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VNIVERSITAT DE VALÈNCIA

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Analytical approaches to the calculation of $f_+^{K^0\pi^-}(0)$

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FlaviA net

Summary

- Introduction and definitions
- $f_+^{K^0\pi^-}(0)$ from the scalar Kπ form factor
- $f_+^{K^0\pi^-}(0)$ from large-N_C QCD
- Comparison
- Outlook

Introduction and definitions

$$\left\langle \pi^-(p_\pi)\left|\bar{s}\gamma_\mu u\right|K^0(p_K)\right\rangle = f_+^{K^0\pi^-}(q^2)\left[(p_K+p_\pi)_\mu - \frac{\Delta_{K\pi}}{q^2}q_\mu\right] + f_0^{K^0\pi^-}(q^2)\frac{\Delta_{K\pi}}{q^2}q_\mu$$

$$q_\mu = (p_K - p_\pi)_\mu \quad , \qquad \Delta_{K\pi} = M_K^2 - M_\pi^2 \quad , \qquad f_+^{K^0\pi^-}(0) = f_0^{K^0\pi^-}(0)$$

$$f_{+,0}^{K^0\pi^-}(q^2) = f_+^{K^0\pi^-}(0)\left[1 + \lambda'_{+,0}\frac{q^2}{M_{\pi^+}^2} + \dots\right]$$

Introduction and definitions

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$$f_{+,0}^{K^0\pi^-}(q^2) = f_+^{K^0\pi^-}(0)\left[1 + \lambda'_{+,0}\frac{q^2}{M_{\pi^+}^2} + \ldots\right]$$

$$\Gamma\left(K^0_{\ell 3[\gamma]}\right) \sim \left|V_{us}\, f_+^{K^0\pi^-}(0)\right|^2 I^{K\ell}\left(\lambda'_{+,0},\ldots\right)\left[1 + \Delta_{SU(2)} + \Delta_{EM}\right]$$

What do we know?

$$f_{+}^{K^0\pi^-}(0) \; < \; 1$$

[Furlan et al, 1965]

$$f_{+}^{K^0\pi^-}(0) \; = \; 1 \; - \; \mathrm{O}\left[\left(m_s - m_u\right)^2\right]$$

[Ademollo & Gatto, 1964]

What do we know?

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$$f_+^{K^0\pi^-}(0) = 1 - O\left[(m_s - m_u)^2\right]$$ [Ademollo & Gatto, 1964]

What we want to know?

Chiral Perturbation Theory

$$f_+^{K^0\pi^-}(0) = 1 + \underbrace{f_{p^4} + f_{p^6} + \ldots}_{\text{SU(3) breaking}}$$

$$f_{p^4} = -0.0227$$ [Leutwyler & Roos, 1984] [Gasser & Leutwyler,1985]

One-loop, no local contribution

Chiral Perturbation Theory at O(p^6)

[Post & Schilcher, 2002] [Bijnens & Talavera, 2003]

$$f_{p^6} = f_{p^6}^{2-loops}(\mu) + f_{p^6}^{L_i \times \; loop}(\mu) + f_{p^6}^{tree}(\mu)$$

$$\left[f_{p^6}^{LR} = -0.016 \pm 0.008 \right]$$ (Quark Model) [Leutwyler & Roos,1984]

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$$f_{p^6} = \Omega\left(M_\rho, L_i^r\right) - 8\frac{\Delta_{K\pi}^2}{F_\pi^2}\left[C_{12}^r\left(M_\rho\right) + C_{34}^r\left(M_\rho\right)\right]$$

$$\Omega\left(M_\rho, L_i^r\right) = 0.0146\ \underbrace{(57)}_{loops}(28)$$ ← $O(p^4)$ LEC's, L_i^r

[Bijnens & Talavera, 2003]

LEC's in Chiral Perturbation Theory

$$L_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$$

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\left\langle u_\mu u^\mu + \chi_+ \right\rangle$$

($N_F = 3$)

Even-intrinsic-parity sector only

$$\mathcal{L}^{(4)} = L_1 \left\langle u_\mu u^\mu \right\rangle^2 + \ldots = \sum_{i=1}^{10} L_i \; O_i^4$$

+ 2 local

[Gasser & Leutwyler, 1985]

$$\mathcal{L}^{(6)} = C_1 \left\langle u_\alpha u^\alpha h_{\mu\nu} h^{\mu\nu} \right\rangle + \ldots = \sum_{i=1}^{90} C_i \; O_i^6$$

+ 4 local

[Fearing & Scherer, 1996]

[Bijnens, Colangelo & Ecker, 1999,2000]

High-precision predictions within χPT : $F^\pi_{V,S}$, $K_{\ell 3}$, ... → L_i , C_i ?

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$O(p^4)$ LEC's, L_i^r

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Large scale dependence!!

Ω contains a tree-level term

$f_+^{K^0\pi^-}(0)$ from the scalar Kπ form factor

[Bijnens & Talavera, 2003] [Jamin, Oller & Pich, 2004,2006]

$$f_0^{K^0\pi^-}(t) = f_+^{K^0\pi^-}(0) + \bar{\Delta}(t) + \left(\frac{F_K}{F_\pi} - 1\right)\frac{t}{\Delta_{K\pi}} + 8\,\frac{M_K^2 + M_\pi^2}{F_\pi^2}\left(2C_{12}^r + C_{34}^r\right)t - \frac{8}{F_\pi^2}C_{12}^r t^2$$

$\bar{\Delta}(0) = 0,$ $\bar{\Delta}(t)$ → **2-loops + 1-loop L_i + L_i^2**

$$C_{12}^r \sim \frac{d^2}{dt^2} f_0^{K^0\pi^-}(0) \,, \qquad 2C_{12}^r + C_{34}^r \sim \frac{d}{dt} f_0^{K^0\pi^-}(0)$$

$f_+^{K^0\pi^-}(0)$ from the scalar Kπ form factor

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[Jamin, Oller & Pich, 2000,2002,2004] **Scalar Kπ form factor from a coupled-channel dispersion-relation analysis**

$$f_{p^6} = -0.0034\ (57)(28)(90)$$

C_i^r → (90)

$$\left[\, f_{p^6}^{LR} = -0.016 \pm 0.008 \,\right]$$

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$f_+^{K^0\pi^-}(0)$

$f_+^{K^0\pi^-}(\Delta_{K\pi})$

needed !!

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$f_+^{K^0\pi^-}(0)$ from large-N_C QCD

[Cirigliano et al, 2005,2006]

1) Low energy expansion of Green Functions (Ward Id.)
2) Large N_C limit of QCD:
   - Infinite tower of non-decaying hadronic states
   - Description through a meromorphic function
3) Operator Product Expansion (OPE) at high energies
4) Brodsky-Lepage behaviour of form factors of hadron currents : $F_H \sim \frac{1}{Q^2} \quad , \; Q^2 \to \infty$

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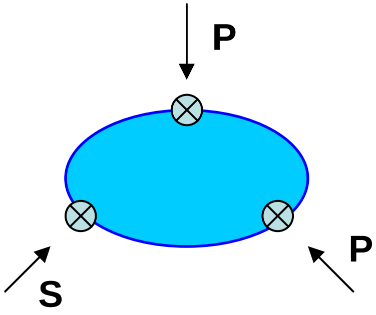
Matching procedure [Moussallam, 1997] [Peris et al, 1998] [Knecht & Nyffeler,2001] [Bijnens et al,2003] [Cirigliano et al, 2004, 2005,2006]

$$\Pi^{\chi}_{ijk}\left(L_i^r, C_i^R\right) \xrightarrow[s,t,u \ll M_R^2]{} \Pi^{R}_{ijk}\left(c_{ijk}, M_R\right) \xrightarrow[s,t,u \gg M_R^2]{} \Pi^{OPE}_{ijk}\left(\alpha_S\right)$$

Order parameters of χ-symmetry breaking

$L_5^r, C_{12}^r, C_{34}^r$

$$i^2 \int dx dy e^{ipx+iqy+irz} \left\langle 0 \left| T S^a(x) P^b(y) P^c(z) \right| 0 \right\rangle = d^{abc} \Pi_{SPP}\left(p^2, q^2, r^2\right)$$



$$\Pi_{SPP}^{\chi}(s,t,u) = \frac{(2B_0)^3}{tu}\left\{F^2 + 4L_5 s + 4\left(4L_8 - L_5\right)(t+u) - 8C_{12}s^2 + \right.$$
$$+ 8\left(2C_{12} + C_{34} + C_{38}\right)s(t+u) -$$
$$\left. -8\left(C_{12} + C_{34} - C_{38}\right)\left(u^2 + t^2\right)\right\} -$$
$$-(4B_0)^3\left(2C_{12} - 4C_{31} - 2C_{34} + 2C_{38} - C_{94}\right) + O(p^2)$$

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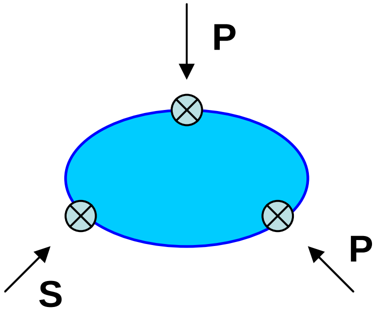

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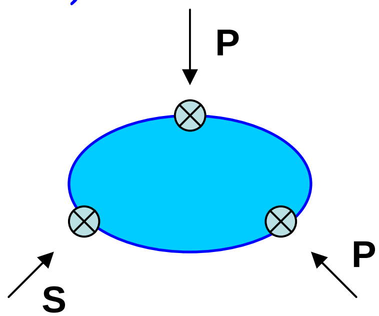
$$-(4B_0)^3 (2C_{12} - 4C_{31} - 2C_{34} + 2C_{38} - C_{94}) + O(p^2)$$

$$\Pi_{SPP}^{R}(s,t,u) = \mathcal{N}_{\chi} \frac{P_0 + P_1 + P_2 + P_3 + P_4}{[M_S^2 - s][-t][-u][M_P^2 - t][M_P^2 - u]}$$

$$P_n = \sum_{k=0}^{n} \sum_{l=0}^{k} c_{n-k,k-l,l} \; s^{n-k} t^{k-l} u^l$$

$L_5^r, C_{12}^r, C_{34}^r$

$$i^2 \int dx dy e^{ipx+iqy+irz} \left\langle 0 \left| T S^a(x) P^b(y) P^c(z) \right| 0 \right\rangle = d^{abc} \Pi_{SPP}\left(p^2, q^2, r^2\right)$$



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L_5^{SP}, L_8^{SP}

$C_{12}^{SP}, C_{34}^{SP}, C_{38}^{SP}$

$$\Pi_{SPP}^{R}(s,t,u) = \mathcal{N}_{\chi} \frac{P_0 + P_1 + P_2 + P_3 + P_4}{[M_S^2 - s][-t][-u][M_P^2 - t][M_P^2 - u]}$$

$$P_n = \sum_{k=0}^{n} \sum_{l=0}^{k} c_{n-k,k-l,l} \; s^{n-k} t^{k-l} u^l$$

$\Pi_{SPP}^{OPE}\left(\alpha_S^0\right)$

+

Asymptotic behaviour of form factors

$$f_{p^6} = f_{p^6}^{2-loops}(\mu) + f_{p^6}^{L_i\times\ loop}(\mu) + f_{p^6}^{tree}(\mu)$$

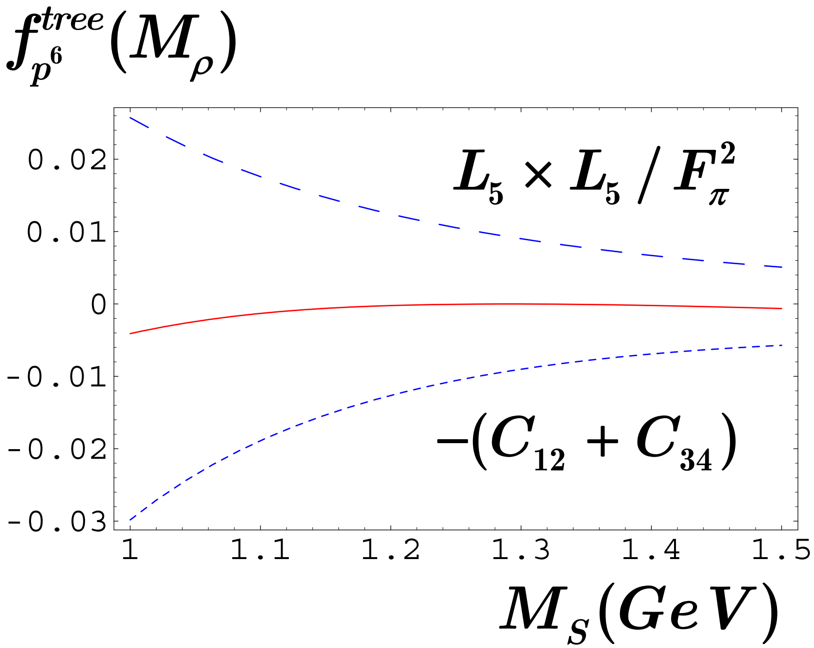
$$f_{p^6}^{tree}(M_\rho) = 8\frac{\Delta_{K\pi}^2}{F_\pi^2}\left[\frac{(L_5^r(M_\rho))^2}{F_\pi^2} - C_{12}^r(M_\rho) - C_{34}^r(M_\rho)\right]$$

$$= -\frac{\Delta_{K\pi}^2}{2M_S^4}\left(1-\frac{M_S^2}{M_P^2}\right)^2$$

Large cancellation

Less scale dependent

Large-N_C matching



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Large cancellation

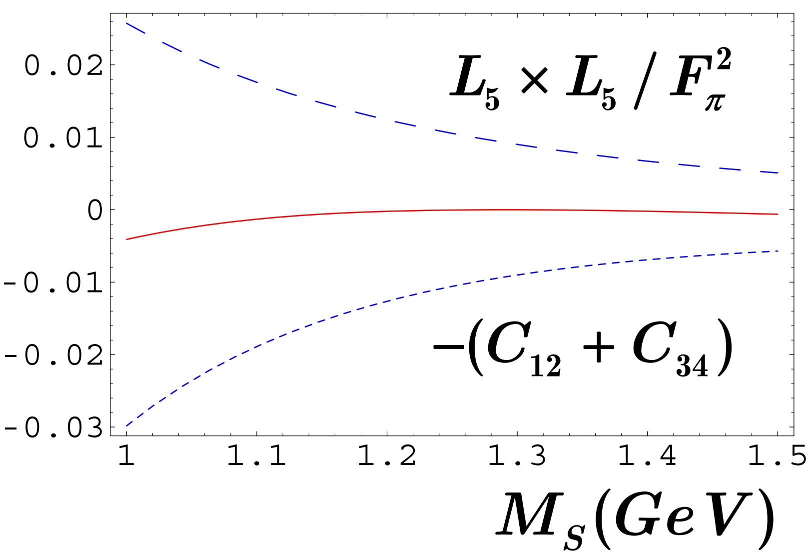
Less scale dependent

$$= -\frac{\Delta_{K\pi}^2}{2M_S^4}\left(1-\frac{M_S^2}{M_P^2}\right)^2$$

← Large-N_C matching

$$f_{p^6}^{tree}(M_\rho) = -0.002 \pm 0.008_{1/N_C} \pm 0.002_{M_S}\,{}^{+0.000}_{-0.002}\ \mathrm{P'}$$

$f_{p^6}^{tree}(M_\rho)$



$$f_{p^6} = 0.007 \pm 0.012$$

$$\left[f_{p^6}^{LR} = -0.016 \pm 0.008\right]$$

Comparison

	Reference	$f_+^{K^0\pi^-}(0)$
Quark model	**[Leutwyler & Roos, 1984]**	0.961 (8)
Lattice	**[Becirevic et al, 2005]**	0.960 (9)
	[MILC Collab., 2005]	0.962 (11)
	[Dawson et al, 2006]	0.968 (11)
	[UKQCD/RBC Collab., 2006]	0.968 (2) **!**
	[Bijnens & Talavera, 2003]	0.976 (10)
$K\pi$ scalar f.f.	**[Jamin, Oller & Pich, 2004]**	0.974 (11)
Large-N_C	**[Cirigliano, Neufeld & Pichl, 2004]**	0.981 (10)
	[Cirigliano et al, 2005]	0.984 (12)

$$f_+^{K^0\pi^-}(0) \cdot |V_{us}| = 0.2167\ (6)$$ [KLOE, 2005]

Reference	$f_+^{K^0\pi^-}(0)$	$\lvert V_{us}\rvert$
[Leutwyler & Roos, 1984]	0.961 (8)	0.2255 (20)
[Becirevic et al, 2005]	0.960 (9)	0.2257 (22)
[MILC Collab., 2005]	0.962 (11)	0.2253 (27)
[Dawson et al, 2006]	0.968 (11)	0.2239 (27)
[UKQCD/RBC Collab., 2006]	0.968 (2)	0.2239 (8)
[Bijnens & Talavera, 2003]	0.976 (10)	0.2220 (24)
[Jamin, Oller & Pich, 2004]	0.974 (11)	0.2225 (26)
[Cirigliano, Neufeld & Pichl, 2004]	0.981 (10)	0.2209 (23)
[Cirigliano et al, 2005]	0.984 (12)	0.2202 (28)

[PDG,2006] $|V_{us}|^{unitarity} = 0.2275\ (12)$

Outlook

Kπ scalar form factor

- Scalar strange spectral function
- Self-consistency check ($f_+^{K^0\pi^-}(0)$ needed)
- New parameterization [Bernard et al, 2006]
- Rely essentially on experimental improvements

  (S-wave Kπ scattering data, $K_{\mu2}$, $\pi_{\mu2}$)

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$K\pi$ scalar form factor

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Large-N_C inspired

- Discrepancy with lattice results is most disturbing
- Inclusion of an extra multiplet of scalar resonances
- Can we check C_{12}^r and C_{34}^r in other observables?

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New parameterizations

- Improve the convergence of the form factor [Hill, 2006]
- Already used by KTeV [KTeV collab., 2006]

Parsifal

(**PARticleS and Interactions : Flavour And coLour dynamics**)

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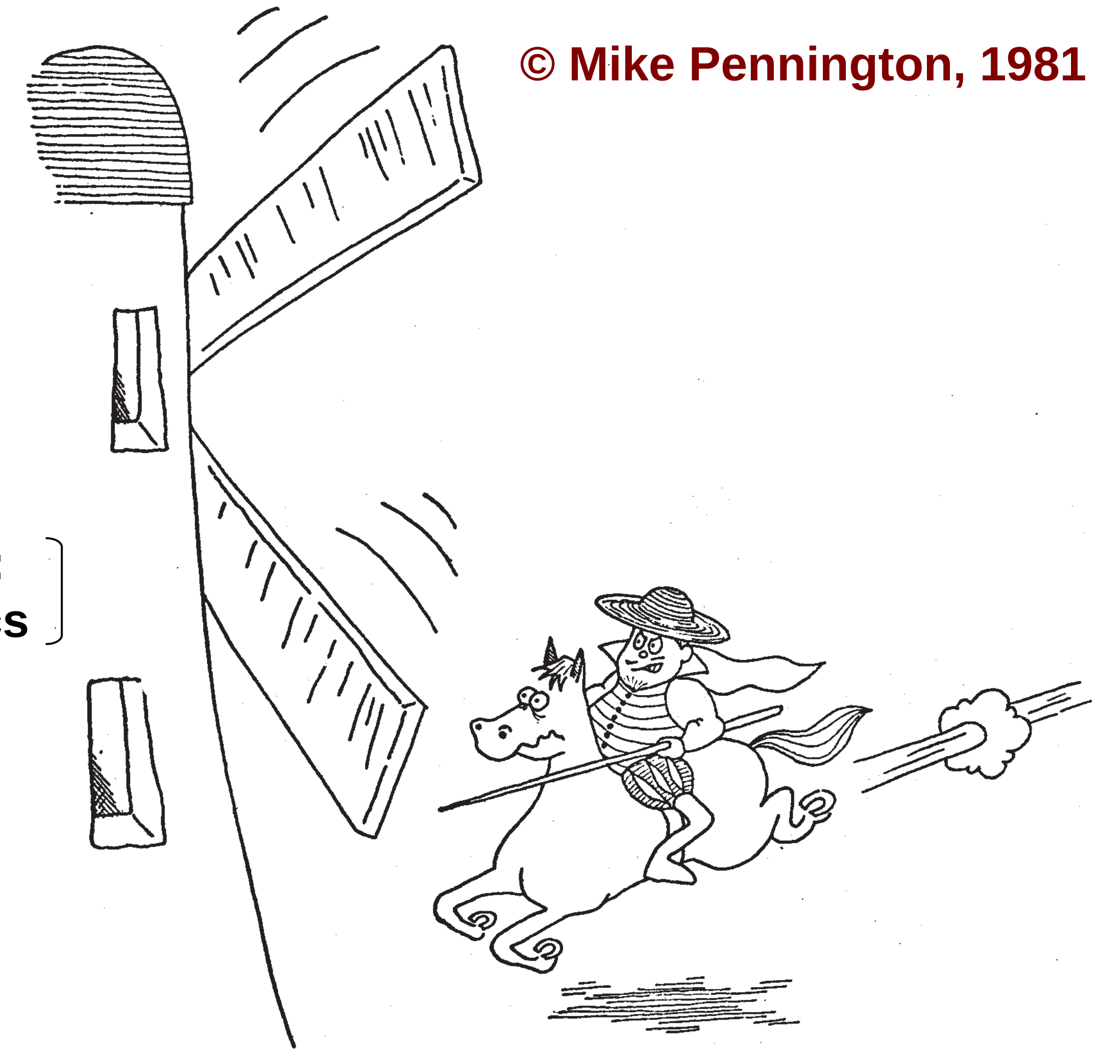

Stubbornly

Testing QCD

Backup

New parameterization of form factors [Hill, 2006]

Semileptonic form factors $f_+^{K^0\pi^-}(t)$ and $f_0^{K^0\pi^-}(t)$ are analytic functions throughout the complex t plane, except along a branch cut on the positive real axis starting at Kπ production threshold.

$$f(t) = \frac{1}{\phi(t, t_0, Q^2)} \sum_{k=0}^{\infty} a_k(t_0, Q^2)\, z(t, t_0)^k, \qquad t_0 \in (-\infty, t_+)$$

$$z(t, t_0) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}, \qquad t_+ = (M_K + M_\pi)^2$$

The form factors are now analytic in $|z| < 1$

The function $z(t, t_0)$ sums an infinite number of terms, transforming the original series into ones with a much smaller expansion parameter