MFV
(Minimal Flavor Violation)

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References

MFV:

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R.S.Chivukula, H.Georgi, Phys. Lett. B**188**, 99(1987)
A.J.Buras, *et al.*, Phys. Lett. B**500**, 16(2001)
➠G.D'Ambrosio, *et al.*, Nucl. Phys. B**645**, 155(2002)

MFLV
V. Cirigliano *et al,* Nucl.Phys.B**728**, 121(2005); Nucl.Phys.B**752**, 18(2006)

MFV-GUT
R. Barbieri, *et al.*, Phys.Lett. B**338**, 212(1994); Nucl.Phys. B**445**, 219(1995)
V. Cirigliano *et al,* hep-ph/0608123

Outline

- MFV (quarks)
- MFLV (leptons)
- MFV-GUT (quarks + leptons)
- Conclusions and Future

Rationale for MFV

- Effective Field Theory: <u>Model Independent Approach</u>
- Buchmuller & Wyler: order $1/\Lambda^2$ terms in H_{eff}

  Nucl.Phys. **B268**,621(1986)
- If Λ is related to EW-breaking, or higgs hierarchy problem, expect $\Lambda \sim 10^{3\text{-}4}$ GeV
- All fine, except processes with FCNC
- To avoid FCNC need small coefficients $C \sim 10^{-(2\text{-}3)}$, or large scale $\Lambda \sim 10^{6\text{-}7}$ GeV
- If $\Lambda \sim 10^{6\text{-}7}$ GeV then no relation to hierarchy
- Can we make C naturally small?

Minimal Flavor Violation (MFV)

- Quark sector in SM, in absence of masses has large flavor (global) symmetry: $G_F = SU(3)^3 \times U(1)^2$
- Premise: Unique source of flavor braking
- Symmetry is only broken by Yukawa interactions, parametrized by couplings λ_U and λ_D
- MFV: all breaking of G_F must transform as these
- When going to mass eigenstate basis, all mixing is parametrized by CKM and GIM is automatic

MFV by example: consider $K_L \to \pi\nu\nu$

SM: $$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$\mathcal{C}^\ell = \frac{\alpha X(\frac{m_t}{M_W})}{2\pi \sin^2\theta_W} V_{ts}^* V_{td}$$

CKM factor

1 loop factor,

$X \sim 1$

$$V_{\rm CKM} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda(1+iA^2\lambda^4\bar\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2(1+i\lambda^2\bar\eta) & 1 \end{pmatrix} + \mathcal{O}(\lambda^6).$$

$\lambda \simeq 0.22$ $\qquad |V_{ts}V_{td}| \sim A^2\lambda^5$

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \; \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

New physics

$$\mathcal{H}_{\text{eff}} = \frac{1}{\Lambda_F^2} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell_{\text{new}} \bar{s}_L \gamma_\mu d_L \; \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

with $\mathcal{C}^\ell_{\text{new}} \sim 1$

Assume sensitivity to fractional deviation *r* from SM rate:

$$1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{A^2\lambda^5/(16\pi^2)} \right|^2$$

For example, r = 4% gives sensitivity to Λ $\sim 10^6$ GeV

(r = 4% is KOPIO's would-be sensitivity)

- Small factor $A^2\lambda^5$ comes from CKM
- Generalized GIM mechanism (suppress FCNCs)
  - old GIM: smallness of $V_{cs}V_{cd}^*(m_c^2 - m_u^2)/M_W^2$
  - new GIM: smallness of 1-to-3 generation jump

$$|V_{ts}V_{td}^*| = A^2\lambda^5|1 - \bar{\rho} - i\bar{\eta}|$$

- If new physics respects this then the same small CKM factor appears. New estimate

$$1 + r \sim \left|1 + \frac{(M_W/\Lambda)^2}{1/(16\pi^2)}\right|^2$$

And now r = 4% gives sensitivity to $\Lambda \sim 10^{3\text{-}4}$ GeV

How does this work? (what is "new physics respects this"?)

In our example, $K_L \to \pi\nu\bar{\nu}$

Recall, $G_{\rm F}$ breaking from: $\mathcal{L}_{\rm Yuk} = H\bar{q}_L\lambda_U u_R + \tilde{H}\bar{q}_L\lambda_D d_R$

$G_{\rm F}$, using spurion method:

$$q_L \to V_L q_L \qquad \lambda_U \to V_L \lambda_U V_u^\dagger$$
$$u_R \to V_u u_R \qquad \lambda_D \to V_L \lambda_D V_d^\dagger$$
$$d_R \to V_d d_R$$

Effective lagrangian $\mathcal{L}_{\rm eff} = \frac{1}{\Lambda^2}\sum C_i O_i$

where the operator is, for example

$$O = \bar{q}_L(\lambda_U\lambda_U^\dagger)\gamma_\mu q_L\,\bar{\nu}_L\gamma^\mu\nu_L$$

In mass basis $\Rightarrow \left(\sum_{q=u,c,t} V_{qs}^* V_{qd}\frac{m_q^2}{v^2}\right)\bar{s}_L\gamma_\mu d_L\,\bar{\nu}_L\gamma^\mu\nu_L$

Since $|V_{ts}^* V_{td}| m_t^2/v^2 \approx A^2\lambda^5 \approx 5\times 10^{-4}$, effectively $C \sim 10^{-3}$

❄ Various types of processes can be sensitive to the presence of MFV interactions

❄ Characterized by specific operators

❄ Operators built up of basic FCNC bilinears:

$$\bar{q}_L \lambda_U \lambda_U^\dagger q_L \quad \text{and} \quad \bar{d}_R \lambda_D^\dagger \lambda_U \lambda_U^\dagger q_L$$

(recall: $q_L \to V_L q_L$, $u_R \to V_u u_R$, $d_R \to V_d d_R$, $\lambda_U \to V_L \lambda_U V_u^\dagger$, $\lambda_D \to V_L \lambda_D V_d^\dagger$)

Kept at most one power of λ_D and one power of $\lambda_U \lambda_U^\dagger$ (which after going to mass basis gives $(m_t^2/v^2) V_{ti} V_{tj}^*$)

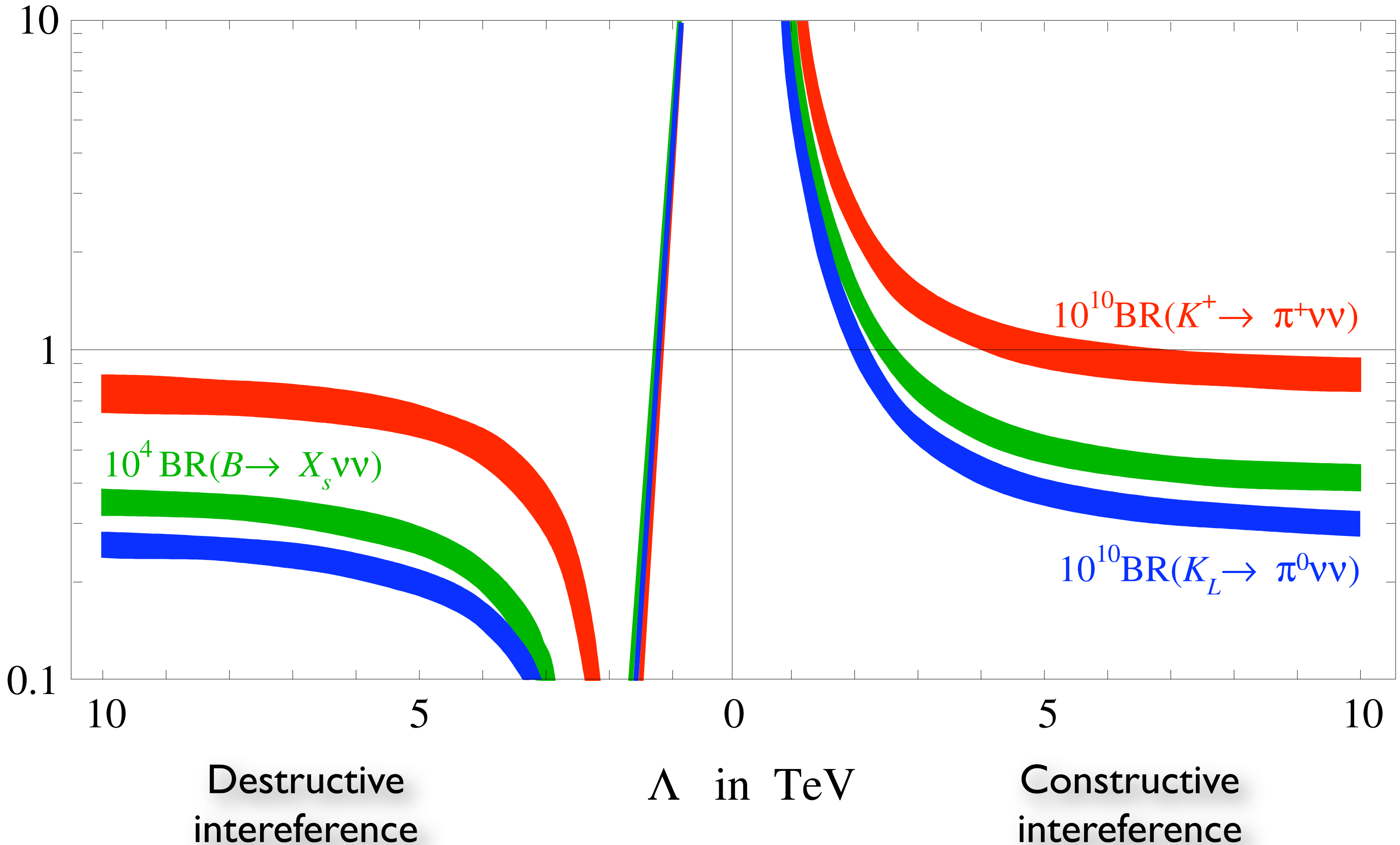
Bounds on Λ (99% CL)

Minimally flavour violating dimension six operator		main observables	Λ [TeV] $-$	$+$
$\mathcal{O}_0 =$	$\frac{1}{2}(\bar{q}_L \lambda_U \lambda_U^\dagger \gamma_\mu q_L)^2$	$\epsilon_K, \quad \Delta m_{B_d}$	6.4	5.0
$\mathcal{O}_{F1} =$	$H^\dagger \left(\bar{d}_R \lambda_D \lambda_U \lambda_U^\dagger \sigma_{\mu\nu} q_L \right) F_{\mu\nu}$	$B \to X_s \gamma$	9.3	12.4
$\mathcal{O}_{G1} =$	$H^\dagger \left(\bar{d}_R \lambda_D \lambda_U \lambda_U^\dagger \sigma_{\mu\nu} T^a q_L \right) G^a_{\mu\nu}$	$B \to X_s \gamma$	2.6	3.5
$\mathcal{O}_{\ell 1} =$	$(\bar{q}_L \lambda_U \lambda_U^\dagger \gamma_\mu q_L)(\bar{L}_L \gamma_\mu L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.1	2.7
$\mathcal{O}_{\ell 2} =$	$(\bar{q}_L \lambda_U \lambda_U^\dagger \gamma_\mu \tau^a q_L)(\bar{L}_L \gamma_\mu \tau^a L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.4	3.0
$\mathcal{O}_{H1} =$	$(\bar{q}_L \lambda_U \lambda_U^\dagger \gamma_\mu q_L)(H^\dagger i D_\mu H)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	1.6	1.6
$\mathcal{O}_{q5} =$	$(\bar{q}_L \lambda_U \lambda_U^\dagger \gamma_\mu q_L)(\bar{d}_R \gamma_\mu d_R)$	$B \to K\pi, \quad \epsilon'/\epsilon, \ldots$	~ 1	

- One operator at a time
- C = 1
- circa 2002, little change, don't expect much (best chance in $\ell\ell$ and $\nu\nu$ modes)

How bounds evolve.

Example: branching fractions $s \to d$ & $b \to s$ (1σ bands)



For $\Lambda \geq 5$ TeV, precision physics

Updated figure courtesy of Gino Isidori

Two Higgs Doublets

- More parameters: $v_2/v_1 = \tan\beta, m_{H^\pm}$
- Interest: Large D-Yukawas (large tan β)
- Flavor Symmetry G_F has extra $U(1)_{PQ}$
- Break $U(1)_{PQ} \Rightarrow$ FCNCs in H^0 tree-level exchange
- H^+ gives radiative FCNCs (e.g., $b \to s\gamma$ "penguin")
- In addition, operators as in one Higgs case

Moral: MFV can be formulated, breaking of $U(1)_{PQ}$ has restricted flavor structure, scale Λ as before

MLFV
(Minimal Lepton Flavor Violation)

- Rationale: elevate MFV to a principle
- If leptons acquire Dirac masses (mimic quark sector) copy from above. But uninteresting: flavor violation proportional to tiny neutrino masses
- More attractive/interesting: Small neutrino masses from see-saw mechanism. Question then: What are the restrictions from MLFV in see-saw models?
- Two cases. Field content (below LNV scale):
  - Minimal: three L_i and e_{Ri}
  - Extended: three L_i, e_{Ri}, and ν_{Ri}

A Note on: LN vs LF

- Distinguish Lepton Number (LN) violating interactions from Lepton Flavor (LF) violating interactions
- LN is a $U(1)$ symmetry, assigning unit charge to all leptons.
  - Neutrino Majorana mass breaks LN
- LF is an $SU(3)$ symmetry, mixing different flavors
  - Commutes with $U(1)_{\text{LN}}$, *i.e.*, preserves LN charge
- No analogue in quark sector: BN not violated

MLFV: Minimal Field Content

Ex: SUSY Triplet Model, A. Rossi, PRD**66**(2002)075003

Assumptions:

1. The breaking of the $U(1)_{\text{LN}}$ is independent from the breaking of lepton flavor G_{LF}, with large Λ_{LN} (associated with see-saw)

2. There are only two irreducible sources of G_{LF} breaking, λ_{e} and g_ν, defined by

$$\mathcal{L}_{\text{Sym.Br.}} = -\lambda_e^{ij}\,\bar{e}_R^i(H^\dagger L_L^j) - \frac{1}{2\Lambda_{LN}}\,g_\nu^{ij}(\bar{L}_L^{ci}\tau_2 H)(H^T\tau_2 L_L^j) + \text{h.c.}$$

3. The scale of LNV is large compared to the scale of LFV, $\Lambda_{\text{LF}} \gg \Lambda_{\text{LN}}$

$$\mathcal{L}_{\text{LFV}} = \frac{1}{\Lambda_{\text{LFV}}^2}\sum C_i O_i$$

Implementation of MLFV in Minimal Field Content Case

- Add all possible terms to the lagrangian consistent with assumptions (also: Lorentz invariance, gauge symmetry, locality, ...)

- Characterize allowed operators by spurion method

$$L_L \to V_L\, L_L \qquad e_R \to V_R\, e_R$$

$$\lambda_e \to V_R\, \lambda_e V_L^\dagger \qquad g_\nu \to V_L^*\; g_\nu V_L^\dagger$$

(recall: $\mathcal{L}_{\rm Sym.Br.} = -\lambda_e^{ij}\, \bar{e}_R^i (H^\dagger L_L^j) - \frac{1}{2\Lambda_{LN}} g_\nu^{ij} (\bar{L}_L^{ci} \tau_2 H)(H^T \tau_2 L_L^j) + {\rm h.c.}$)

- All ops for $\mu \to e\gamma, \quad \mu + N \to e + N'$, have at least one factor of $\Delta \equiv g_\nu^\dagger g_\nu$ (neglect Δ^2); $\quad m_\nu \sim v^2 g_\nu / \Lambda_{LN}$

- Amplitudes are given in terms of
  - Λ_{LN} and Λ_{LFV} (actually only ratio $\Lambda_{LN}/\Lambda_{LFV}$)
  - Coefficients, C, of order 1
  - Low energy measured (or measurable) masses and mixing angles
- In particular, the following two combinations appear in the operators (δ appears in 4ℓ ops, eg, $\mu \to ee\bar{e}$):

$$\Delta = \frac{\Lambda_{LN}^2}{v^4} U m_\nu^2 U^\dagger \qquad \delta = \delta^T = \frac{\Lambda_{LN}}{v^2} U^* m_\nu U^\dagger$$

U = PMNS mixing matrix, m_ν diagonal

MLFV: Extended Field Content

Ex: SUSY with RH degenerate N's, J. Hisano et al, Phys. Rev. D **53**, 2442–2459 (1996)

Recall, now we include RH neutrinos, flavor group has additional $SU(3)_{\nu R}$ factor

Assumptions:

1. The right handed neutrino mass is flavor neutral, *ie*, it breaks $SU(3)_{\nu R}$ to $O(3)_{\nu R}$. Denote $M_\nu^{ij} = M_\nu \delta^{ij}$
2. The right handed neutrino mass is the only source of LN breaking and $M_\nu \gg \Lambda_{LFV}$
3. Remaining LF-symmetry broken only by λ_e and λ_ν defined by

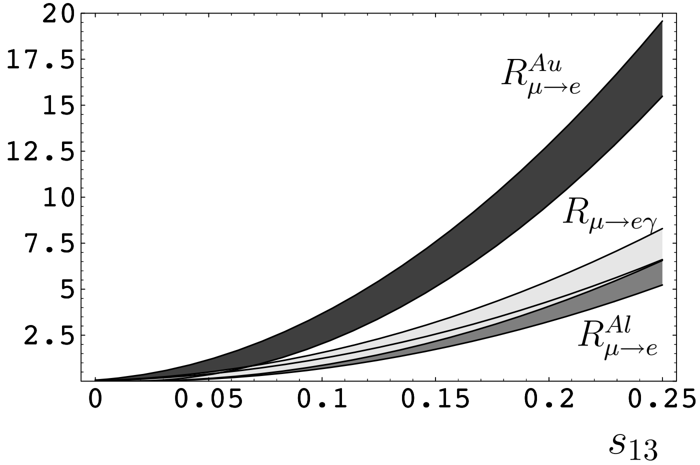
$$\mathcal{L}_{\mathrm{Sym.Br.}} = -\lambda_e^{ij}\, \bar{e}_R^i (H^\dagger L_L^j) + i\lambda_\nu^{ij} \bar{\nu}_R^i (H^T \tau_2 L_L^j) + \mathrm{h.c.}$$

MLFV: Phenomenology

- Future experiments will (continue to) look for flavor changing neutral interactions in the charged lepton sector:
  - MECO ... was cancelled
  - PRIME at the PRISM muon facility at JPARC will measure μ-to-e conversion at 10^{-18} sensitivity
  - MEG at PSI looks for $\mu^+ \rightarrow e^+ \gamma$ at 10^{-13} single event sensitivity

COBRA(Constant Bending Radius Spectrometer)

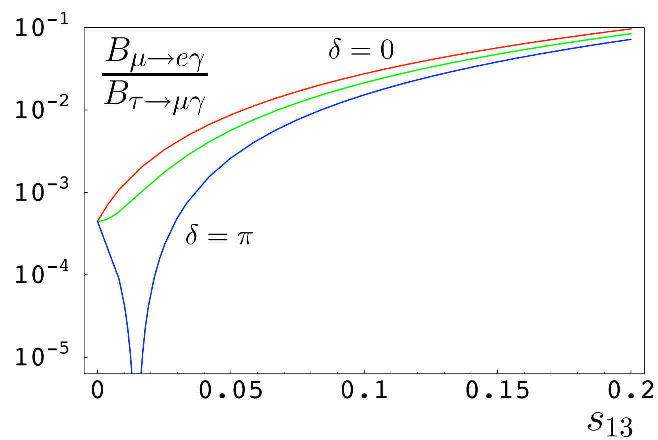
Sample results, minimal field content

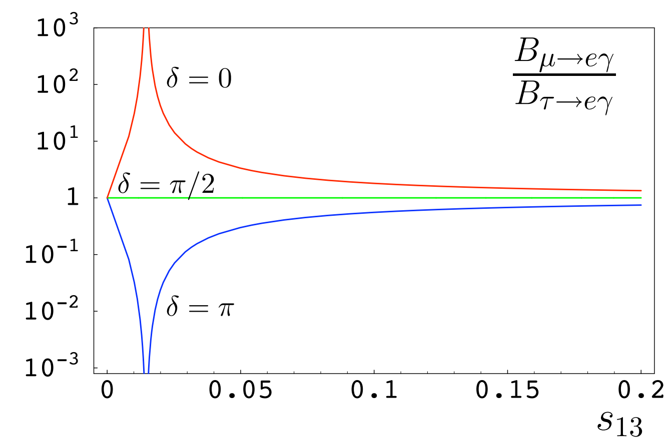


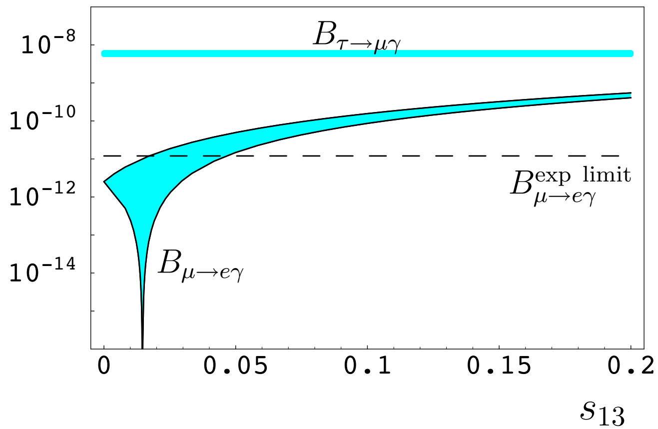
$$B_{\ell_i\to\ell_j(\gamma)} = 10^{-50}\left(\frac{\Lambda_{\rm LN}}{\Lambda_{\rm LFV}}\right)^4 R_{\ell_i\to\ell_j(\gamma)}(s_{13},\delta;c^{(i)})$$

- s_{13} and δ unknown PMNS parameters (scan on δ)
- choose $c^{(i)}$ of order one for the estimate
- ratio of scales can be large
  perturbative $g_\nu \Rightarrow \Lambda_{\rm LN} \lesssim 3\times 10^{13}(1\ {\rm eV}/m_\nu)$ GeV
  so $\Lambda_{\rm LFV}\sim 1$ TeV $\Rightarrow \Lambda_{\rm LN}/\Lambda_{\rm LFV} \lesssim 10^{10}$

Predictive:
patterns are independent
of unknown input parameters







If s_{13} is small, look at tau modes.
Here $\Lambda_{LN}/\Lambda_{LFV} = 10^{10}$ and $c^{(1)}_{RL} - c^{(2)}_{RL} = 1$

MFV-GUT

$m_d,\, m_e$ $\qquad$ m_u

- Quarks/leptons in same reps of $G_{\rm GUT}$, eg., $\psi \sim \bar{5}, \chi \sim 10$
- Flavor symmetry <u>smaller</u>: $SU(3)^2$
- Broken by Yukawas, including trans-PL term for light quark/lepton mass relations $(H \sim 5, \Sigma \sim 24)$

$$\mathcal{L}_{\rm sym.br.} = \psi^T \lambda_5 \chi H^* + \chi^T \lambda_{10} \chi H + \frac{1}{M} \psi^T \lambda_5' \Sigma \chi H^*$$

- Right handed neutrino for see-saw (not mandatory); extends $G_{\rm F}$ to $SU(3)^3$, and

$$\Delta\mathcal{L}_{\rm sym.br.} = N^T \lambda_1 \psi H + N^T M_R N$$

- MFV-GUT: $G_{\rm F}$ broken only by

$$\lambda_5, \lambda_5', \lambda_{10}, \lambda_1, M_R$$

Implementation similar,

$$Q_L \to V_{10}\ Q_L \qquad \lambda_{10} \to V_{10}^*\ \lambda_{10}\ V_{10}^\dagger$$
$$u_R \to V_{10}^*\ u_R \qquad \lambda_5 \to V_{\bar 5}^*\ \lambda_5\ V_{10}^\dagger$$
$$d_R \to V_{\bar 5}^*\ d_R \qquad \lambda_5' \to V_{\bar 5}^*\ \lambda_5'\ V_{10}^\dagger$$
$$L_L \to V_{\bar 5}\ L_L \qquad \lambda_1 \to V_1^*\ \lambda_1\ V_{\bar 5}^\dagger$$
$$e_R \to V_{10}^*\ e_R \qquad M_R \to V_1^*\ M_R\ V_1^\dagger$$

Trade parameters for low energy ones, e.g.,

$$\lambda_u = a_u\left[\lambda_{10} + \epsilon_{1u}\lambda_{10}\lambda_{10}^\dagger\lambda_{10} + \epsilon_{2u}^{('')}\lambda_{10}(\lambda_5^{(')})^\dagger\lambda_5^{(')} + \ldots\right]$$
$$\lambda_d = a_d\left[\left(\lambda_5 + \lambda_5'\right) + \epsilon_{d1}'\lambda_5' + \epsilon_{d2}^{(')}\lambda_5^{(')}\lambda_{10}^\dagger\lambda_{10} + \epsilon_{d3}^{(')}\lambda_1^T\lambda_1^*\lambda_5^{(')} + \ldots\right]$$
$$\lambda_e^T = a_e\left[\left(\lambda_5 - \frac{3}{2}\lambda_5'\right) + \epsilon_{e1}'\lambda_5' + \epsilon_{e2}^{(')}\lambda_5^{(')}\lambda_{10}^\dagger\lambda_{10} + \epsilon_{e3}^{(')}\lambda_1^T\lambda_1^*\lambda_5^{(')} + \ldots\right]$$

Note, no problem with m_e/m_μ vs m_d/m_s, while m_τ/m_b fine, because λ_5' is automatically small

FCNCs: Bilinears as building blocks of operators

Old

quarks: $\bar{Q}_L \lambda_u^\dagger \lambda_u Q_L \,, \qquad \bar{d}_R \lambda_d \lambda_u^\dagger \lambda_u Q_L$

leptons: $\bar{L}_L \lambda_1^\dagger \lambda_1 L_L \,, \qquad \bar{e}_R \lambda_e \lambda_1^\dagger \lambda_1 L_L$

New (plus replace anywhere $\lambda_e \leftrightarrow \lambda_d^T$)

quarks:

$$\bar{Q}_L (\lambda_e \lambda_e^\dagger)^T Q_L$$

$$\bar{d}_R \lambda_e^T (\lambda_e \lambda_e^\dagger)^T Q_L \,, \qquad \bar{d}_R (\lambda_e \lambda_1^\dagger \lambda_1)^T Q_L$$

$$\bar{d}_R (\lambda_e^\dagger \lambda_e)^T d_R \,, \qquad \bar{d}_R (\lambda_1^\dagger \lambda_1)^T d_R$$

leptons:

$$\bar{L}_L (\lambda_d \lambda_d^\dagger)^T L_L$$

$$\bar{e}_R (\lambda_d \lambda_d^\dagger \lambda_d)^T L_L, \qquad \bar{e}_R \lambda_u \lambda_u^\dagger \lambda_d^T L_L$$

$$\bar{e}_R \lambda_u \lambda_u^\dagger e_R \,, \qquad \bar{e}_R (\lambda_d^\dagger \lambda_d)^T e_R$$

More new: leptoquark bilinears

$$\bar{Q}_L \lambda_u^\dagger e_R, \qquad \bar{Q}_L \lambda_u^\dagger \lambda_e e_R, \qquad \bar{Q}_L \lambda_u^\dagger \lambda_d^T e_R$$

giving rise to new 4-fermion operators, e.g.,

$$(\bar{Q}_L \lambda_u^\dagger e_R)\ (\bar{e}_R \lambda_u Q_L)$$

The suppression scale of these exotic FCNC operators is not forced to be the GUT scale by symmetry arguments, although in realistic frameworks (such as Barbieri et al's SUSY) they are usually generated only at the GUT scale.

quick example (probably out of time by now):

$$\tau \to \mu\gamma, \quad \tau \to e\gamma \quad \& \quad \mu \to e\gamma$$

$$\Delta\mathcal{L}_{\text{eff}} = \frac{v}{\Lambda^2}\bar{e}_R \left[c_1 \lambda_e \lambda_1^\dagger \lambda_1 + c_2 \lambda_u \lambda_u^\dagger \lambda_e + c_3 \lambda_u \lambda_u^\dagger \lambda_d^T \right] \sigma^{\mu\nu} e_L F_{\mu\nu}$$

just like MLFV above

Generalizes Barbieri & Hall

New mixing structures

$$C = V_{e_R}^T V_{d_L}$$

$$G = V_{e_L}^T V_{d_R}$$

Independent of M_ν

Hierarchical

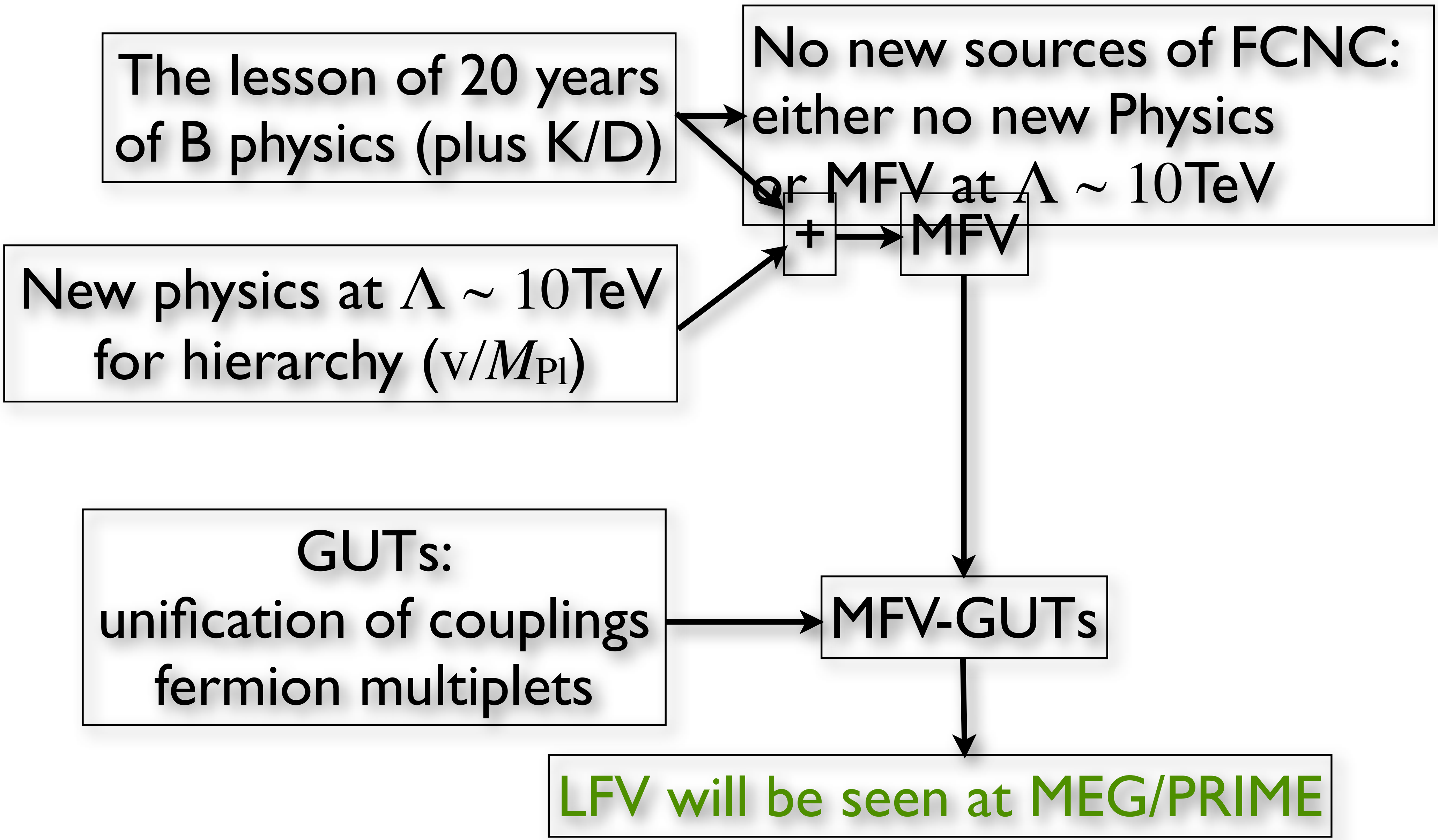
$$\left(\frac{m_t^2}{v^2}\right) \times \begin{cases} \lambda^2 (m_\tau/v), & (\tau \to \mu) \\ \lambda^3 (m_\tau/v), & (\tau \to e) \\ \lambda^5 (m_\mu/v), & (\mu \to e) \end{cases}$$

$$(\lambda = 0.22)$$

Large: for Λ=10TeV

$$\mathrm{Br}(\mu \to e\gamma) \sim 10^{-12}$$

Conclusions and Future



In 20 years:

- the VLHC and NILC will begin to study the new physics at $\Lambda \sim 10$ TeV (maybe sooner, $\Lambda \sim M/g$)

- we will have CKM-PMNS meetings, mostly discussing CPV in neutrino interactions (we will still have talks on SCET, pQCD and QCD factorization)

- we will be entering the era of precision measurement of LFV processes, establishing patterns, e.g.,

$$\tau \to e\bar{e}e : \tau \to e\bar{\mu}e : \tau \to e\bar{e}\mu : \tau \to \mu\bar{\mu}e : \tau \to \mu\bar{\mu}\mu : \mu \to e\bar{e}e$$

  and we will begin to sort out MFV-GUTs

The End