CKM 2006 – Dec 12-16, 2006 – Nagoya University, Japan

EXTRACTING V_{us} FROM HYPERONS

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Unitary constraints

$$V_{us}^2 + V_{ud}^2 - 1 = (3.3 \pm 1.5) \cdot 10^{-3}$$ *PDG '04*

$$V_{us}^2 + V_{ud}^2 - 1 = (0.8 \pm 1.1) \cdot 10^{-3}$$ *PDG '06*

mainly from K_{l3}

$V_{us} = 0.2200(26)$ ('04)
$V_{us} = 0.2257(21)$ ('06)

<u>*Importance of a independent estimation*</u>

Hyperon decays?

Ademollo Gatto theorem

$$\langle B_2|V_\mu|B_1\rangle=\overline{B_2}\left[f_1(q^2)\gamma^\mu-i\,f_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{M_1+M_2}+f_3(q^2)\frac{q^\mu}{M_1+M_2}\right]B_1$$

$$\langle B_2|A_\mu|B_1\rangle=\overline{B_2}\left[g_1(q^2)\gamma^\mu-i\,g_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{M_1+M_2}+g_3(q^2)\frac{q^\mu}{M_1+M_2}\right]\gamma_5 B_1$$

not protected by symmetries

Not so bad: $\frac{g_1}{f_1}$ *can be extracted from experiments*

Cabibbo, Swallow and Winston '03 and '04

Experiments can extract $|V_{us}\cdot f_1(0)|^2$

with % accuracy!

Decay Process	Rate (μsec^{-1})	g_1/f_1	V_{us}
$\Lambda \to p e^- \overline{\nu}$	3.161(58)	0.718(15)	0.2224 ± 0.0034
$\Sigma^- \to n e^- \overline{\nu}$	6.88(24)	−0.340(17)	0.2282 ± 0.0049
$\Xi^- \to \Lambda e^- \overline{\nu}$	3.44(19)	0.25(5)	0.2367 ± 0.0099
$\Xi^0 \to \Sigma^+ e^- \overline{\nu}$	0.876(71)	1.32(+.22/ − .18)	0.209 ± 0.027
Combined	—	—	0.2250 ± 0.0027

Cabibbo, Swallow, Winston '03-'04
(see also ***Mateu and Pich '05****)*

No SU(3)-breaking corrections?

$$f_1(0) = CG\left(1 + O(m_s - m_d)^2\right)$$

Ademollo-Gatto Theorem

Compatible with K_{l3} and unitarity!

further study of the SU(3)-breaking needed...

SU(3) corrections:

old/new estimates:

- $1/N_c$ expansion;
- quark models;
- (H)BChPT.

do not agree between themselves

(not even the sign)

hint: *different expansions, different contributions? (large cancellations?)*

Need for model independent computations:

Lattice QCD!

Hyperon v.f.f. on the lattice

need to extract $f_1(0)$ at % level!

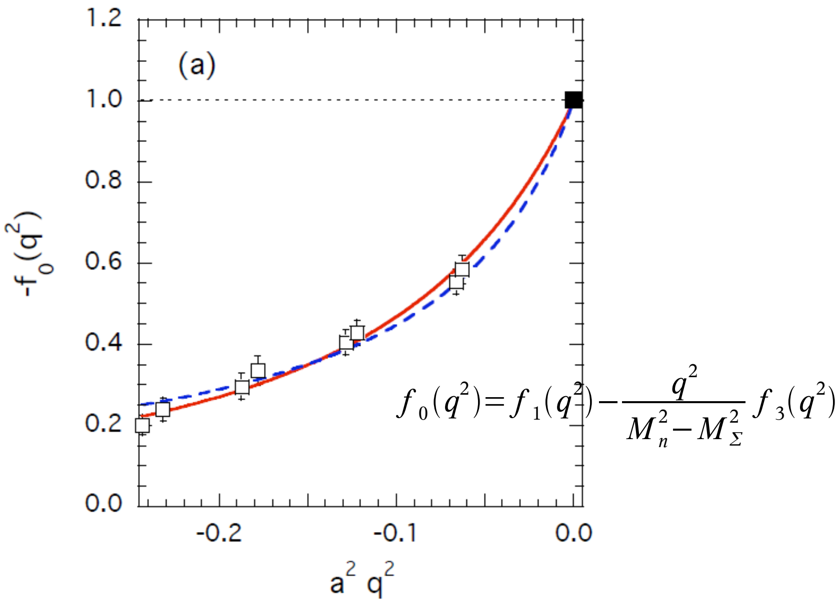
Guadagnoli *et al.* **'04-'06**

same strategy as in the K_{l3} case

The 3-steps procedure: (***Becirevic*** *et al.* **'04**)

- $f_0(q^2_{max})$ *from double ratios (very precise!)*
- $f_0(q^2)$->$f_1(0)$ *from double ratios (good accuracy)*
- *Chiral Extrapolation*

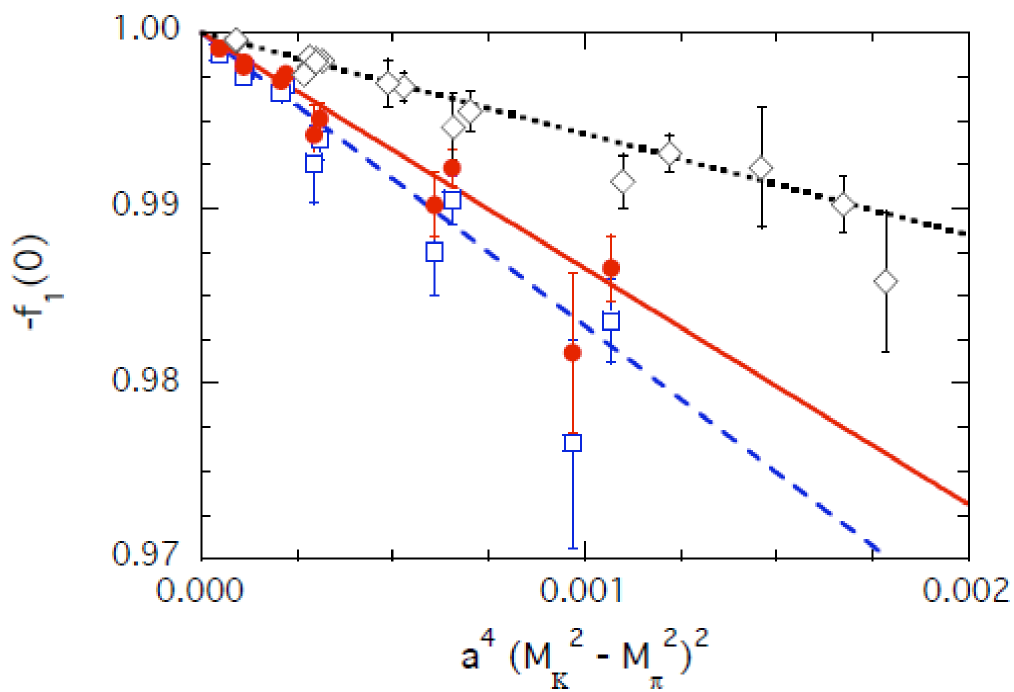
1) double ratio correlations give $f_0(q^2{}_{max})$ at *0.2%*!



2) double ratios give $f_0(q^2)$ with good precision

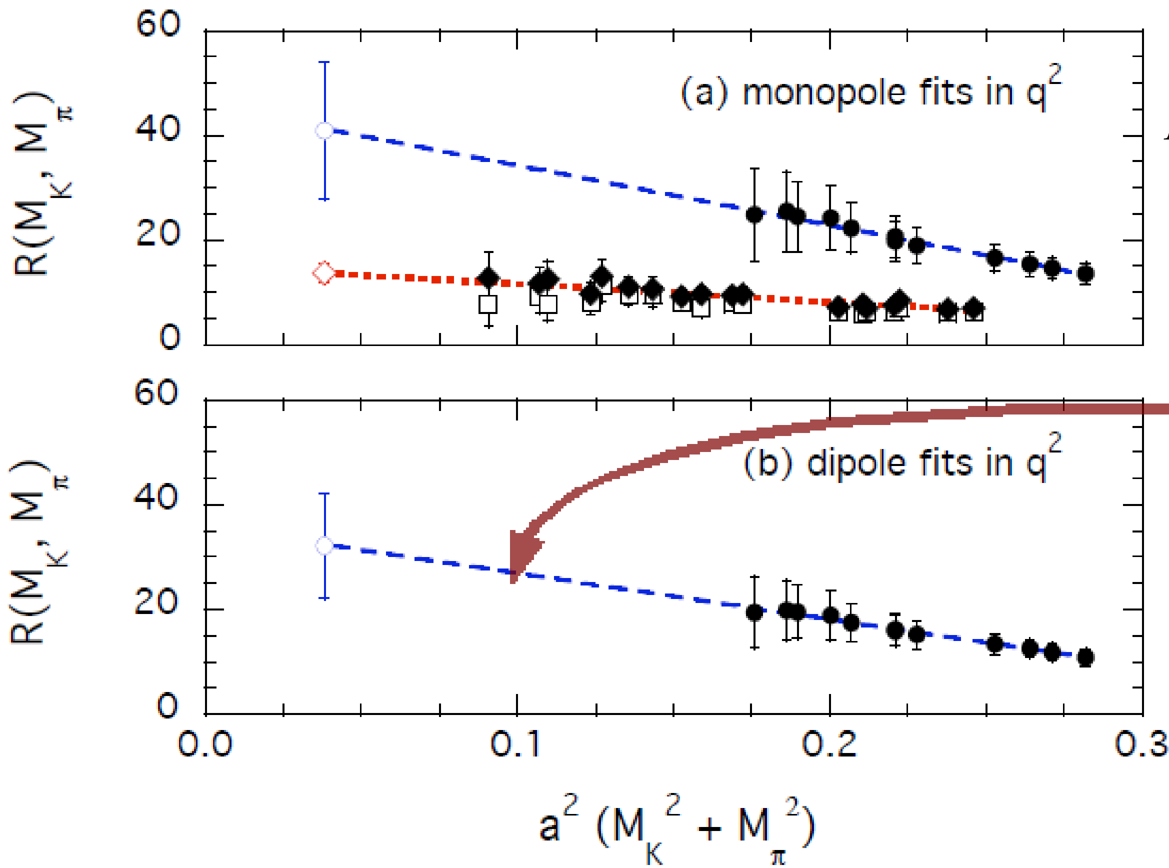
$\Rightarrow$

extrapolation of $f_0(0)=f_1(0)$



– good agreement with Ademollo-Gatto Theorem

$1+O(m_K{}^2-m_\pi{}^2)^2$



$$R(M_K, M_\pi) \stackrel{\text{def}}{=} \frac{f_1(0)-(-1)}{(a^2 M_K^2 - a^2 M_\pi^2)^2}$$

not sensitive to meson loops !

$$f_1^{\Sigma\to n}(0) = -1[1-0.052(29)]$$ *up to quenching error!*

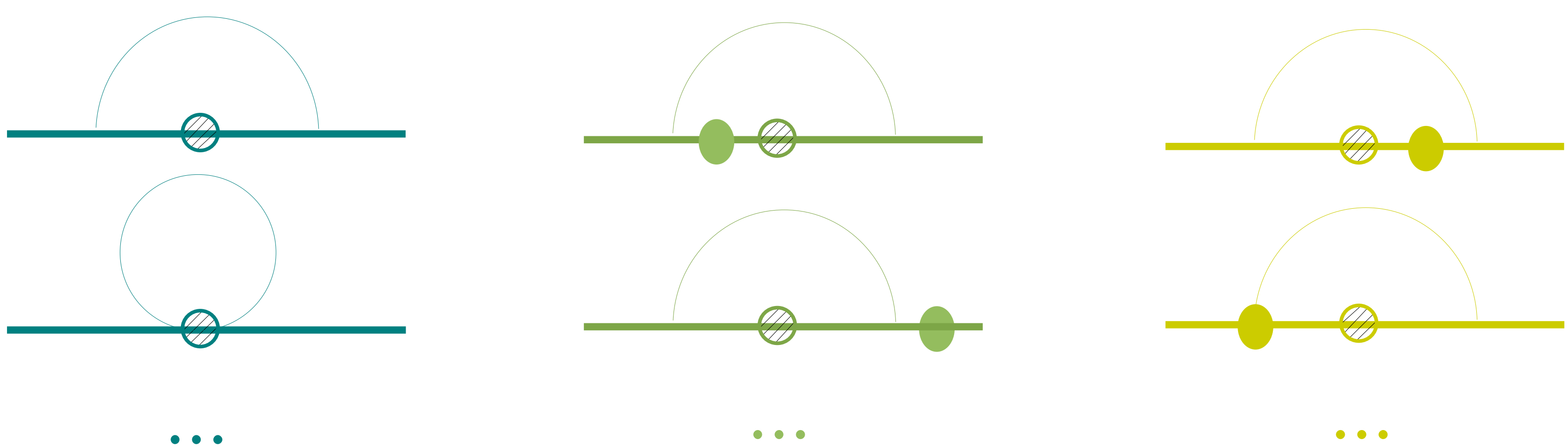
need for ChPT to drive the chiral extrapolation

SU(3)-breaking corrections in HBChPT

Villadoro '06

$$f_1(0) = CG\left[1 + \frac{p^2}{\Lambda_\chi^2} + \left(\frac{p^3}{\Lambda_\chi^3} + \frac{p^3}{\Lambda_\chi^2 M}\right) + \ldots\right]$$

$$\frac{m_K^2}{(4\pi f_\pi)^2} \sim 0.2 \qquad \frac{m_K^2}{(4\pi f_\pi)^2}\frac{\pi\,\delta M_B}{m_K} \sim 0.2 \qquad \frac{m_K^2}{(4\pi f_\pi)^2}\frac{\pi\, m_K}{M_B} \sim 0.2$$



$O(p^2)$ corrections:

Ademollo Gatto ⇒

$$H_{1,2} = m_1^2 + m_2^2 - 2\frac{m_1^2 m_2^2}{m_2^2 - m_1^2}\log\frac{m_2^2}{m_1^2}$$

⇒ no LECs !

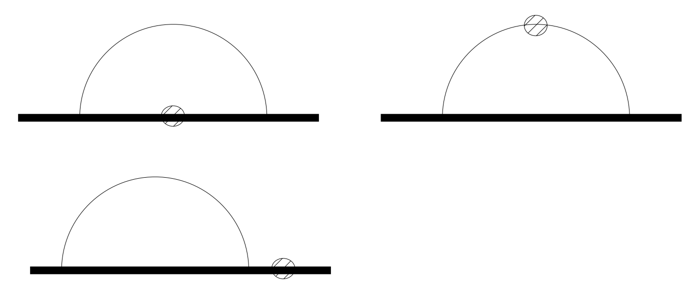
2 contributions:

Tadpoles

- universal contribution ⇔ to that of $K^0 \to \pi^-$
- independent of LO LECs *"D"* and *"F"*

~ -2.3%

Sunsets



- non-universal
- dependent on *"D"* & *"F"*

~ ±(2÷7)%

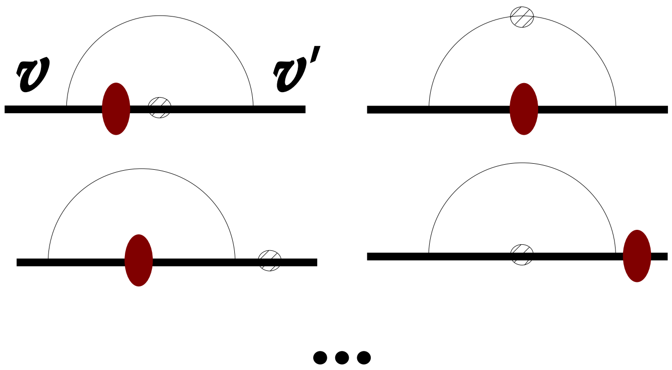
$O(p^3)$ corrections:

At this order: many operators with unknown LECs BUT in this process ONLY baryon mass shifts operators contribute!!!

$\Rightarrow$ no LECs

$$\frac{i}{k\cdot v-\delta M_B+i\epsilon}\simeq\frac{i}{k\cdot v+i\epsilon}+\frac{i}{k\cdot v+i\epsilon}(-i\,\delta M_B)\frac{i}{k\cdot v+i\epsilon}$$

$O(p)$ $O(p^2)$



Results:

- depend on F,D
- important contributions

$\sim +(2\div 6)\%$

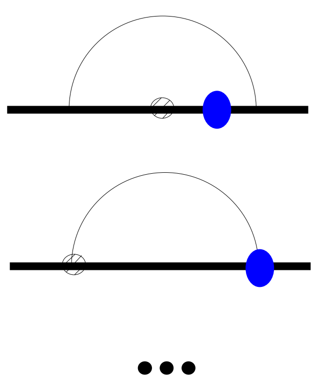
Disagreement with Anderson and Luty '93
difference $\propto (q_{max})^2=(M_{B1}-M_{B2})^2$

1/M corrections:

<u>All coefficients fixed</u>
by *Lorentz invariance*
No new unknown parameters

$\Rightarrow$ no LECs

$$\frac{i}{\not{p}-M_0+i\epsilon} \simeq \frac{i}{k\cdot v+i\epsilon} + \frac{i}{k\cdot v+i\epsilon}\left[i\left(\frac{(k\cdot v)^2-k^2}{2M_0}\right)\right]\frac{i}{k\cdot v+i\epsilon}$$



Results:

- important contributions
- sensitive to central value M_0
- tendency to cancel *D & F* $O(p^2)$ contributions
- agree with *1/M* expansion of relativistic BChPT by Krause '90

$$\sim \pm(3\div 8)\%$$

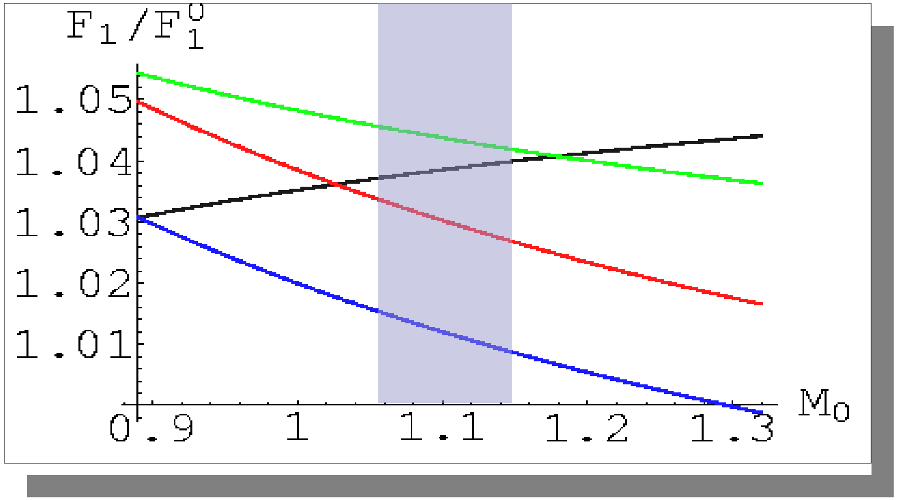
Final Results in HBChPT

Ademollo-Gatto ⇨ All contributions up to 2-loop $O(p^4)$ depend only on LO LECs

$f_1(0)/f_1(0)^{SU(3)}$	$O(p^2)$	$O(p^3)$	$1/M_0$	All
$\Sigma^- \to n$	+0.7%	+6.5%	-3.2%	+4.1%
$\Lambda \to p$	-9.5%	+4.3%	+8.0%	+2.7%
$\Xi^- \to \Lambda$	-6.2%	+6.2%	+4.3%	+4.3%
$\Xi^- \to \Sigma^0$	-9.2%	+2.4%	+7.7%	+0.9%

Parameter used:

- *physical masses and decay constants*
- *D=0.80, F=0.46*
- *M_0=1.1 GeV*



Dynamical Decuplet...

Rarita-Schwinger field

$$L = i\,\bar{T}^{\mu}\, v\cdot D\, T_{\mu} + \ldots$$

$$\frac{i\,P^{\mu\nu}_{3/2}}{k\cdot v - \Delta + i\,\epsilon}$$

Octet-Decuplet
mass splitting

Jenkins and Manohar '91

Decoupling Limit -- $\Delta \gg \Lambda_{QCD}$
Local effects $O(p^4)$
(into the LECs)

Small Scale Exp. -- $\Delta \ll \Lambda_{QCD}$
Model – not an Effective Theory
Δ is not a chiral parameter
⇒ no control on higher order operators

Hemmert, Holstein and Kambor '98

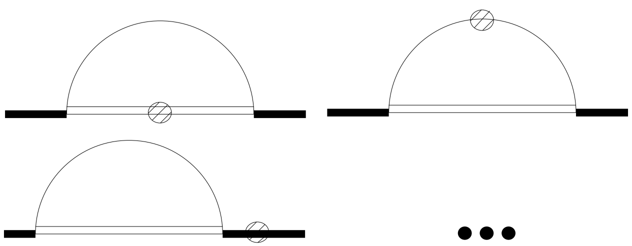
Ademollo Gatto ⇒

no LECs in Δ-contributions to

$$f_1(0) = f_1^{SU(3)}(0)\left(1 + O(p^2) + O(p^3) + O(1/M) + \ldots\right)$$

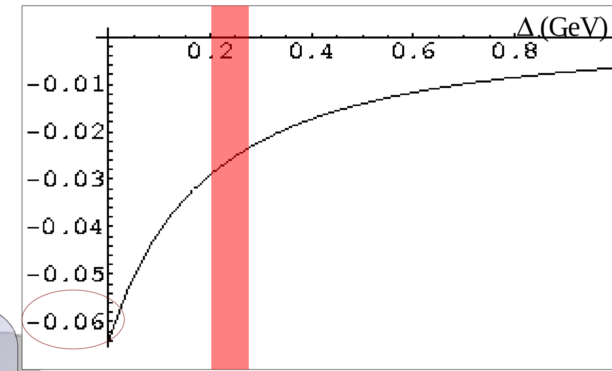
...and the breaking of the Chiral Expansion!

$O(p^2)$ contributions:



e.g. $\Sigma^- \to n$ channel

Results ~ -3.1%

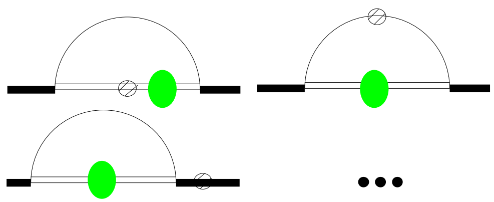


$\propto \mathcal{C}^2 \sim (1.6)^2 \quad (g_{\pi N\Delta})$

with $\Delta \simeq 230$ MeV

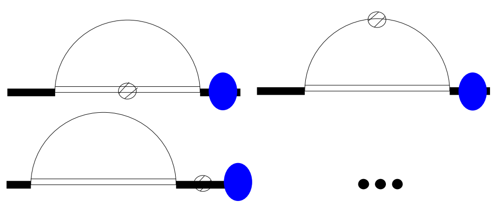
$O(p^3)$ contributions:

δM_Δ

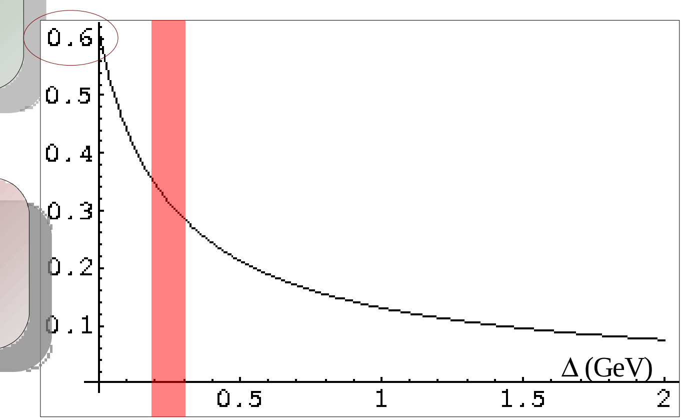


Results ~ -1.8%

δM_B



Results ~ +38%



Speculations...

neglecting Δ contributions and "believing" HBChPT

$$f_1^{\Sigma\to n}(0) = -1\left(1 + 0.041_{HBChPT} - 0.052_{lattice} \pm 0.029_{lattice}\right)$$
$$= -1\left(1 - 0.011 \pm 0.029_{lattice}\right)$$

SU(3)-breaking compatible with 0

(⇒ compatible with K_{l3})

even neglecting errors from HBChPT

…seriously speaking

- *ChPT unreliable for baryons*
- *need to go near the physical point for quark masses*
- *need to remove the quenching "approximation"*

When such simulations will be available
hyperons will represent an important cross-check
to V_{us} from kaon decays

looking forward for lattice improvements…