Sides of the Unitarity Triangle

Benjamin Grinstein

CKM2006

Nagoya, Dec 2006

The talks are thought to introduce the Workshop, but more than being simply reviews and "state-of-art", are meant to be "provocative," i.e., ... What have we learnt, what should we do to learn more, are really the errors we quote under control? Which are the tests to be done...

-CKM2006 IAC charge

Known Unknowns

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The Unknown

As we know,
There are known knowns.
There are things we know we know.
We also know
There are known unknowns.
That is to say
We know there are some things
We do not know.
But there are also unknown unknowns,
The ones we don't know
We don't know.

Donald Rumsfeld
—Feb. 12, 2002, Department of Defense news briefing

The talks are thought to introduce the Workshop, but more than being simply ~~reviews and "state-of-art"~~, are meant to be "provocative," i.e., ... What have we learnt, what should we do to learn more, are really the errors we quote under control? Which are the tests to be done...

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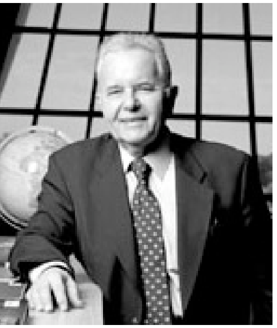
provoke /prəˈvoʊk/ [pruh-vohk]
–verb (used with object), -voked, -voking.
1. to anger, enrage, exasperate, or vex.
2. ...

I will have no friends by the end of the talk
but maybe someone young will be inspired



The CKM Matrix

(it is not just a triangle)

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Diagonalize quark mass terms by unitary transformations

$$\bar{U}_L \lambda_U U_R + \bar{D}_L \lambda_D D_R \longrightarrow \bar{U}_L (V^\dagger_{U_L} \lambda_U V_{U_R}) U_R + \bar{D}_L (V^\dagger_{D_L} \lambda_D V_{D_R}) D_R$$

Charged current

$$\bar{U}_L \gamma^\mu D_L \rightarrow \bar{U}_L \gamma^\mu (V^\dagger_{U_L} V_{D_L}) D_L, \qquad V_{\mathrm{CKM}} = V^\dagger_{U_L} V_{D_L}$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM is inevitable. The question is not whether CKM is correct.
It has to be there.
The question is: is it sufficient?

What can we exclude?
This should dictate some of the goals in this field.
For example:

1. Fourth generation?
   More generally, is the CKM unitary?

2. New CP violating interactions?
   Needed for lepto/baryo-genesis

3. Other new interactions?
   Particularly those related to EW-SB (TeV scale)

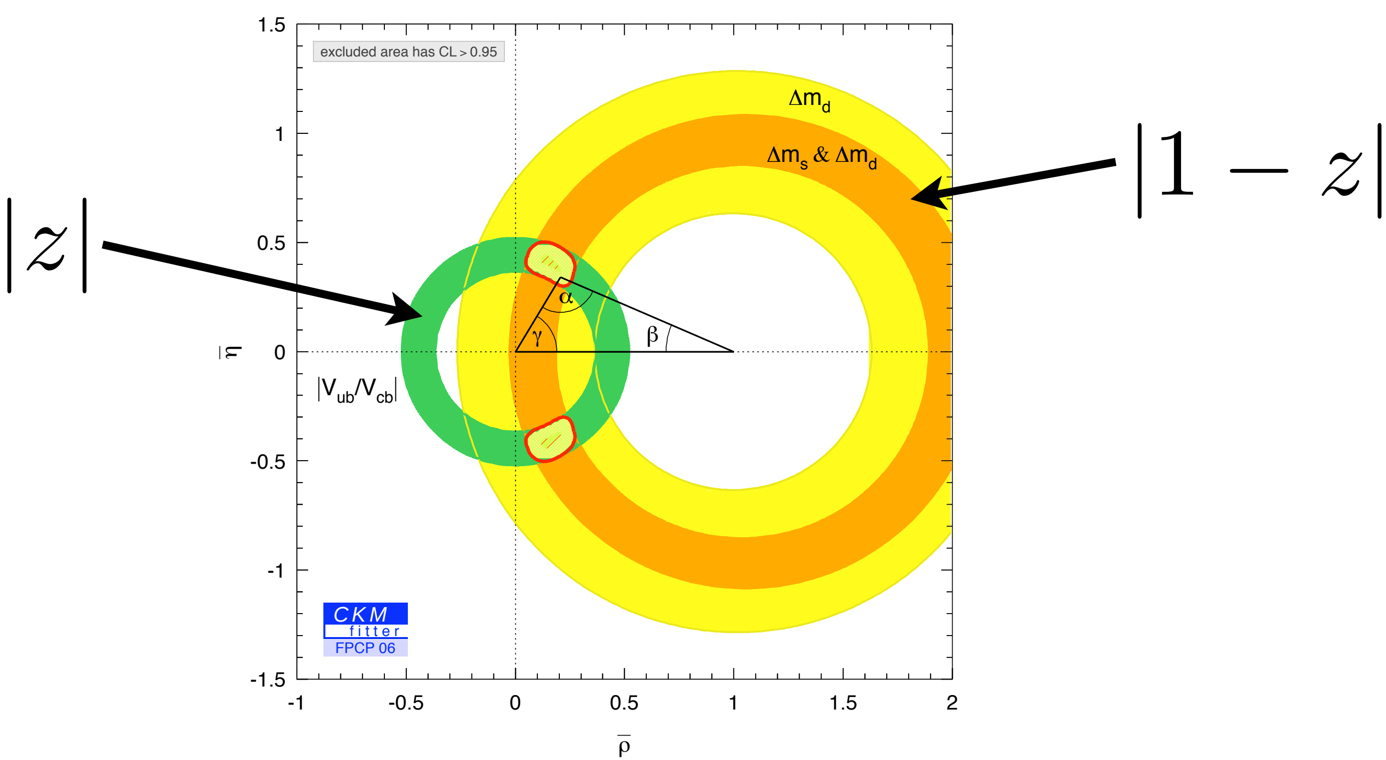
Answer: sadly, we cannot exclude much.
  But we may be able to set useful constraints

Wolfenstein parametrization

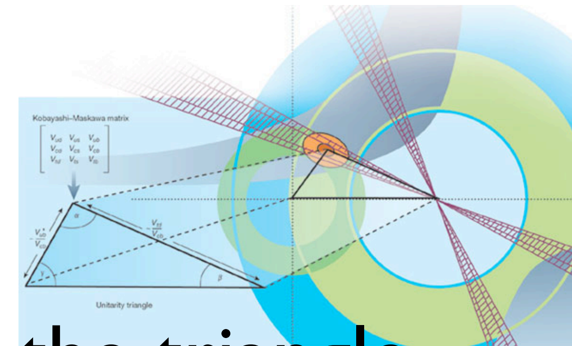
$$V_{\mathrm{CKM}} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda(1+iA^2\lambda^4\bar\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2(1+i\lambda^2\bar\eta) & 1 \end{pmatrix} + \mathcal{O}(\lambda^6).$$

CKM06 (in SM): determine four parameters
CKM06 “sides:” determine modulus of complex $z = \bar\rho + i\bar\eta$

Sides of triangle? Circles!!



provocative a la Andy Rooney?
(with apologies to non-US TV watchers)



Have you noticed that talks on "sides" of the triangle are really on circles of rho-eta plane?

Yes, and not really

And doesn't it bother you that a conformal map w=f(z) could change these shapes (preserving areas and relative angles at intersections!)

No, and what's the point?

And why do we call "angles" the wedges in that plane?

C'mon, give me a break, we know (and alpha is not a wedge)

And isn't it a disservice to all to exclude K physics data even if it is not wedges and circles? Shouldn't we include it precisely because it is not wedges and circles?

Well, yes, we have been self-centered and arrogant...

So what do we learn from measuring V_{CKM} precisely? How precise should we aim at measuring it?

This is neither rhetorical nor simply killing time:
Governments are trusting us with (US)-billions of dollars to do determine V_{cb} to three significant figures. Why would anyone care?: Explain this to your son's grade school's science teacher (let alone the history teacher).

Three yardsticks:

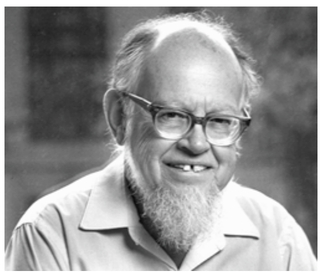
1. Can we exclude a fourth generation?
2. Can we exclude/limit new TeV physics?
3. Can we exclude models of CKM (limit the landscape)?

I will dispose of 3 quickly. By models of CKM I mean models that predict mass matrices of quarks/leptons at some short distance scale, typically using some discrete symmetries to give matrices a "texture." To the extent that there is no grand principle behind these models, model-builders have the freedom to tweak their models to ever more closely reproduce the CKM. No real yardstick here.

yardstick1:

Testing unitarity (or fourth generation) can give us an idea of what to aim for, as follows. (BTW, I know Z-width implies only 3 light neutrinos)

Wolfenstein reminds us of the texture of the CKM matrix



$$V_{\mathrm{CKM}}^{(3)} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

To see what to expect, guess what would go in fourth row and column.

Guess #1

$$V_{\mathrm{CKM}}^{(4)} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 & \lambda^5 \\ \lambda & 1 & \lambda^2 & \lambda^4 \\ \lambda^3 & \lambda^2 & 1 & \lambda^2 \\ \lambda^5 & \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

Then expect,

$$V_{\rm CKM}^{(4)} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 & \lambda^5 \\ \lambda & 1 & \lambda^2 & \lambda^4 \\ \lambda^3 & \lambda^2 & 1 & \lambda^2 \\ \lambda^5 & \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$1-\left(|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2\right)\sim\lambda^{10}\sim 3\times10^{-7}$$

$$1-\left(|V_{cd}|^2+|V_{cs}|^2+|V_{cb}|^2\right)\sim\lambda^{8}\sim 5\times10^{-6}$$

$$\underline{1-\left(|V_{td}|^2+|V_{ts}|^2+|V_{tb}|^2\right)\sim\lambda^{4}\sim 2\times10^{-3}}$$

According to PDG

$$\sum_{i=d,s,b}|V_{ui}|^2 = 0.9992\pm0.0011$$ first row

$$\sum_{i=d,s,b}|V_{ui}|^2 = 0.968\pm0.181$$ second row

$$\sum_{i=u,c,t}|V_{id}|^2 = 1.001\pm0.005$$ first column

$$\sum_{u,c,d,s,b}|V_{ij}|^2 = 2.002\pm0.027$$ first + second rows (W-decay)

Not good enough! (Postpone Q: do you believe this?)
Shouldn't we

i. concentrate on third row !! ?

ii. for first and second concentrate on $|V_{qi}|$ with $i \neq b$!!?

Guess #2

$$V_{\mathrm{CKM}}^{(4)} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 & \lambda^3 \\ \lambda & 1 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \cos\theta_G & \sin\theta_G \\ \lambda^3 & \lambda^2 & -\sin\theta_G & \cos\theta_G \end{pmatrix}$$

This is just as in "Guess #1" but allowing large mixing in third-fourth generation. It is allowed by data! PDG gives

$$|V_{tb}| > 0.78 \quad \text{and} \quad |V_{tb}| = 0.77^{+0.18}_{-0.24}$$

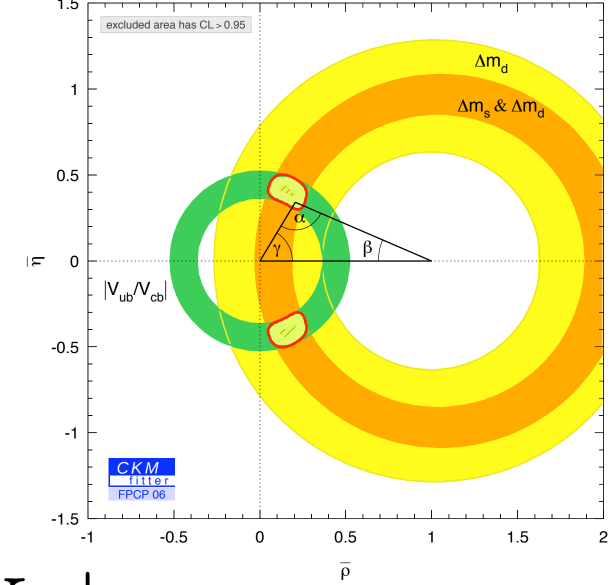
First and second rows still demanding, but row three could be within reach.

Note: If you don't like $\cos\theta_G$ and $\sin\theta_G$ replace by 1 and λ

Now the aimed accuracy is as small as it could possibly be, in all rows.
Lesson: for test of unitarity of CKM to be meaningful need to be sensitivity of order of smallest entry in each of rows/columns 1 and 2.

Should think hard on direct determination of third row elements

Remarkably unexplored.



For example, given

$$a = |V_{td}V_{ts}|,\ \ b = |V_{td}V_{tb}|,\ \ c = |V_{ts}V_{tb}|$$

which could be extracted from mixing of B and K mesons (it is a that is harder, requires more than mixing), one can determine (directly, no assumption of unitarity):

$$|V_{td}| = \frac{ab}{c}, \quad |V_{ts}| = \frac{ac}{b}, \quad |V_{tb}| = \frac{bc}{a}$$

Note: if anyone here knows if this has been proposed/done please let me know

Beware that "Guess #1-2" are subjective prejudices

Amusing numerology

$$A \approx 1 - \lambda, \qquad \bar{\rho} \approx \lambda, \qquad \bar{\eta} \approx \frac{3}{2}\lambda$$

More seriously, if we want Wolfenstein to suggest orders of magnitude, with A, ρ, η of order 1, perhaps we should write

$$V_{\rm CKM}^{(3)} \sim \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$V_{\rm CKM}^{(3)} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A(1-\lambda)\lambda^4(\bar{\rho}-i\bar{\eta}) \\ -\lambda & 1-\lambda^2/2 & A(1-\lambda)\lambda^2 \\ A(1-\lambda)\lambda^4(1-\bar{\rho}-i\bar{\eta}) & -A(1-\lambda)\lambda^2 & 1 \end{pmatrix}$$

yardstick 2: Can we exclude/limit new TeV physics?
Q: how precise do we need V_{CKM} to distinguish CKM from new physics at TeV scale?

A1:

$$\mathcal{A} = \mathcal{A}_{\rm SM} + \mathcal{A}_{\rm New}$$

$$\mathcal{A}_{\rm SM} \sim \frac{g^2}{M_W^2} \times {\rm CKM} \qquad\qquad \mathcal{A}_{\rm New} \sim \frac{1}{\Lambda^2}$$

need roughly, at least

$$\frac{\delta({\rm CKM})}{\rm CKM} \sim \frac{1}{\rm CKM}\frac{1/\Lambda^2}{g^2/M_W^2} \sim \frac{1}{\rm CKM}\frac{v^2}{\Lambda^2} \sim 1\% \times \left(\frac{0.03}{\rm CKM}\right)\left(\frac{10\ {\rm TeV}}{\Lambda}\right)^2$$

A2: Use process which are at least one loop in SM
e.g., Flavor Changing Neutral Currents (FCNC)

Restate answer "A1:"
determination of CKM through SM-tree level process does not get New Physics contamination (to 1% accuracy)

Now

$$\mathcal{A}_{\rm SM} \sim \frac{\alpha}{4\pi \sin^2\theta_w}\frac{g^2}{M_W^2} \times {\rm CKM} \qquad \mathcal{A}_{\rm New} \sim \frac{1}{\Lambda^2}$$

$$\frac{\delta({\rm CKM})}{\rm CKM} \sim \frac{1}{\rm CKM}\frac{1/\Lambda^2}{\alpha/(4\pi\sin^2\theta_w)(1/v^2)} \sim 400\% \times \left(\frac{0.03}{\rm CKM}\right)\left(\frac{10\ {\rm TeV}}{\Lambda}\right)^2$$

Don't even need ~10% (tree level) determination of CKMs to be sensitive to new physics from 10 TeV scale, if we use FCNCs as probes!!

Ready, Aim, Fire...

Sides determination

Could we do better?

1. $|V_{td}|$ (B_d/B_s mixing)
2. $|V_{ub}|$ ($B \to X_u \, \mu\nu$)
3. $|V_{cb}|$ ($B \to X_c \, \mu\nu$)

Inclusive+Exclusive
More emphasis on exclusives
(tend to get neglected)

$|V_{cb}|$ ain't a circle. Needed for extraction of z

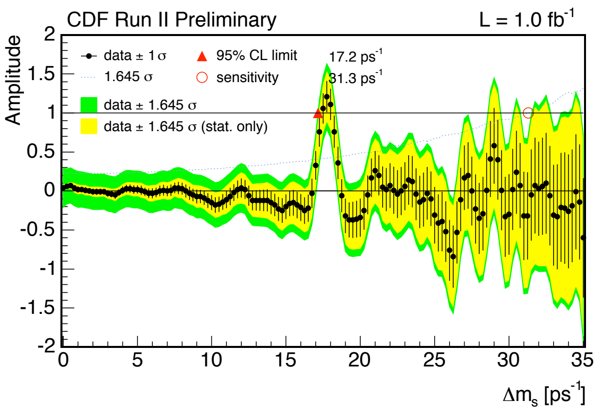
$$|z| = \frac{|V_{ub}|}{|V_{cb}V_{us}|}$$

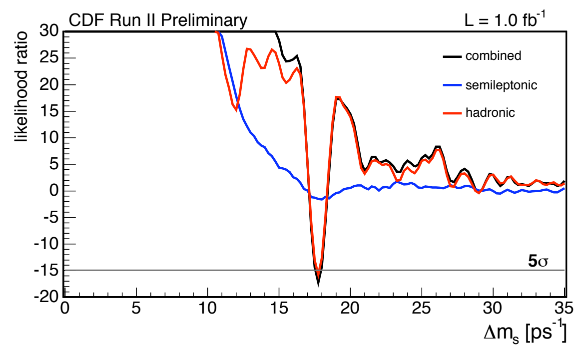
Similarly, $|V_{us}|$ ($K \to \pi e\nu$) needed, but not covered here.
And, of course, should check rest (like magical 1-2% precision in exclusive D decays).

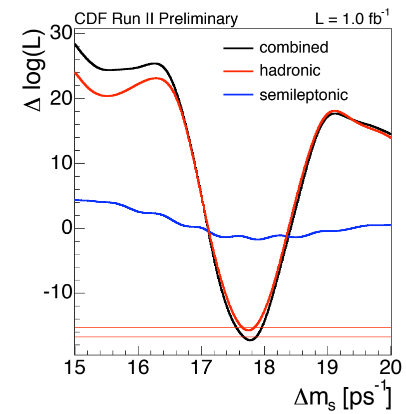
Won't give a compendium of latest numbers (quote only when tension)

$|V_{td}|$

This year's sweetheart







Theory: $$\frac{|V_{td}|}{|V_{ts}|} = \xi \sqrt{\frac{\Delta m_s \; m_{B_s}}{\Delta m_d \; m_{B_d}}} \qquad \xi^2 = \frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2}$$

Lattice: $$\xi = 1.21\,^{+0.047}_{-0.035}$$

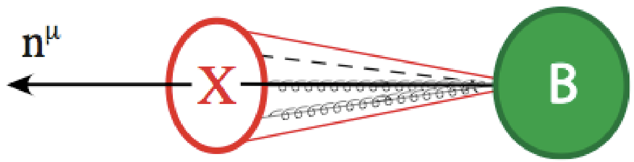
I'll believe a 3% lattice theory error when the lattice has produced one successful prediction and several 3% postdictions
However, here the calculation is really of ξ^2-1, and th error is 16%
Chiral lag gives only chiral logs, so error in $\xi^2-1\approx 1.3$ is 100%

$$\boxed{\frac{|V_{td}|}{|V_{ts}|} = 0.2060 \pm 0.0007(\mathrm{exp})\,^{+0.0081}_{-0.0060}(\mathrm{theory})}$$

Rating: Experiment ★★★★★
Theory ★★ (it's a factor of 10 behind experiment and only one method)

apologies to I.S.

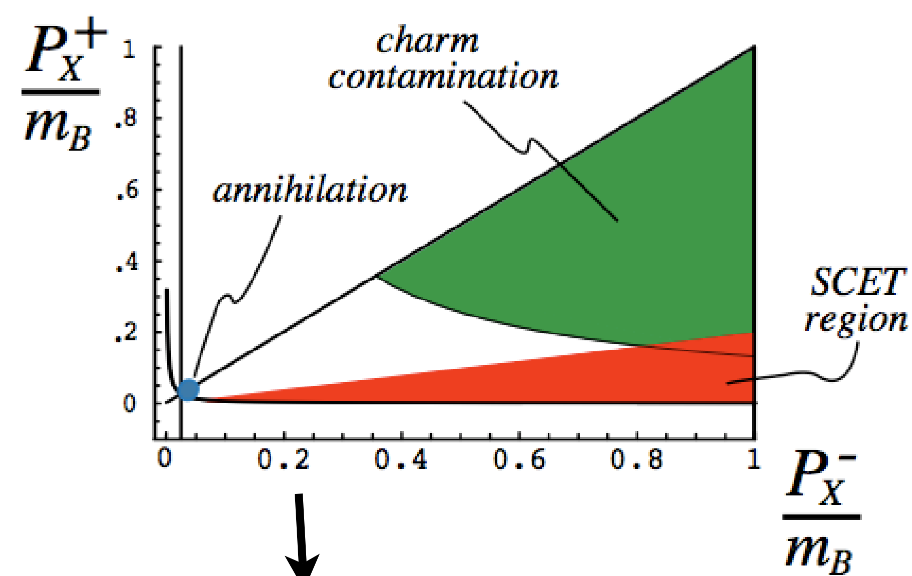
$|V_{ub}|$ inclusive



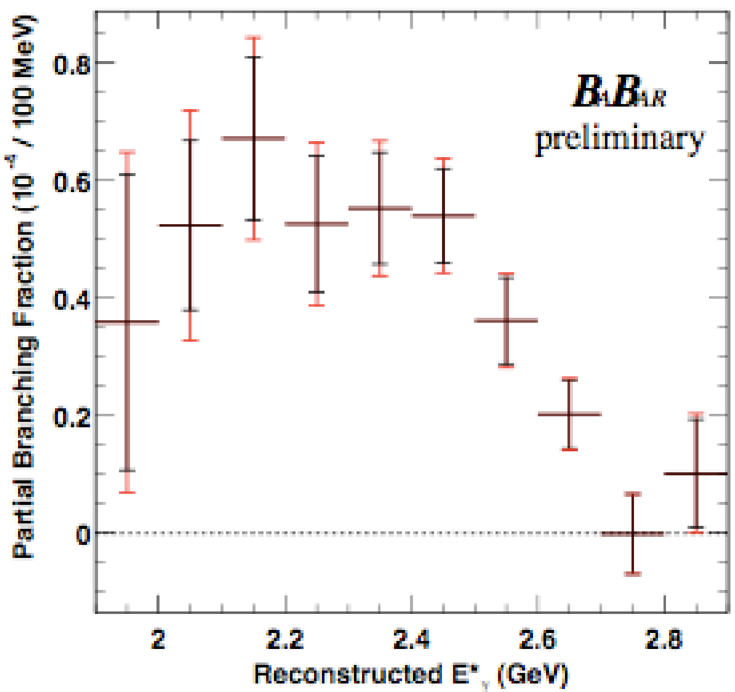
$m_X^2 \sim m_b \Lambda$
$P_X^- \gg P_X^+$

factorization
SCET

$B \to X_u e \bar{\nu}$



$B \to X_s \gamma$



not fully inclusive
need non-pert from exp

the problem is

$$W_i^{(2)} =$$

$$\boxed{\begin{aligned}
& \frac{h_i^{0f}(\bar{n}\cdot p)}{2m_b} \int_0^{p_X^+} dk^+\, \mathcal{J}^{(0)}(\bar{n}\cdot p\, k^+,\mu)\, f_0^{(2)}(k^+ + r^+,\mu) \\
+ & \sum_{r=1}^{2} \frac{h_i^{rf}(\bar{n}\cdot p)}{m_b} \int_0^{p_X^+} dk^+\, \mathcal{J}^{(0)}(\bar{n}\cdot p\, k^+,\mu)\, f_r^{(2)}(k^+ + r^+,\mu) \\
+ & \sum_{r=3}^{4} \frac{h_i^{rf}(\bar{n}\cdot p)}{m_b} \int dk_1^+\, dk_2^+\, \mathcal{J}_{1\pm 2}^{(-2)}(\bar{n}\cdot p\, k_j^+,\mu)\, f_r^{(4)}(k_j^+ + r^+,\mu) \\
+ & \sum_{r=5}^{6} \frac{h_i^{rf}(\bar{n}\cdot p)}{\bar{n}\cdot p} \int dk_1^+ dk_2^+ dk_3^+\, \mathcal{J}_1^{(-4)}(\bar{n}\cdot p\, k_{j'}^+,\mu)\, f_r^{(6)}(k_{j'}^+ + r^+,\mu)
\end{aligned}}$$

$$\begin{aligned}
+ & \frac{h_i^{00f}(\bar{n}\cdot p)}{m_b} \int_0^{p_X^+} dk^+\, \mathcal{J}^{(0)}(\bar{n}\cdot p\, k^+,\mu)\, g_0^{(2)}(k^+ + r^+,\mu) \\
+ & \sum_{r=3}^{4} \frac{h_i^{rf}(\bar{n}\cdot p)}{m_b} \int dk_1^+\, dk_2^+\, \mathcal{J}_{3\pm 4}^{(-2)}(\bar{n}\cdot p\, k_j^+,\mu)\, g_r^{(4)}(k_j^+ + r^+,\mu) \\
+ & \sum_{r=5}^{6} \frac{h_i^{rf}(\bar{n}\cdot p)}{\bar{n}\cdot p} \int dk_1^+ dk_2^+ dk_3^+\, \mathcal{J}_2^{(-4)}(\bar{n}\cdot p\, k_{j'}^+,\mu)\, g_r^{(6)}(k_{j'}^+ + r^+,\mu) \\
+ & \sum_{r=7}^{8} \frac{h_i^{rf}(\bar{n}\cdot p)}{\bar{n}\cdot p} \int dk_1^+ dk_2^+ dk_3^+\, \big[\mathcal{J}_3^{(-4)}(\bar{n}\cdot p\, k_{j'}^+,\mu)\, g_r^{(6)}(k_{j'}^+ + r^+,\mu) \\
& \qquad\qquad + \mathcal{J}_4^{(-4)}(\bar{n}\cdot p\, k_{j'}^+,\mu)\, g_{r+2}^{(6)}(k_{j'}^+ + r^+,\mu)\big] \\
+ & \sum_{m=1,2} \int dz_1 dz_2\, \frac{h_i^{[2b]m+8}(z_1,z_2,\bar{n}\cdot p)}{m_b} \int_0^{p_X^+} dk^+\, \mathcal{J}_m^{(2)}(z_1,z_2,p_X^- k^+)\, f^{(0)}(k^+ + \bar{\Lambda} - p_X^+) \\
+ & \sum_{m=3,4} \frac{h_i^{[2c]m+8}(\bar{n}\cdot p)}{m_b} \int_0^{p_X^+} dk^+\, \mathcal{J}_m^{(2)}(p_X^- k^+)\, f^{(0)}(k^+ + \bar{\Lambda} - p_X^+) \\
+ & \sum_{m=5}^{10} \int dz_1\, \frac{h_i^{[2c]m+8}(z_1,\bar{n}\cdot p)}{m_b} \int_0^{p_X^+} dk^+\, \mathcal{J}_m^{(2)}(z_1,p_X^- k^+)\, f^{(0)}(k^+ + \bar{\Lambda} - p_X^+) \\
+ & W_i^{[2La]f}[g_{11,12}^{(2)}] + W_i^{[2Lb]f}[g_{13,14}^{(2)}] + W_i^{[2LL]f}[g_{15-26}^{(4)}] + W_i^{[2Ga]f}[f_{3,4}^{(4)}]
\end{aligned}$$

+ phase space & kinematic corrections

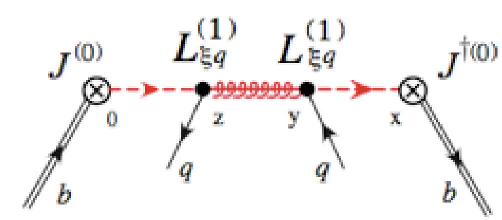
(triple differential spectra)

$h_i(\bar{n}\cdot p)$: $\alpha_s(m_b^2)$

$\mathcal{J}(\bar{n}\cdot pk_j^+)$: $\alpha_s(m_X^2) \sim \alpha_s(m_b\Lambda)$

A brick wall: $\alpha_s \dfrac{\Lambda}{m_b}$

- keep $\dfrac{\Lambda}{m_b}$ and $4\pi\alpha_s \dfrac{\Lambda}{m_b}$

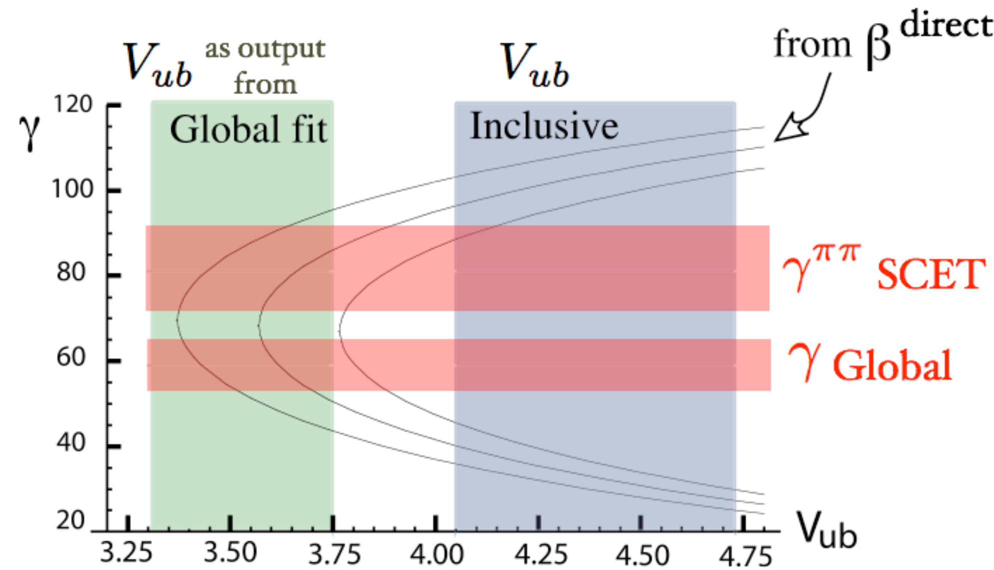


- model these subleading shape functions to get uncertainties

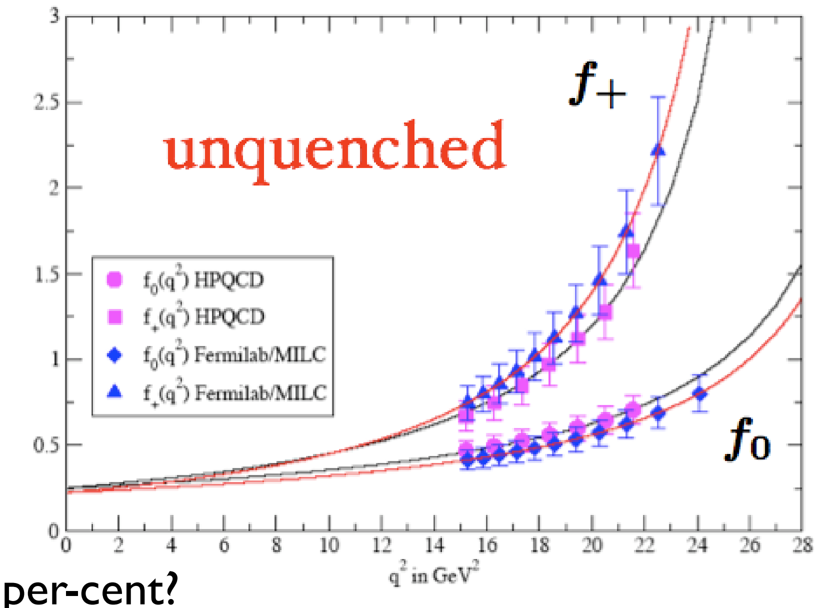
(& interpolate to local OPE)

$|V_{ub}|$ inclusive brown muck

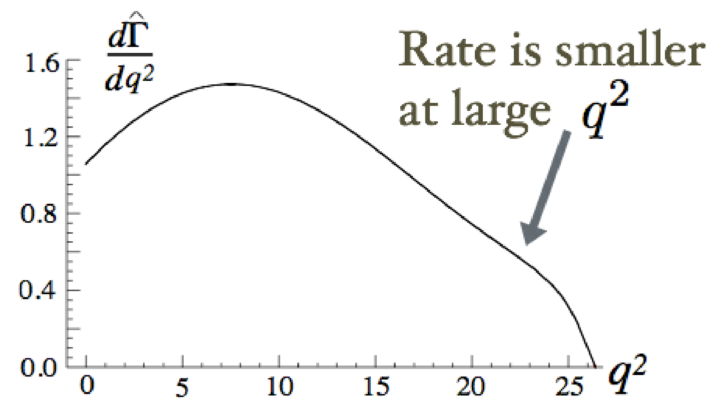
- $\alpha_s(\sqrt{\Lambda m_b})*\Lambda/m_b$ “brick wall”
  - numerics: $\alpha_s(\sqrt{\Lambda m_b})*\Lambda/m_b$ at least 5% but there are ~10 terms so guesstimate $\sqrt{(10)}*5\% = 15\%$
- shape function fit dependence: avoid by using Leibovich, Low, and Rothstein, but slightly larger errors
  (why do we still use parametrized fits???)
- subleading-shape functions
- data



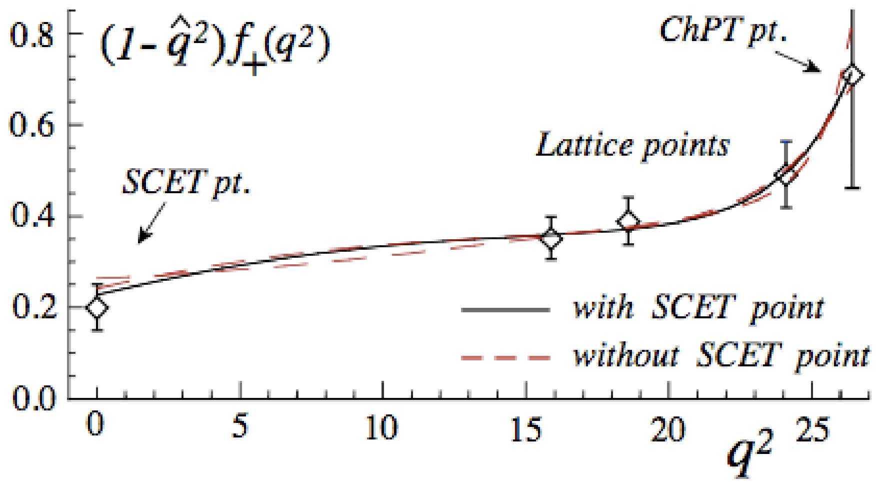
$|V_{ub}|$ exclusive

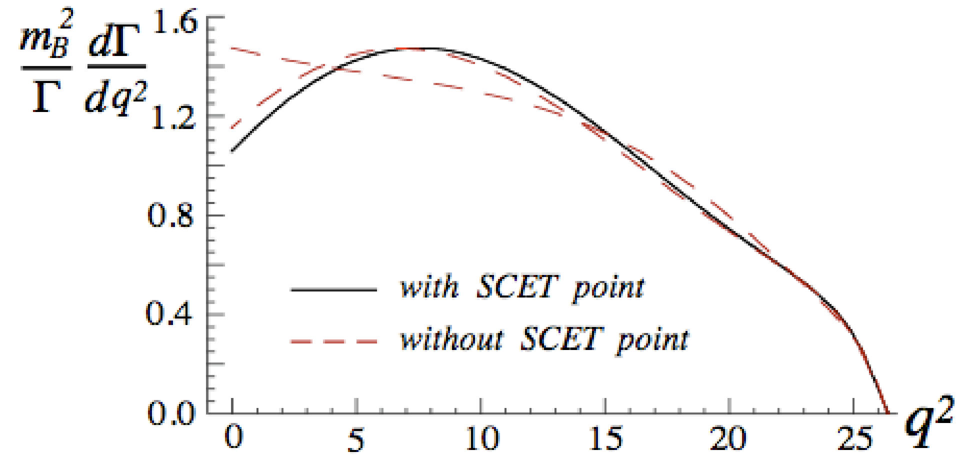


- Br(exp) to 8%, shouldn't we have $|V_{ub}|$ to 4%?
- Normalization of form factor ($f(0)$) from $B \to D\pi$ (SCET)?
  - Will never be better than 10% accurate
- Form factors from lattice: can we trust the lattice to few per-cent?
  - Need a number of successful lattice predictions (vs postdictions)
  - Eventual agreement between lattice groups with full dynamical fermions is not enough
- Lattice only at $q^2 > 16$ GeV^2. Need either
  - high precision experiment at $q^2 > 16$ GeV^2 where rate is smallest (even though ff is largest)
  - theory of shape of form factor
    - models?
    - QCD sum rules: uncontrolled, not good to few %
    - dispersion relations



Dispersion relations + lattice





Type of Error	Variation From	$\delta\lvert V_{ub}\rvert^{q^2}$
Input Points	1-σ correlated errors	$\pm 13\%$
Bounds	F_+ versus F_-	$< 1\%$
m_b^{pole}	4.88 ± 0.40	$< 1\%$
OPE order	2 loop $\rightarrow$ 1 loop	$< 1\%$

$\chi^2/(dof) \sim 1.0$ expt. & theory

$10^3 \times |V_{ub}| = 3.72 \pm 0.52$ FNAL

$10^3 \times |V_{ub}| = 4.11 \pm 0.52$ HPQCD

Error in V_{ub} is ~13% (only 4% experimental)

Challenge: Need third method!

One idea out there: double ratios.
Example of "double ratio:"

- $SU(3)$ flavor symmetry $\Rightarrow$

$$\frac{f_{B_s}}{f_B} = 1 \qquad \text{and} \qquad \frac{f_{D_s}}{f_D} = 1$$

- Heavy Quark Flavor Symmetry $\Rightarrow$

$$\frac{f_{B_s}}{f_{D_s}} = \sqrt{\frac{m_c}{m_b}} \qquad \text{and} \qquad \frac{f_B}{f_D} = \sqrt{\frac{m_c}{m_b}}$$

- Ratio of ratios[a] ("double ratio")

$$R_1 = \frac{f_{B_s}/f_B}{f_{D_s}/f_D} = \frac{f_{B_s}/f_{D_s}}{f_B/f_D} = 1 + \mathcal{O}\left(m_s\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right)$$

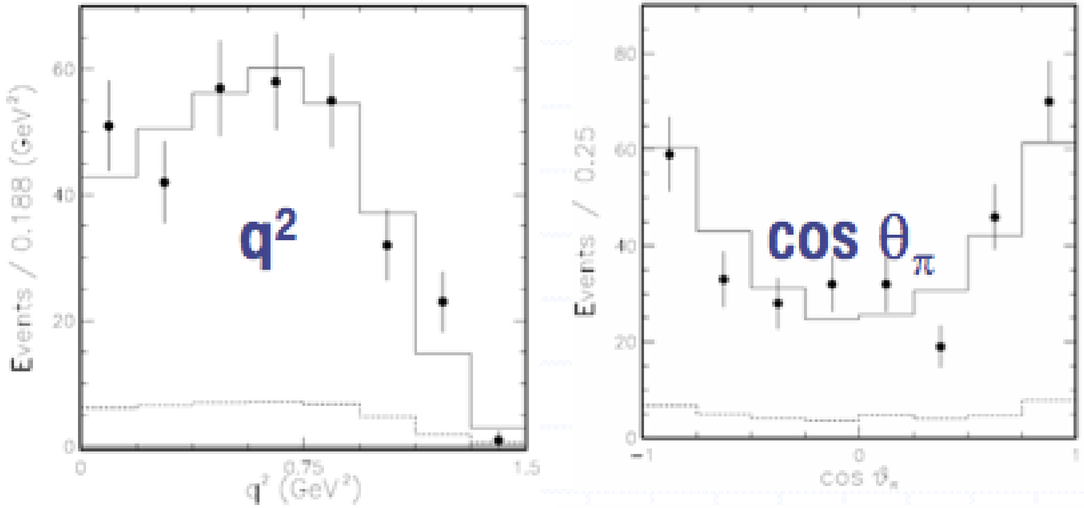
Error is ~ 5%

This is a Unknown Known!

something we don't know
that we know
(although we probably don't
not know it well enough, yet)

$\boxed{D \to \rho \ell \nu}$

CLEO



1. Measure, for q^2 above charmonium resonance region

$$\frac{\mathrm{d}\Gamma(\bar{B}\to\rho e\nu)/\mathrm{d}q^2}{\mathrm{d}\Gamma(\bar{B}\to K^*\ell^+\ell^-)/\mathrm{d}q^2} = \frac{|V_{ub}|^2}{|V_{tb}V_{ts}^*|^2}\cdot\frac{8\pi^2}{\alpha^2}\cdot\frac{1}{|C_9^{\rm eff}(1+\delta(q^2))|^2+|C_{10}|^2}\,\frac{\sum_\lambda |H_\lambda^{B\to\rho}(q^2)|^2}{\sum_\lambda |H_\lambda^{B\to K^*}(q^2)|^2}$$

2. Measure decays spectra for $D\to\rho\ell\nu$ and $D\to K^*\ell\nu$
3. Express all as functions of $y = E_V/m_V$ $(V=\rho, K^*)$
4. Use double-ratio. Let

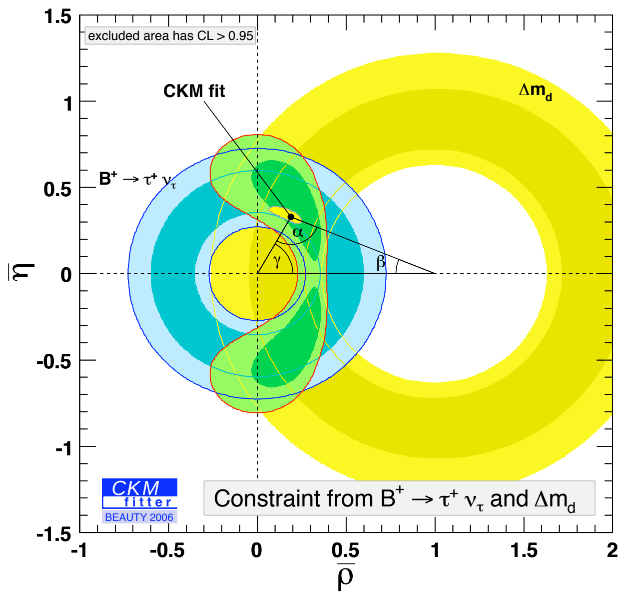
$$R_{B\to V}(y) \equiv \frac{\sum_\lambda |H_\lambda^{B\to\rho}(y)|^2}{\sum_\lambda |H_\lambda^{B\to K^*}(y)|^2} \qquad R_{D\to V}(y) \equiv \frac{\sum_\lambda |H_\lambda^{D\to\rho}(y)|^2}{\sum_\lambda |H_\lambda^{D\to K^*}(y)|^2}$$

then

$$R_{B\to V}(y) = R_{D\to V}(y)\left(1+\mathcal{O}(m_s(\frac{1}{m_c}-\frac{1}{m_b}))\right)$$

5. Given $N_{\rm eff}(q^2) = |C_9^{\rm eff}(1+\delta(q^2))|^2+|C_{10}|^2$ obtain $|V_{ub}|^2/|V_{tb}V_{ts}^*|^2$

Know how to do this. Not known (not done).



Fourth method:

$$\mathrm{Br}(B_u \to \tau\nu) \sim |V_{ub}|^2 f_{B_u}^2$$

$$\mathcal{B}(B^+ \to \tau^+\nu_\tau) = (0.88^{+0.68}_{-0.67}(\mathrm{stat.}) \pm 0.11(\mathrm{syst.})) \times 10^{-4},$$ BaBar

Can you trust the lattice for f_B ?

Could also use double ratio here

$$\frac{\dfrac{\Gamma(B_u \to \tau\nu)}{\Gamma(B_s \to \ell\bar\ell)}}{\dfrac{\Gamma(D_d \to \ell\nu)}{\Gamma(D_s \to \ell\nu)}} \sim \left(\frac{f_{B_u}/f_{B_s}}{f_{D_d}/f_{D_s}}\right)^2 \frac{|V_{ub}|^2}{|V_{ts}V_{tb}|^2}$$

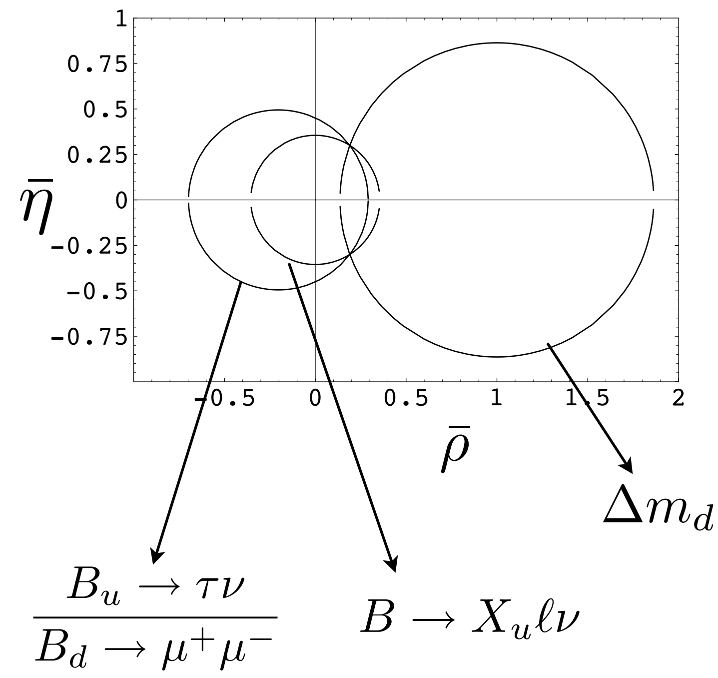
Not an unknown known (yet!):

$$Br(B_s \to \mu^+\mu^-) = 3.5 \cdot 10^{-9} \left[\frac{\tau(B_s)}{1.6\mathrm{ps}}\right] \left[\frac{F_{B_s}}{210\,\mathrm{MeV}}\right]^2 \left[\frac{|V_{ts}|}{0.040}\right]^2 \left[\frac{\overline{m}_t(m_t)}{170\,\mathrm{GeV}}\right]^{3.12}$$

$$Br(B_s \to \mu^+\mu^-) < 2.3 \times 10^{-8} \qquad (\mathrm{CDF})$$

Fifth method??:

$$\frac{\mathrm{Br}(B_u \to \tau\nu)}{\mathrm{Br}(B_d \to \mu^+\mu^-)} \sim \frac{f_{B_u}^2}{f_{B_d}^2} \frac{|V_{ub}|^2}{|V_{td}V_{tb}|^2}$$



(Almost) no hadronic uncertainty!
(use only isospin symmetry)

Unusual circle (centered at ~ (–0.2,0), radius ~ 0.5)

A challenge for experiment (seems impossible)

Sixth method????:

$$\bar{B}_{d,s} \to \bar{D}^0 X \qquad (b\bar{q} \to u\bar{c})$$

Wrong charm decays:

-Exclusive: interesting connection to $B_{d,s}$ mixing matrix elements (lattice check)
-Inclusive: challenge for experiment?

As it happens
I often hear
"We know $|V_{ub}|$ to 4% ..."

Which reminds me of ...

Happenings

You're going to be told lots of things.
You get told things every day that don't happen.

It doesn't seem to bother people, they don't—
It's printed in the press.
The world thinks all these things happen.
They never happened.

Everyone's so eager to get the story
Before in fact the story's there
That the world is constantly being fed
Things that haven't happened.

All I can tell you is,
It hasn't happened.
It's going to happen.

Donald Rumsfeld—Feb. 28, 2003, DoD briefing

$|V_{cb}|$ inclusive - moments

well known

$$\frac{\mathrm{d}\Gamma(B\to X_c\ell\nu)}{\mathrm{d}x\mathrm{d}y} = |V_{cb}|^2 f(x,y)$$

In full QCD rate given in terms of four parameters: $|V_{cb}|^2, \alpha_s, m_b, m_c$

drops out of normalized moments

fix by moments

Get $|V_{cb}|$ from rate

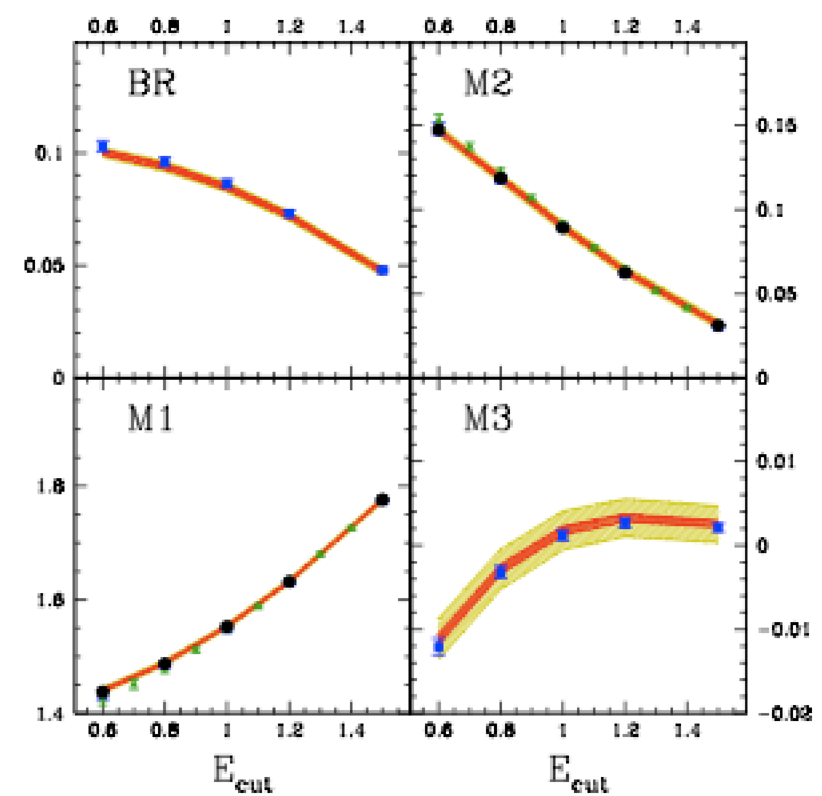
Problem: can't get f(x,y) in QCD
Solution: Use $1/m_b$ expansion (ie Λ/m_b)

$$\begin{array}{ccccc} \Gamma_0+ & \Gamma_1+ & \Gamma_2+ & \Gamma_3+ & \ldots \\ \downarrow & \downarrow & \downarrow & \downarrow & \\ m_b & & \lambda_1 & \mathcal{T}_{1,2,3},\rho_1 & \end{array}$$

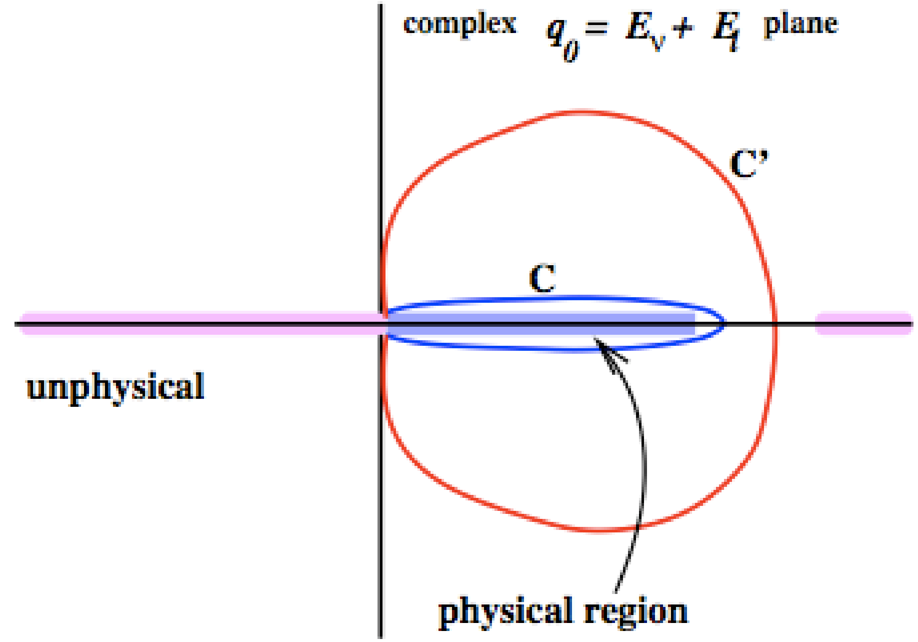
$\leftarrow$ fit these too

Error: 2%, Understand without magic:
use last term used in expansion to estimate

$$\beta_0\alpha_s^2,\ \alpha_s\Lambda_{\rm QCD}/m_b,\ (\Lambda_{\rm QCD}/m_b)^3$$



desperately looking for unknown unknowns?



possible fly in the ointment: duality

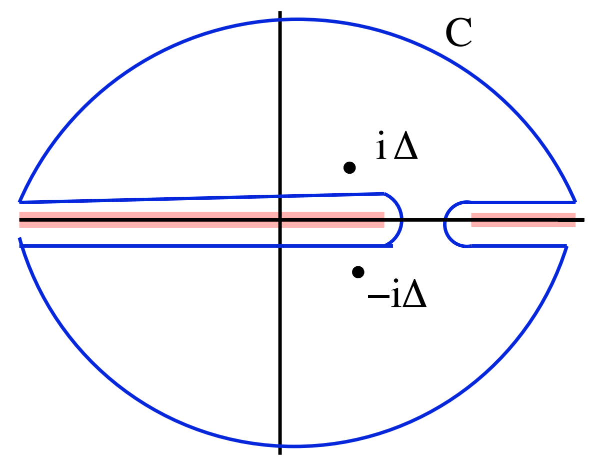
rate/moments from dispersion relation

$$\frac{d\Gamma}{dq^2} = \mathrm{const} \times \int dq_0 \;\; L^{\mu\nu}\, \mathrm{Im} T_{\mu\nu}$$

$$= \mathrm{const} \times \oint_C dq_0 \;\; L^{\mu\nu}\, T_{\mu\nu}$$

this could be studied by considering a contour around whole cut and introducing a pole

$$\oint_C dq_0 \;\; \frac{L^{\mu\nu} T_{\mu\nu}}{(q_0 - M)^2 + \Delta^2}$$



but we don't study this because ...

Note: duality works in SV limit, explicitly (to order $1/m^2$)

$|V_{cb}|$ exclusive

- Good to confirm inclusive
- HQET-inspired parametrization

$$\frac{\mathrm{d}\Gamma(B\to D^*\ell\bar\nu)}{\mathrm{d}w} = \frac{G_F^2 m_B^5}{48\pi^3} r_*^3 (1-r_*)^2 \sqrt{w^2-1}\,(w+1)^2 \times \left[1+\frac{4w}{1+w}\frac{1-2wr_*+r_*^2}{(1-r_*)^2}\right] |V_{cb}|^2 \mathcal{F}_*{}^2(w)$$

$$\frac{\mathrm{d}\Gamma(B\to D\ell\bar\nu)}{\mathrm{d}w} = \frac{G_F^2 m_B^5}{48\pi^3} r^3 (1+r)^2 (w^2-1)^{3/2} |V_{cb}|^2 \mathcal{F}^2(w)$$

- $\mathcal{F}$, $\mathcal{F}_*$: combination of form factors of $V-A$
- At lowest order in HQET $\mathcal{F}(1)=\mathcal{F}_*(1)=1$
- Luke's Theorem: $\mathcal{F}_*(1)-1=\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_c)^2$ (get from lattice)
- Measure at $w>1$, extrapolate
- Extrapolation uncertainty reduced by theory/dispersion relations

“Good to confirm inclusive” ??

$$|V_{cb}| = 37.6 \pm 0.3 \pm 1.3 \pm 1.5 \times 10^{-3}$$ Exclusive (BABAR Phys.Rev.D74:092004,2006)

$$|V_{cb}| = 41.6 \pm 0.6 \times 10^{-3}$$ Inclusive (PDG)

Form factor tension with theory?

theory

$$\begin{aligned} R_1(w) &= 1.25 - 0.10(w-1) \\ R_2(w) &= 0.81 + 0.09(w-1) \end{aligned}$$

experiment

$$\begin{aligned} R_1 &= 1.396 \pm 0.060 \pm 0.035 \pm 0.027 \\ R_2 &= 0.885 \pm 0.040 \pm 0.022 \pm 0.013 \end{aligned}$$

And, whatever happened to problem with slopes (D* vs D)?

$$\begin{aligned} \rho^2_{\mathcal{F}} - \rho^2_{\mathcal{F}_*} &= 0.203 + 0.053\,\epsilon - 0.013\,\epsilon^2_{\mathrm{BLM}} + 0.075\,\eta(1) + 0.14\,\eta'(1) \\ &\quad + 1.0\,\chi_2(1) - 3.0\,\chi'_3(1) - 0.018\,\lambda_1/\mathrm{GeV}^2 \simeq 0.19 \end{aligned}$$ theory

$$\rho^2_{\mathcal{F}} - \rho^2_{\mathcal{F}_*} \simeq -0.22 \pm 0.20$$ experiment

Opportunity for lattice to show they can postdict quantities to 3% and predict slope difference to 3%.

Conclusions

So what?

Recall "yardstick 2:"

Can we exclude/limit new TeV physics?

or

How precise $V_{\rm CKM}$ to distinguish CKM from new physics at TeV scale?

Recall "A2:"

A ~10% (tree level) determination of CKMs to be sensitive to new physics from 10 TeV scale, if we use FCNCs as probes.

If you believe we know V_{td} and V_{ub} to 3-4% precision, then new physics at the TeV scale is already excluded/limited

(we have not seen any deviations form SM in FCNC processes, like radiative decays)

Recall we had

$$\frac{\delta(\mathrm{CKM})}{\mathrm{CKM}} \sim \frac{1}{\mathrm{CKM}} \frac{1/\Lambda^2}{\alpha/(4\pi \sin^2\theta_w)(1/v^2)} \sim 400\% \times \left(\frac{0.03}{\mathrm{CKM}}\right)\left(\frac{10\ \mathrm{TeV}}{\Lambda}\right)^2$$

Assuming we know the error in CKM
and using realistic CKM for FCNC

$$\Lambda > v \sqrt{\frac{1}{\frac{\delta(\mathrm{CKM})}{\mathrm{CKM}}} \frac{1}{\mathrm{CKM}} \frac{4\pi \sin^2\theta_w}{\alpha}} \sim 10^3\ \mathrm{TeV} \times \left(\frac{10\%}{\frac{\delta(\mathrm{CKM})}{\mathrm{CKM}}}\right)^{\frac{1}{2}} \left(\frac{0.0002}{\mathrm{CKM}}\right)^{\frac{1}{2}}$$

Amazing!
Does this mean LHC is not going to find anything
(at least, not any new physics with flavor)??

Let's consider this with slightly more care:

- Needed: Model independent approach to effects of new physics
- Effective Theory: Buchmuller & Wyler give order $1/\Lambda^2$ terms in H_{eff}
- If Λ is related to EW-breaking, or higgs hierarchy problem, expect $\Lambda \sim 1\text{-}10$ TeV
- With $\Lambda \sim 1\text{-}10$ TeV all is fine, except processes with FCNC
- To avoid FCNC need either small coefficients of operators $C \sim 10^{-(2\text{-}3)}$, or large scale $\Lambda \sim 10^{3\text{-}4}$ TeV
- If $\Lambda \sim 10^{3\text{-}4}$ TeV then no relation to hierarchy (and no LHC stuff!!)
- Only natural explanation (I know) of small C's is Minimal Flavor Violation (MFV)
- MFV gives no observable deviations from CKM (in, eg, rare decays): have we dig our own grave?

I view MFV as the same type of insight as the GIM was 40 years ago.

It is a great accomplishment of 40 years of flavor physics

No space/time to explain MFV here (see several talks in WG6). In a nutshell:

Symmetry Principle which results in the coefficients C (in $H_{\rm eff}$) include automatic CKM suppression in FCNC

$$\Lambda > v\sqrt{\frac{1}{\frac{\delta(\mathrm{CKM})}{\mathrm{CKM}}}\cancel{\frac{1}{\mathrm{CKM}}}\frac{4\pi\sin^2\theta_w}{\alpha}} \sim 10\ \mathrm{TeV} \times \left(\frac{10\%}{\frac{\delta(\mathrm{CKM})}{\mathrm{CKM}}}\right)^{\frac{1}{2}}$$

Many implications (with few caveats). List two:

1. Demands on precision in determination of CKM are again in the few % level

(great: keep pushing!)

2. Assume GUT in addition to MFV

$\Rightarrow \mu \to e$ (LFV) expected at MEG & PRISM

!!!

COBRA(Constant Bending Radius Spectrometer)

The End

A Confession

Once in a while,
I'm standing here, doing something.
And I think,
"What in the world am I doing here?"
It's a big surprise.

Donald Rumsfeld
—May 16, 2001, interview with the New York Times