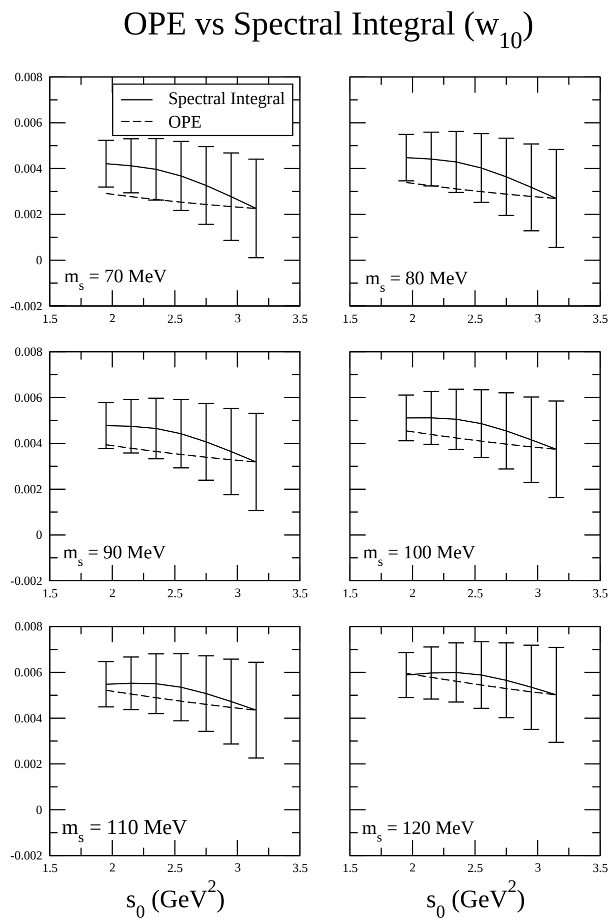


$w^{(0,0)}$ OPE/SPECTRAL INTEGRAL MATCHES



w_{20}, w_{10} OPE/SPECTRAL INTEGRAL MATCHES





- evidence of instability for $(0,0)$ spectral weight, improved stability for non-spectral weights
  - $O(a^N)$-truncated $D=2$ correlator/Adler function difference as alternate estimate of truncation uncertainty
    - $r_k^w(s_0)$: $O(a^k)$ (correlator-Adler)/correlator ratio

Weight	$r_1^w(m_\tau^2)$	$r_2^w(m_\tau^2)$	$r_3^w(m_\tau^2)$	$r_4^w(m_\tau^2)$
$w_{J=0+1}^{(0,0)}$	-0.01	0.06	0.20	0.67
$\hat{w}_{10}$	-0.11	-0.07	-0.05	-0.03
w_{20}	-0.11	-0.08	-0.05	-0.03
w_{10}	-0.10	-0.06	-0.03	-0.01

    - $\Rightarrow$ truncation uncertainty for $w^{(0,0)}$ $\sim$ 2.5 larger than previous $O(a^3)$ [(residual scale dependence) $\oplus$ (last term size)] estimate

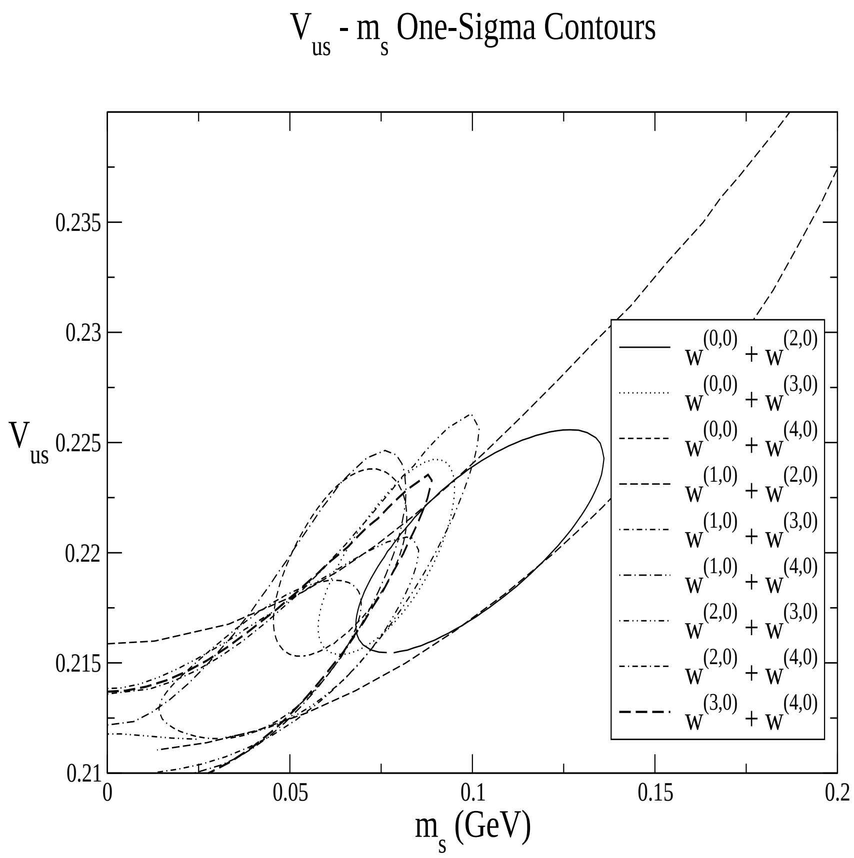
– V_{us} vs. s_0 [GeV^2] stability tests (PDG04 m_s input)

s_0	$w^{(0,0)}$	$\hat{w}_{10}$	w_{20}	w_{10}
2.35	0.2149	0.2220	0.2243	0.2201
2.55	0.2167	0.2218	0.2235	0.2203
2.75	0.2181	0.2218	0.2230	0.2207
2.95	0.2193	0.2220	0.2227	0.2211
3.15	0.2202	0.2223	0.2226	0.2216

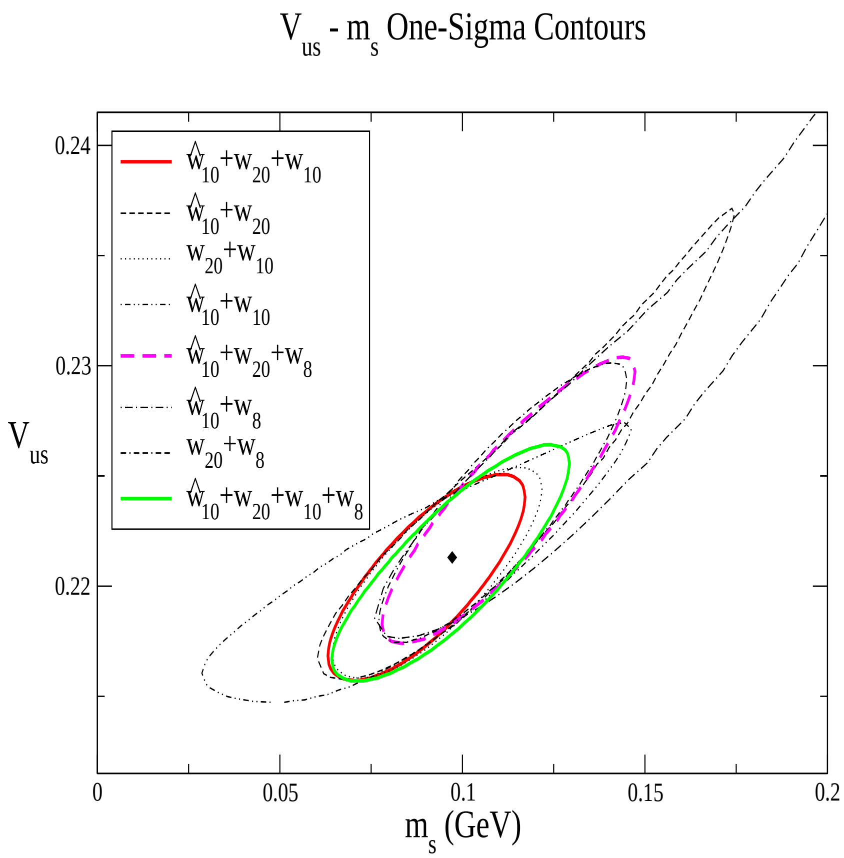
– m_s, $|V_{us}|$ fits with pairs of weights, $s_0 = m_\tau^2$, show poor consistency of 1σ contours for different spectral weight pairs [FIGURE]

– m_s, $|V_{us}|$ fits with pairs of weights, $s_0 = m_\tau^2$, show good consistency of 1σ contours for different non-spectral weight pairs [FIGURE]

FIT CONTOURS, $(k, 0)$ SPECTRAL WEIGHTS



FIT CONTOURS, NON-SPECTRAL WEIGHTS



RESULTS WITH CURRENT DATA

- conventional $s_0 = m_\tau^2$ 1-weight fits for $|V_{us}|$ with $m_s(2\text{ GeV}) = 94 \pm 6$ MeV as input

$$0.2210 \pm 0.0030_{exp} \pm 0.0010_{th} \ (\hat{w}_{10})$$
$$0.2209 \pm 0.0029_{exp} \pm 0.0017_{th} \ (w_{20})$$
$$0.2206 \pm 0.0032_{exp} \pm 0.0007_{th} \ (w_{10})$$
$$0.2218 \pm 0.0037_{exp} \pm 0.0009_{th} \ (w_8)$$

[Combined fits below (strong OPE/spectral integral correlations)]

- central values could still have non-trivial shifts with B factory data (see below)

- Combined $s_0 = m_\tau^2$, w_{20}, $\hat{w}_{10}$, w_{10} fit results

$$m_s(2\ \text{GeV}) = 89 \pm 26\ \text{MeV}$$
$$|V_{us}| = 0.2202 \pm 0.0046$$

- c.f. 4-fold non-spectral weight fit result

$$m_s(2\ \text{GeV}) = 96 \pm 31\ \text{MeV},\ |V_{us}| = 0.2208 \pm 0.0052$$

  - w_8: less suppression of high error us region, hence larger spectral integral errors at present
  - adding w_8 (and other similar weights) not helpful at present, but becomes so with reduced us errors

- m_s result in excellent agreement with recent average, (94 ± 6 MeV) of strange scalar, strange PS, $n_f = 2+1$ lattice results [Gamiz et al. hep-ph/0610246]

- 3-fold fit with $m_s(2\text{ GeV}) = 94 \pm 6$ MeV input:

$$|V_{us}| = 0.2209 \pm 0.0031$$

(c.f. $|V_{us}| = 0.2221 \pm 0.0026$ with expt'l errors only, central OPE input)

- NOTE: true us spectral errors actually somewhat smaller (OPAL, final version of ALEPH analysis) but relevant us covariance matrices not yet publicly available

ERRORS AND THE FUTURE

- us data errors soon to be much reduced by B factory experiment analyses ($>10^3$ times LEP statistics each for BABAR and BELLE)

- need all strange modes with B to sub-10^{-4} level
  - missing modes: higher multiplicity, higher s region
  - total $B_{us} \sim 3\% \Rightarrow$ neglected 10^{-4} mode lowers $|V_{us}|$ by ~ 0.0004 for $w^{(0,0)}$, somewhat less for $w(s)$ with stronger high-s suppression
  - definitely feasible (preliminary results for modes with $B \sim$ a few $\times 10^{-5}$ already reported)

- ud data also relevant [e.g. slightly lower BELLE (e^+e^-) central $B_{\pi\pi}$ would raise $|V_{us}|$ by ~ 0.0004 (0.0018)]

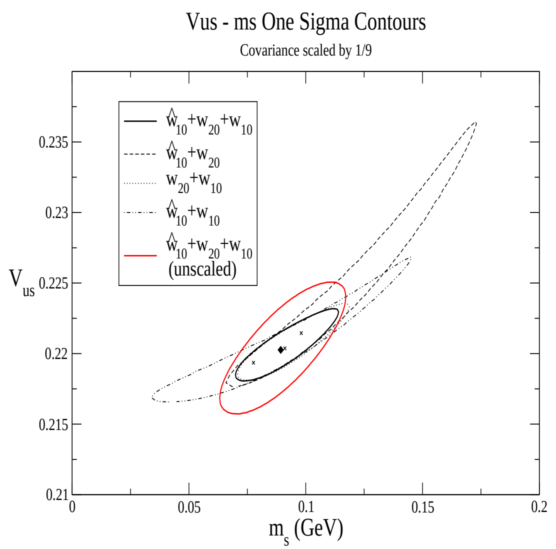
- $D = 2$ truncation dominates $(0, 0)$ theory error

  - no obvious way to improve, and hard to estimate error conservatively (*importance of s_0 stability tests*)

  - unfortunate since improved strange BF's enough to improve $s_0 = m_\tau^2$ us spectral errors for $(0, 0)$ spectral weight analysis (c.f. full re-measurement of us spectral distribution needed for other (spectral or non-spectral) weights and/or other s_0)

- in contrast, m_s uncertainty ($D = 2$ overall scale) dominates theory error for non-spectral weights

- Impact of reduced us experimental errors
  - *existing non-spectral* $w(s)$, $m_s(2\text{ GeV})$ to ± 5 MeV, us errors scaled by 1/5 (already being achieved) $\Rightarrow$ δV_{us} below 0.0010
  - impact on combined fit (consistency check)



- additional possibilities for exploration, improvement of both m_s, $|V_{us}|$ once BABAR, BELLE us data (with reduced errors above K^*) available

- *EXPERIMENTALISTS NOTE: new weight explorations require availability of covariance matrices, NOT just correlated errors for $s_0 = m_\tau^2$ and some set of existing weights already discussed in the literature*

- correlations and errors for $s_0 \neq m_\tau^2$ particularly *crucial* for s_0-stability tests (in turn crucial because of the unavoidable slow convergence of the $D = 2$ $J = 0 + 1$ OPE series)

CONCLUSIONS

- *Longitudinal subtraction/modelling unavoidable*

- importance of s_0-stability checks given unavoidable slow convergence of basic $D = 2$ OPE 0+1 correlator series

- *Polemical stance: window of s_0 MUST exist with $|V_{us}|$ instability less than estimated theory uncertainty. If not $\Rightarrow$ theory error estimate insufficiently conservative*

- truncation uncertainty for $(0, 0)$ case sizeable (limit for significant improvement with this approach) $\Rightarrow$ non-spectral $w(s)$, improvements in whole us spectral distribution, NOT just us BR's

- much smaller truncation uncertainties for other $w(s)$

- improvement via additional non-spectral weight choices almost certainly possible

- m_s to ± 5 MeV plus better than $1/5$ reduction in us data errors would yield $|V_{us}|$ to sub-± 0.0010 accuracy

- combined m_s, V_{us} fit to check consistency of m_s important, given slow convergence of $D = 2$ OPE series

- $|V_{us}|$ uncertainties *completely* independent of those for $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ (lattice chiral extrapolation), hence further improvement by averaging

SUPPLEMENTARY PAGES

- Increase in statistics at B factory experiments
- LEP+CLEO strange hadronic decay mode results
- Details on the handling of potential $D > 6$ OPE contributions

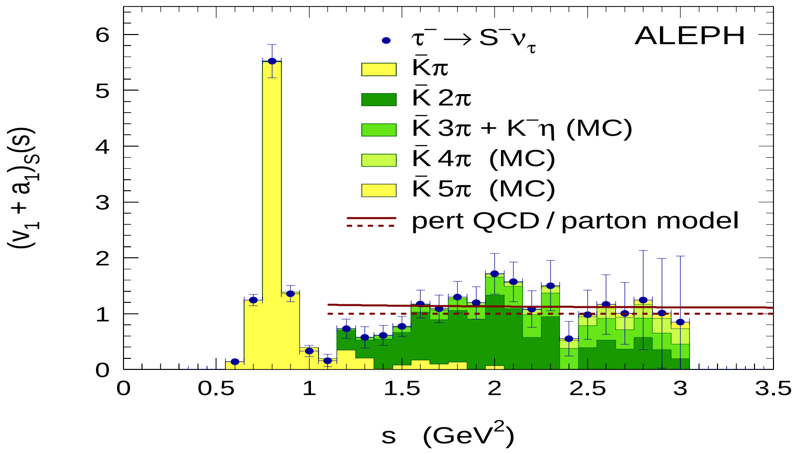
B factory vs. LEP statistics

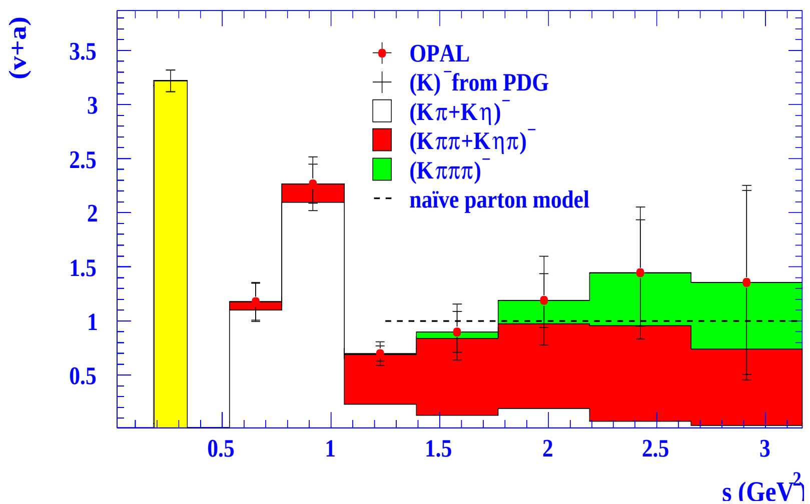
Experiment	# $\tau^+\tau^-$ pairs
LEP	$\sim 3 \times 10^5$
BABAR	$\sim 3 \times 10^8$
BELLE	$\sim 5 \times 10^8$

(plus improved K ID at BABAR, BELLE)

LEP+CLEO us DECAY MODE RESULTS

Mode	$\mathcal{B}$ (%)
K^- [τ decay]	0.681 ± 0.023
(Alternate: K^- [$K_{\mu 2}$])	(0.715 ± 0.003)
$K^-\pi^0$	0.454 ± 0.030
$\bar{K}^0\pi^-$	0.878 ± 0.038
$K^-\pi^0\pi^0$	0.058 ± 0.024
$\bar{K}^0\pi^0\pi^-$	0.360 ± 0.040
$K^-\pi^-\pi^+$	0.330 ± 0.028
$K^-\eta$	0.027 ± 0.006
$(\bar{K}3\pi)^-$ (estimated)	0.074 ± 0.030
$K_1(1270) \rightarrow K^-\omega$	0.067 ± 0.021
$(\bar{K}4\pi)^-$ (estimated) $+ K^*\eta$	0.040 ± 0.012
TOTAL	2.969 ± 0.086 (3.003 ± 0.083)





[Davier, Hocker, Zhang review hep-ph/0507078]

HIGHER D OPE CONTRIBUTIONS

- rough estimates for $D=6$ condensates, $D>6$ combinations unknown, usually assumed negligible

- $w(y) = \sum_m c_m y^m$, $y = s/s_0 \Rightarrow$ integrated $D = 2k+2$ OPE $\propto c_k/s_0^k$ (up to logs) $\Rightarrow$ avoid large c_k, $k \geq 2$

- neglect of non-negligible higher D terms $\Rightarrow$ s_0-instability of output $\Rightarrow$ *need to study output as function of* s_0

- NOTE: growth of coefficients in $(k, 0)$ spectral weights

$$
\begin{aligned}
w^{(0,0)}(y) &= 1 - 3y^2 + 2y^3 \\
w^{(1,0)}(y) &= 1 - y - 3y^2 + 5y^3 - 2y^4 \\
w^{(2,0)}(y) &= 1 - 2y - 2y^2 + 8y^3 - 7y^4 + 2y^5 \\
w^{(3,0)}(y) &= 1 - 3y + 10y^3 - 15y^4 + 9y^5 - 2y^6 \\
w^{(4,0)}(y) &= 1 - 4y + 3y^2 + 10y^3 - 25y^4 + 24y^5 \\
&\quad - 11y^6 + 2y^7
\end{aligned}
$$

- contrast 4 useful *non-spectral weights* to be used below (also normalized to 1 at $y = 0$): w_{20}, $\hat{w}_{10}$, w_{10}, w_8, with largest $k \geq 2$ coefficients $c_4 = 2.087$ (w_{20}), $c_5 = 1.206$ ($\hat{w}_{10}$), $c_5 = 2$ (w_{10}), $c_5 = 1.182$ (w_8)