Nagoya, December 2006

Rare K decays within the Standard Model

UNIVERSITÄT BERN

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- Outline

A- Generalities about FCNC and OPE

B- Anatomy of $K \to \pi\nu\bar{\nu}$ *and* $K_L \to \pi^0 \ell^+ \ell^-$

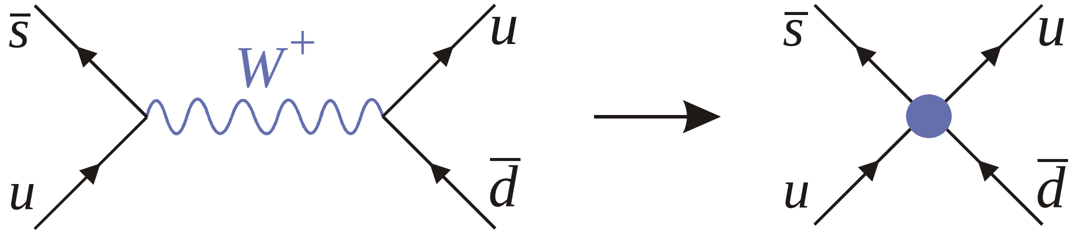
C- A word about $K_L \to \ell^+ \ell^-$

D- Conclusion

FCNC and OPE

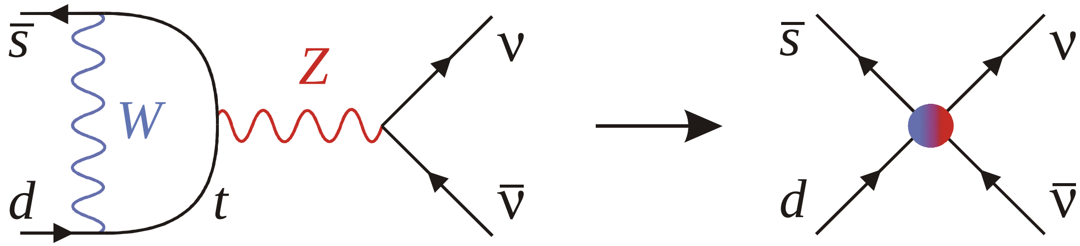
- # Electroweak FCNC

For the *charged current*, the Fermi interaction is obtained by integrating out the W:



$$H_{eff}(\bar{s}u \to \bar{d}u) = \frac{G_F}{\sqrt{2}} \lambda_u \ (\bar{s}u)_{V-A}(\bar{u}d)_{V-A} \qquad\qquad \lambda_q = V_{qd}V_{qs}^*$$

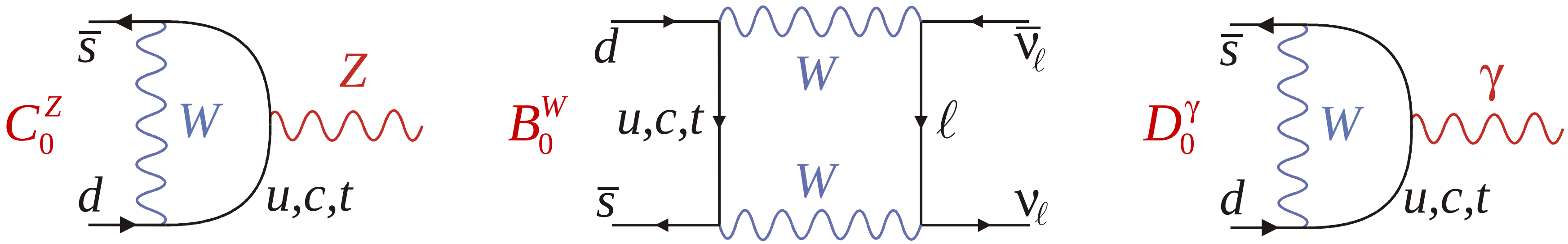
FCNC are generated at one-loop (penguin and box diagrams). Typically:



$$H_{eff}^{Z}(\bar{s}d \to \bar{\nu}\nu) = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \lambda_t C_0^Z(x_t) \ \ (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} \qquad x_q = \frac{m_q^2}{M_W^2}$$

The Inami-Lim function C_0^Z generates a violation of the *GIM mechanism*:

$$\text{if } C_0^Z(x) = C^{st} \quad \Rightarrow \quad \lambda_u C_0^Z(x_u) + \lambda_c C_0^Z(x_c) + \lambda_t C_0^Z(x_t) = 0$$



$$\sqrt{2}K_1 = K^0 - \bar{K}^0,\ \sqrt{2}K_2 = K^0 + \bar{K}^0,\ \langle \pi^0 | (\bar{s}d)_V | K^0 \rangle = -\langle \pi^0 | (\bar{d}s)_V | \bar{K}^0 \rangle$$

If only B_0^W and C_0^Z contribute, light quark effects are suppressed:

$$\langle \pi^0 \nu\bar{\nu} | H_{eff} | K_L \approx K_2 \rangle \sim \mathrm{Im}\,\lambda_u y_u^\nu + \mathrm{Im}\,\lambda_c y_c^\nu + \boxed{\mathrm{Im}\,\lambda_t y_t^\nu}$$

$$\langle \pi^0 \nu\bar{\nu} | H_{eff} | K_S \approx K_1 \rangle \sim \mathrm{Re}\,\lambda_u y_u^\nu + \boxed{\mathrm{Re}\,\lambda_c y_c^\nu + \mathrm{Re}\,\lambda_t y_t^\nu}$$

$$y_q^\nu \supset B_0^W, C_0^Z \sim \frac{m_q^2}{M_W^2}$$

When D_0^γ also contributes, long-distance effects may be significant:

$$\langle \pi^0 \ell^+\ell^- | H_{eff} | K_L \approx K_2 \rangle \sim \mathrm{Im}\,\lambda_u y_u^\ell + \boxed{\mathrm{Im}\,\lambda_c y_c^\ell + \mathrm{Im}\,\lambda_t y_t^\ell}$$

$$\langle \pi^0 \ell^+\ell^- | H_{eff} | K_S \approx K_1 \rangle \sim \boxed{\mathrm{Re}\,\lambda_u y_u^\ell} + \mathrm{Re}\,\lambda_c y_c^\ell + \mathrm{Re}\,\lambda_t y_t^\ell$$

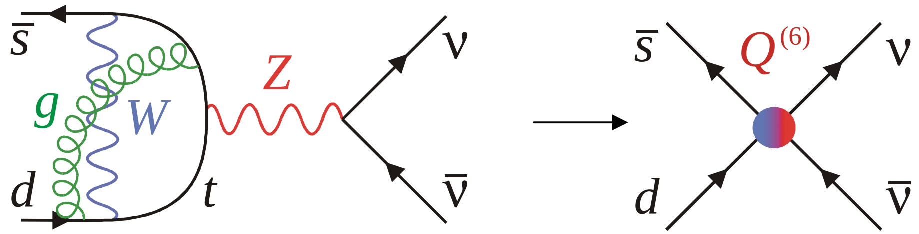
$$y_q^\ell \supset D_0^\gamma \sim \log\left(\frac{m_q}{M_W}\right)$$

Indirect CP-violation: $\langle \pi^0 \nu\bar{\nu}, \pi^0 \ell^+\ell^- | H_{eff} | K_{L(S)} \rangle = \varepsilon \langle \pi^0 \nu\bar{\nu}, \pi^0 \ell^+\ell^- | H_{eff} | K_{1(2)} \rangle$

- QCD corrections

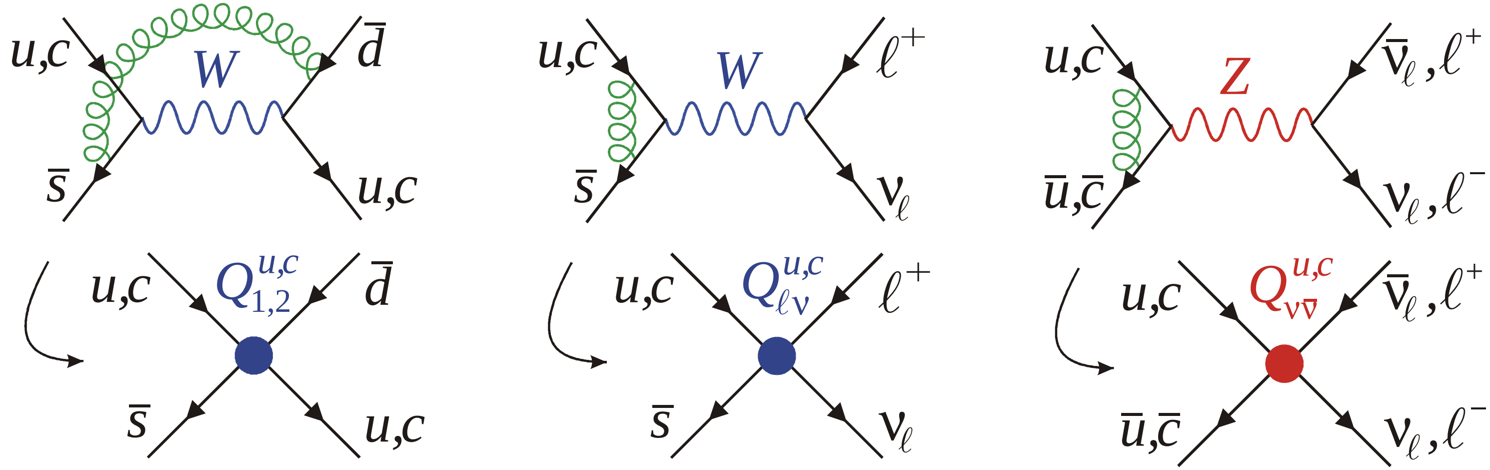
Step 1: integrating out the top, W, Z

Generates local FCNC operators, for example:



$$(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

Generates local Fermi four-fermion operators (all fermions except the top)



...

QCD corrections above M_W are computed perturbatively, and encoded into the Wilson coefficient initial values:

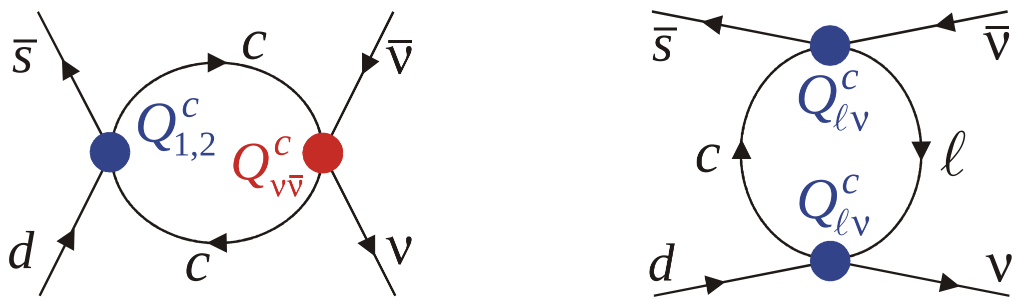
$$H_{eff}(M_W) \sim C_i(M_W)\,Q_i^{u,c} + y_{(6)}^{\nu}(M_W)\,(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

Step 2: crossing the charm quark threshold

QCD corrections are resummed (running down), leading to corrected values for the Wilson coefficients, at lower scales:

$$H_{eff}(m_c) \sim C_i(m_c)\,Q_i^{u,c} + y_{(6)}^{\nu}(m_c)\,(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

Four-fermion operators are combined to integrate out the *c* (similar for b and τ)



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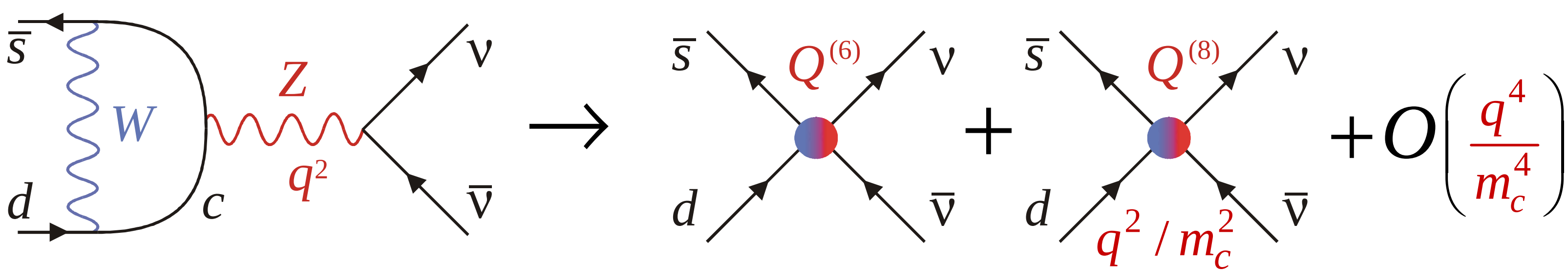
Momentum of external particles (with $q^2 \approx m_K^2$) → Dimension 8, 10,... operators:



$$H_{eff}(m_c) \sim C_i'(m_c)\,Q_i^{u} + y_{(6)}^{\prime\nu}(m_c)\,(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} + y_{(8)}^{\nu}(m_c)\,(\bar{s}d)_{V-A}\,\partial^2(\bar{\nu}\nu)_{V-A} + \dots$$

Step 3: computing matrix elements

$$H_{eff}(\mu) = C_i'(\mu)\,Q_i^u + y_{(6)}^{\prime\nu}(m_c)\,(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} + y_{(8)}^{\nu}(m_c)\,(\bar{s}d)_{V-A}\,\partial^2(\bar{\nu}\nu)_{V-A} + \ldots$$

- *For dim. 6 semi-leptonic operators,* matrix elements extracted from experiment:

$$\langle \pi^0 | (\bar{s}d)_V | K^0 \rangle \approx \langle \pi^0 | (\bar{s}u)_V | K^+ \rangle, \qquad K^+ \to \pi^0 \ell^+ \nu_\ell \; (K_{\ell 3})$$

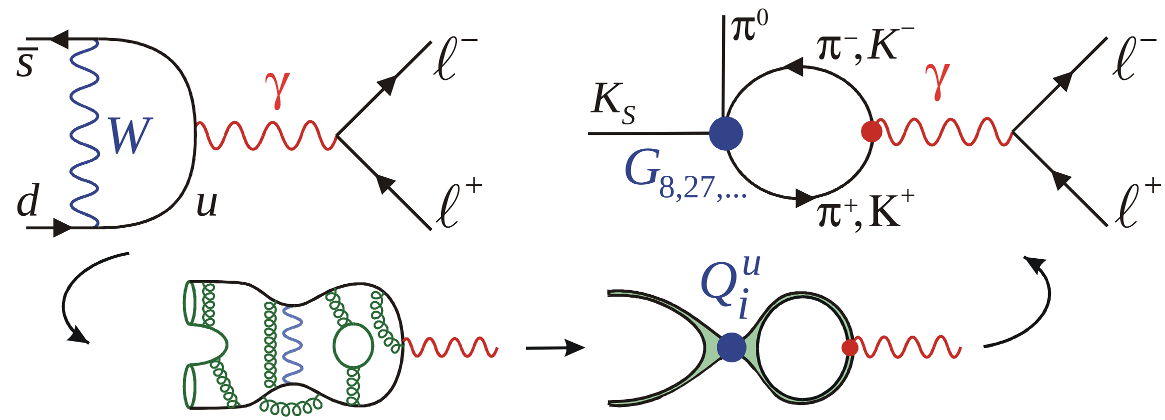
$$\langle 0 | (\bar{s}d)_A | K^0 \rangle \approx \langle 0 | (\bar{s}u)_A | K^+ \rangle, \qquad K^+ \to \ell^+ \nu_\ell \; (K_{\ell 2})$$

- *For dim. 6 four-quark operators*, matrix elements dealt with in ChPT:

$$\langle \pi^0 | \sum_{i=1}^{6} C_i'(\mu)\,Q_i^u \, | K^0 \rangle$$

$$\downarrow$$

$$G_8 (D_\mu U^\dagger D^\mu U)^{ds} + \ldots$$



Give CP-conserving contributions (ε' small), typically through photon penguins.

Low-Energy Constants $G_{8,27,\ldots}$ fixed from experiment ($\Delta I = 1/2$ enhancement).

- *For dim. 8 operators*, matrix elements from approximate matching with ChPT.

The “golden modes”

- The $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ decays

$$Br(K_L \to \pi^0\nu\bar{\nu}) \approx \kappa^0 \left(\,|\,\mathrm{Im}\,\lambda_t X(x_t)\,|^2\, \right)$$

$$Br(K^+ \to \pi^+\nu\bar{\nu}) \approx \kappa^+ \left(\,|\,\mathrm{Im}\,\lambda_t X(x_t)\,|^2 + \overbrace{|\,\underbrace{\mathrm{Re}\,\lambda_t X(x_t)}_{68\%} + \underbrace{\mathrm{Re}\,\lambda_c (P_c + \delta P_{u,c})}_{32\%}\,|^2}^{\sim Br(K_S \to \pi^0\nu\bar{\nu})} \right)$$

From $K_{\ell 3}$, with isospin corr.*

Marciano,Parsa ('96)

1. Precision Physics:

Dimension six *t-quark*: $X(x_t) \overset{NLO}{=} 1.464 \pm 0.041$ *Buchalla,Buras ('93)*

Dimension six *c-quark*: $P_c \overset{NNLO}{=} \lambda^4 (0.37 \pm 0.04)$ *Buras,Gorbahn,Haisch,Nierste ('05)*

Subleading *c-quark* dimension-eight operators
Residual *u-quark* long-distance contributions ($\mathrm{Re}\,\lambda_c \approx -\mathrm{Re}\,\lambda_u$) *Isidori,Mescia,C.S. ('05)*

$$\delta P_{u,c} = \lambda^4 (0.04 \pm 0.02)$$

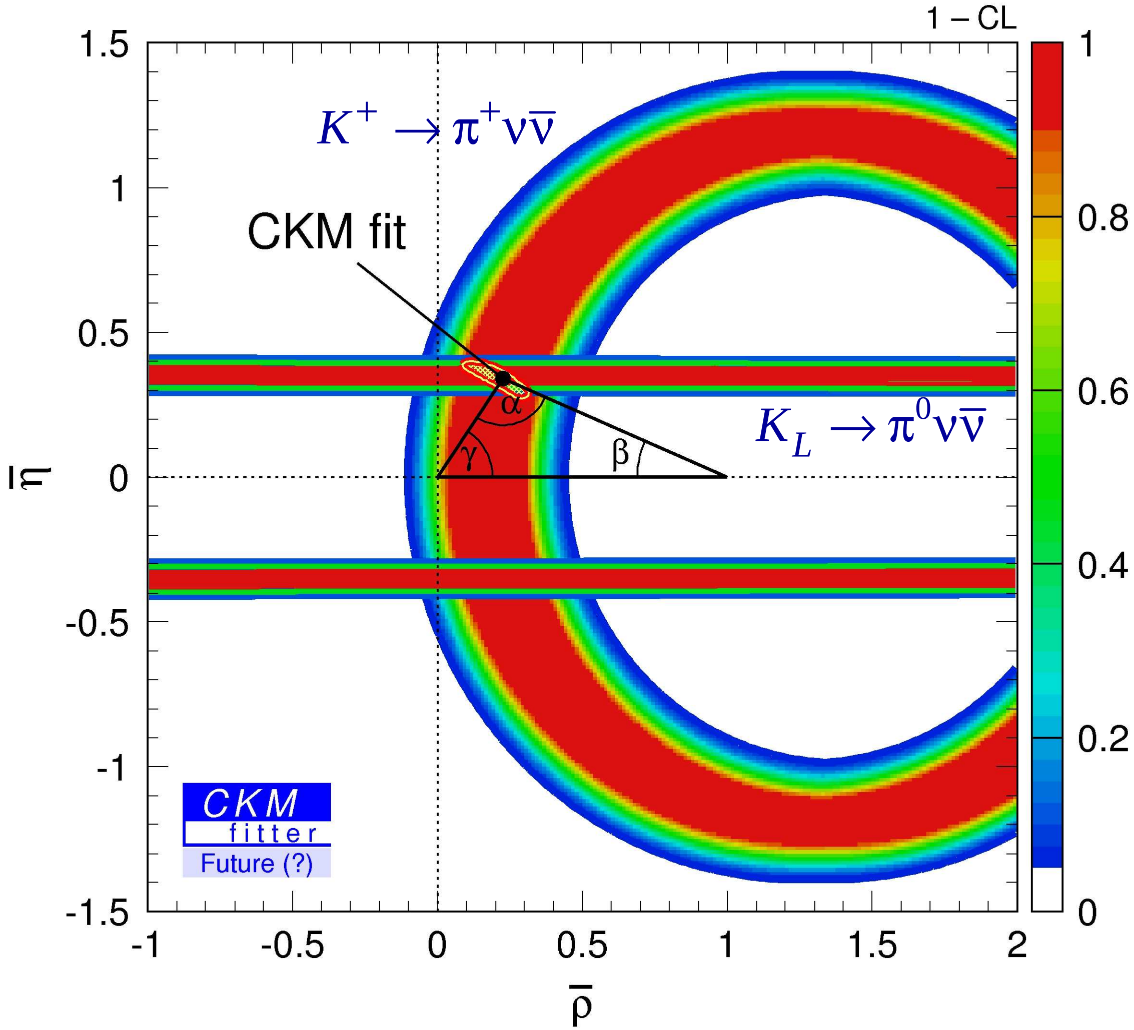
$K_L \to \pi^0\nu\bar{\nu}$: - *Indirect CPV* $\approx 1\%$ *Buchalla,Buras ('96)*

- *CPC* (dim. 8 from box with c,u) $\leq 0.01\%$ *Buchalla,Isidori ('98)*

2. Impact on the CKM determination:

Buras,Gorbahn,Haisch,Nierste ('06)

Future 10% measurements of both $K \to \pi\nu\bar{\nu}$ branching ratios, assuming the SM, would look in the $\bar{\rho}-\bar{\eta}$ plane like:



In particular, the residual error on P_c at NNLO induces:

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \begin{cases} \pm 4.0\% & NLO \\ \pm 1.0\% & NNLO \end{cases}$$

$$\sigma(\sin 2\beta) = \begin{cases} \pm 0.024 & NLO \\ \pm 0.006 & NNLO \end{cases}$$

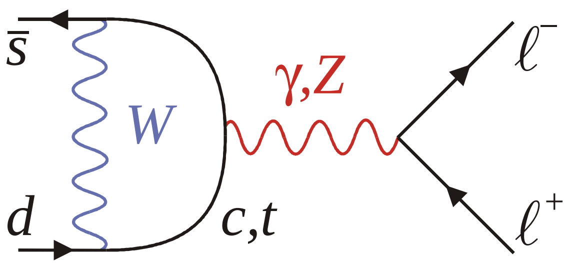
$$\sigma(\gamma) = \begin{cases} \pm 4.7^\circ & NLO \\ \pm 1.2^\circ & NNLO \end{cases}$$

- The $K_L \to \pi^0 \ell^+ \ell^-$ decays

1. Direct CPV: Two structures arise from top & charm integrations (known at NLO):

$$H_{eff}(\bar{s}d \to \ell^+ \ell^-) = \frac{G_F \alpha}{\sqrt{2}} \left(y_{7V} \, (\bar{s}d)_{V-A} (\bar{\ell}\ell)_V + y_{7A} \, (\bar{s}d)_{V-A} (\bar{\ell}\ell)_A \right)$$

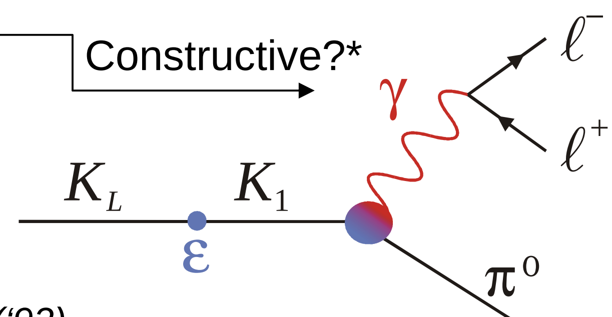
Vector: $C_0^Z, B_0^W, D_0^\gamma \to y_{7V}$, Axial-vector: $C_0^Z, B_0^W \to y_{7A}$



(1^{--})

$(1^{++}, 0^{-+})$

Constructive?*



(1^{--})

* (–) *Bruno,Prades ('92)*

(+) *Buchalla,D'Ambrosio,Isidori ('03)/de Rafael,Friot,Greynat ('04)*

2. Indirect CPV: $A(K_L \to \pi^0 \ell^+ \ell^-)_{ICPV} = \varepsilon \, A(K_1 \approx K_S \to \pi^0 \ell^+ \ell^-), \;\; \varepsilon \approx 10^{-3}$

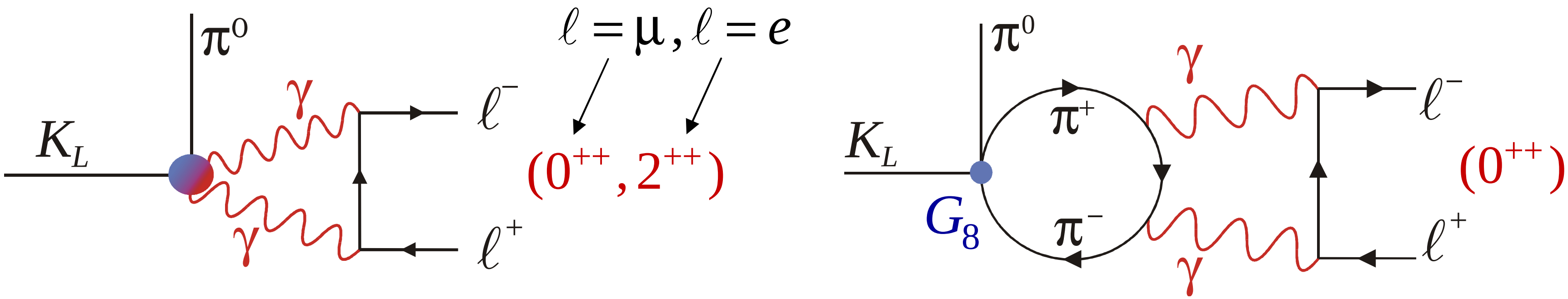
→ Photon penguin, long-distance dominated: to be estimated using *ChPT*

- Meson loops are small; a single counterterm a_S dominates,
- From NA48 measurements of $B(K_S \to \pi^0 \ell^+ \ell^-)$: $|a_S| = 1.2 \pm 0.2$.

D'Ambrosio,Ecker,Isidori,Portolés ('98)

3. CP-conserving:

CP-conserving matrix elements of $Q_1,\ldots,Q_6$ give rise to pure long-distance contributions through $\gamma\gamma$ penguins:



ChPT $O(p^4)$ finite, produces $\ell^+\ell^-$ in a scalar state only (helicity-suppressed).

Higher order effects estimated using the measurements of the $K_L \to \pi^0\gamma\gamma$ rate and spectrum (KTeV & NA48):

- The ratio $R^{\ell}_{\gamma\gamma} = \dfrac{\Gamma(K_L \to \pi^0 \ell^+ \ell^-)_{J=0^{++}}}{\Gamma(K_L \to \pi^0 \gamma\gamma)}$ can be estimated theoretically within 30%.

  Isidori,Unterdorfer,C.S. ('04)

- Production of $(\gamma\gamma)_{J=2^{++}}$ is constrained by the low-energy end of the $\gamma\gamma$ spectrum, and is found negligible.

  Buchalla,D'Ambrosio,Isidori ('03)

4. Complete predictions

$$Br(K_L \to \pi^0 \ell^+ \ell^-) = \left(C_{\rm dir}^{\ell} \kappa^2 \pm C_{\rm int}^{\ell} |a_S| \kappa + C_{\rm ind}^{\ell} |a_S|^2 + C_{\gamma\gamma}^{\ell} \right) \cdot 10^{-12}$$

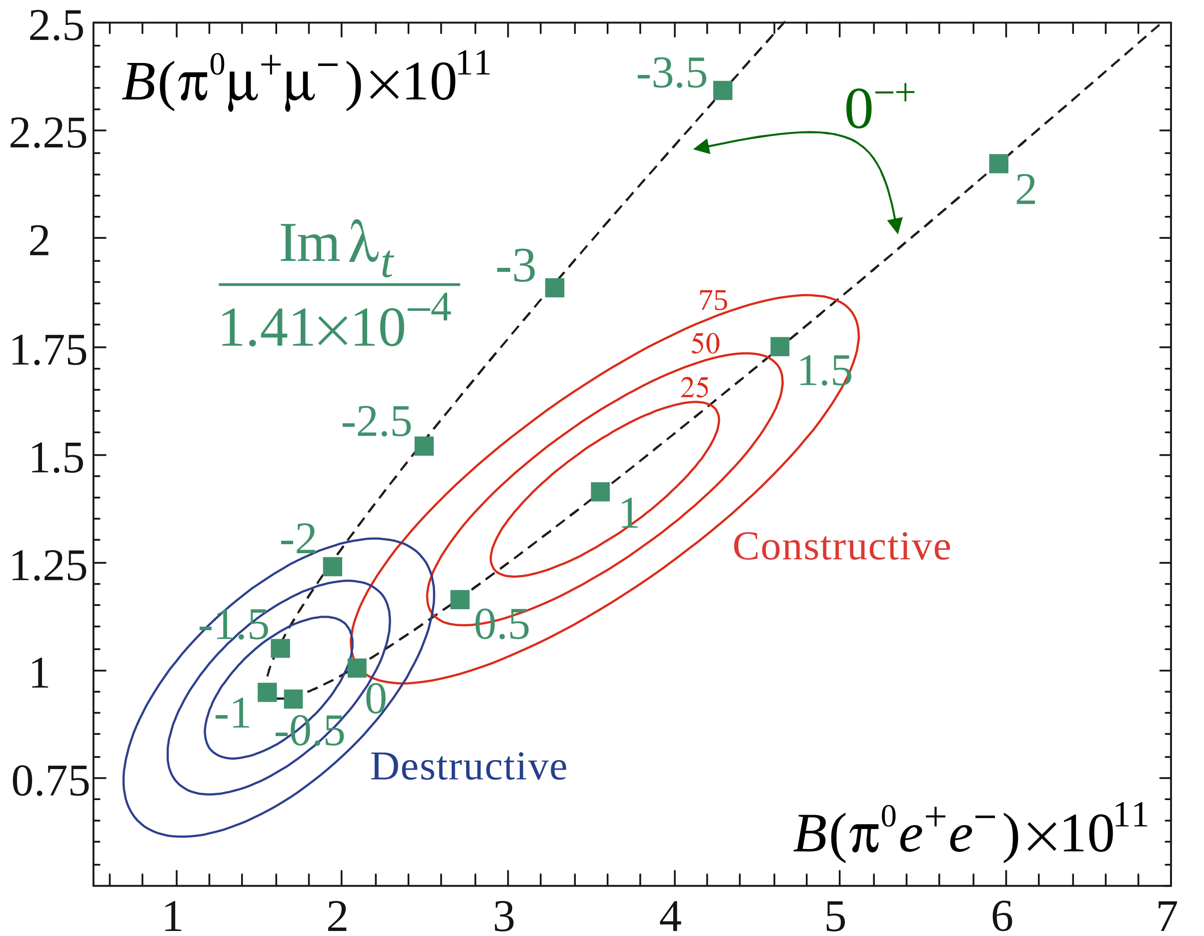
SM: $\kappa = \mathrm{Im}\,\lambda_t \times 10^4 \approx 1.41,\; y_{7A} \approx -0.68,\; y_{7V} \approx 0.73$

$$C_{\rm dir}^{e} \approx 2.3(y_{7V}^2 + y_{7A}^2)$$
$$C_{\rm int}^{e} \approx 8.1 y_{7V}$$
$$C_{\rm ind}^{e} \approx 14.5,\; C_{\gamma\gamma}^{e} \approx 0$$

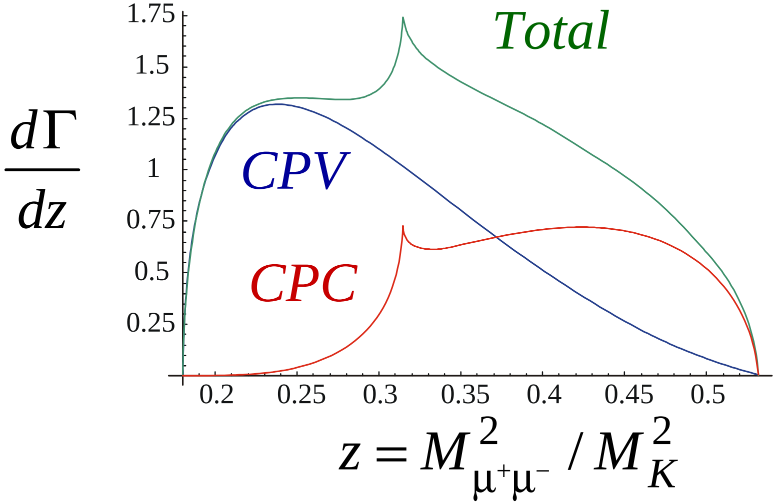
¼ phase-space suppression

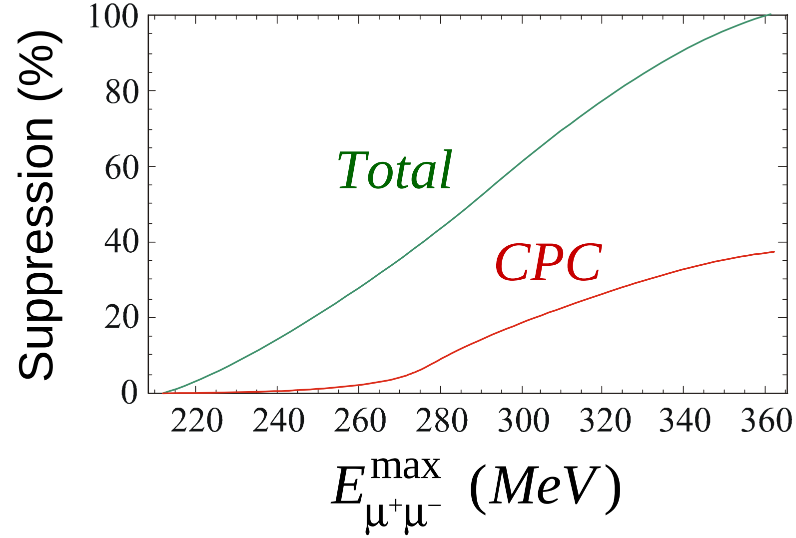
$$C_{\rm dir}^{\mu} \approx 0.55(y_{7V}^2 + 2.33 y_{7A}^2)$$
$$C_{\rm int}^{\mu} \approx 1.9 y_{7V}$$
$$C_{\rm ind}^{\mu} \approx 3.4,\; C_{\gamma\gamma}^{\mu} \approx 5.2$$

Additional helicity-suppressed terms



5. $K_L \to \pi^0\mu^+\mu^-$: *Differential rate and energy cut*



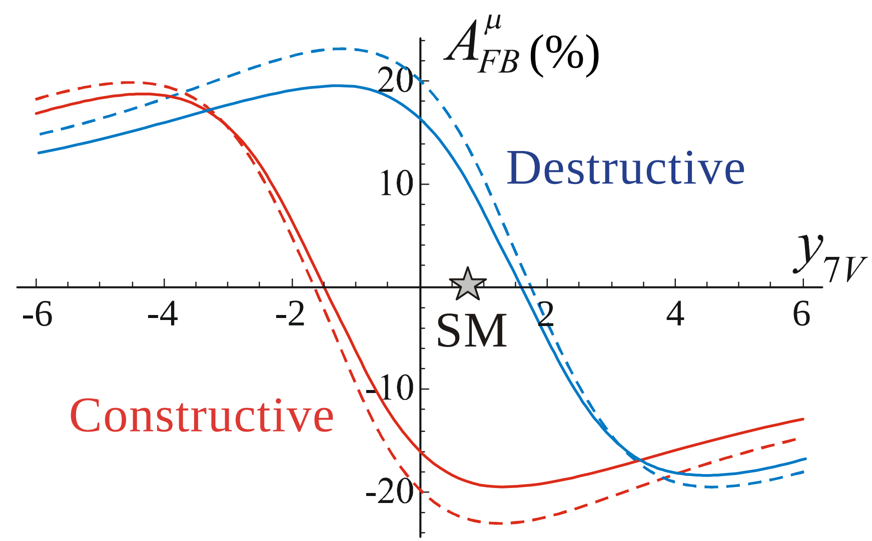


6. $K_L \to \pi^0\mu^+\mu^-$: *Forward-backward CP-asymmetry*

$$A^{\ell}_{FB} = \frac{N(E_- > E_+) - N(E_- < E_+)}{N(E_- > E_+) + N(E_- < E_+)}$$

Helicity-suppressed, proportional to the interference $CPC(0^{++}) \leftrightarrow CPV(1^{--})$

Can be used to fix the sign of a_S

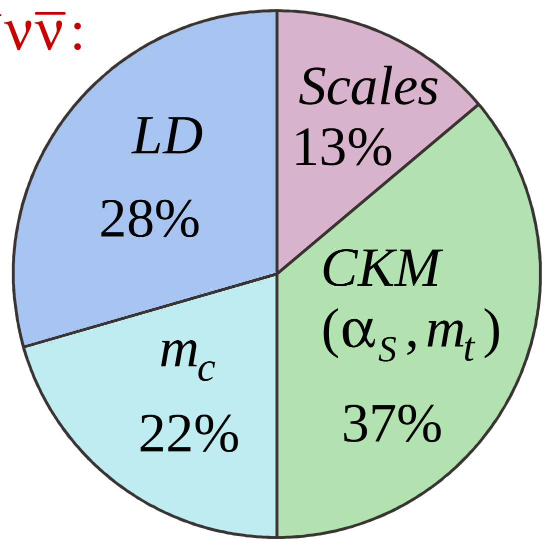


Mescia,Trine,C.S. ('06)

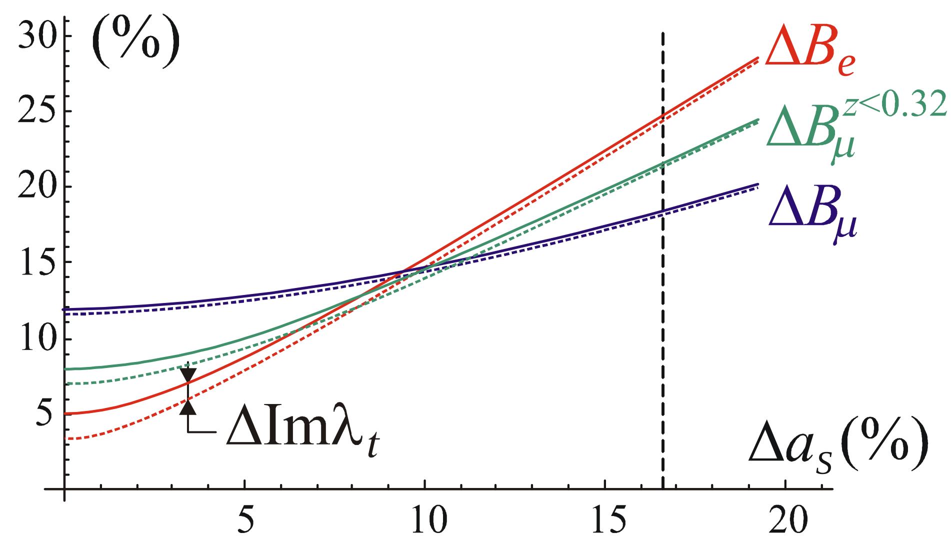
- Summary of current status in the SM:

	V, A	K^0-$\bar{K}^0$	2^{++}	0^{++}	SM $(\times 10^{-11})$	Experiment
$K_L \to \pi^0 \nu\bar{\nu}$	**100%**	(≈ 1%)	–	–	$2.81^{+0.56}_{-0.56}$	$< 2.1 \cdot 10^{-7}$ E391a
$K_L \to \pi^0 e^+ e^-$	**40%**	60%	(<3%)	–	$3.54^{+0.98}_{-0.85}$	$< 2.8 \cdot 10^{-10}$ KTeV
$K_L \to \pi^0 \mu^+ \mu^-$	**30%**	35%	–	35%	$1.41^{+0.28}_{-0.26}$	$< 3.8 \cdot 10^{-10}$ KTeV
$K^+ \to \pi^+ \nu\bar{\nu}$	**100%**	–	–	–	$8.0^{+1.1}_{-1.1}$	$14.7^{+13.0}_{-8.9} \cdot 10^{-11}$ E787 E949

Theory errors for $K^+ \to \pi^+ \nu\bar{\nu}$:



Theory errors for $K_L \to \pi^0 \ell^+ \ell^-$:

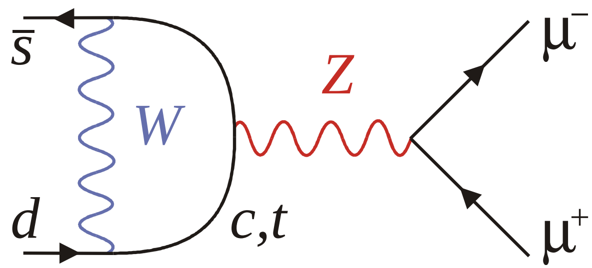


Haisch, FlavLHC ('06)
Buras,Gorbahn,Haisch,Nierste ('05,'06)

A word about $K_L \to \ell^+ \ell^-$

- The $K_L \to \ell^+\ell^-$ decay

1. Short-distance (top & charm quark) is CP-conserving and helicity-suppressed:



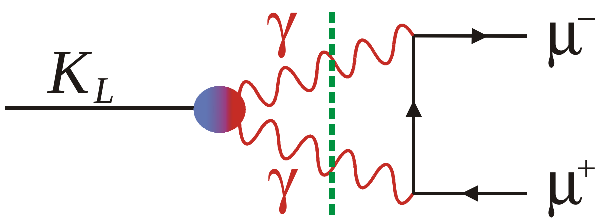
(0^{-+})

(known at NNLO)

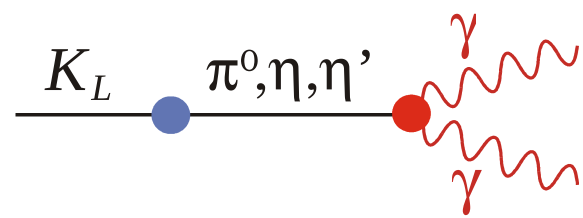
Gorbahn & Haisch ('06)

Good theoretical control (no γ penguin), and indirect CPV very small.

2. Long-distance $\gamma\gamma$ penguin: the absorptive part is known precisely



(0^{-+})



Estimate for the (divergent) dispersive part, which interferes with SD, obtained from experimental data on $K_L \to \gamma^*\gamma^*$ + perturbative behavior of up-quark $\gamma\gamma$ penguin.

Isidori & Unterdorfer ('03)

3. Complete prediction:

$$y_{7A} \approx -0.68$$

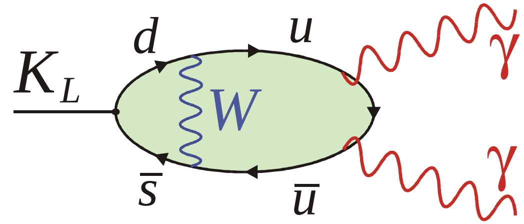
$$Br(K_L \to \mu^+\mu^-) \approx \big((\underbrace{1.1 y_{7A}}_{top} \underbrace{-0.2}_{charm} \underbrace{\pm 0.4^{+0.5}_{-0.5}}_{Disp(\gamma\gamma)})^2 + \underbrace{6.7}_{Abs(\gamma\gamma)}\big) \cdot 10^{-9}$$

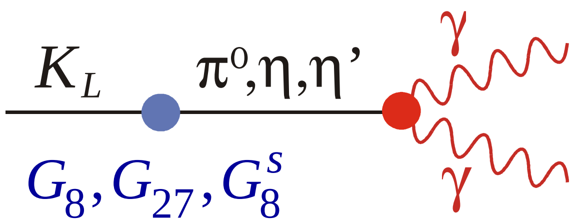
$$Br(e^+e^-) \approx 10^{-12}$$

$K_L \to \ell^+\ell^-$

4. Interference sign? Requires the sign of $A(K_L \to \gamma\gamma)$: *Gérard, Trine, C.S ('05)*

Driven by Q_1 only → vanishes at LO in SU(3) ChPT.
U(3) ChPT needed to disentangle Q_1, Q_2 and Q_6
(partial use of Large N_C: *not* the factorization approx.!)





$$A_{\gamma\gamma} \approx \overbrace{(G_8^S + 2G_{27}/3)}^{\sim C_1(\mu_{hadr.})}\left((0.46)_\pi - (1.83)_\eta - (0.12)_{\eta'}\right)$$

$$B(K_L \to \gamma\gamma)^{\exp} \Rightarrow G_8^S / G_8 \approx \pm 1/3$$

Theoretically, G_8^S can be estimated from the smooth Q_1, Q_2 non-perturbative evolution (with a reasonable penguin fraction in the $\Delta I = 1/2$ rule at the hadr. scale)

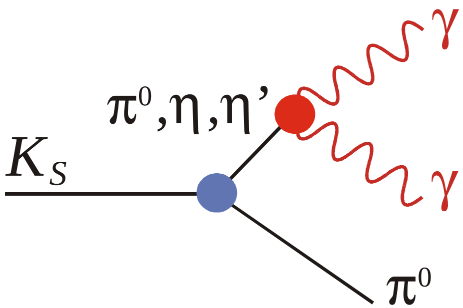
$$(C_1 + C_2)^2 (C_2 - C_1) = 1.0 \pm 0.3 \Rightarrow \begin{cases} G_8^S / G_8 = -0.38 \pm 0.12 \\ F_P \approx 65\%, F_{CC} \approx 35\% \end{cases}$$

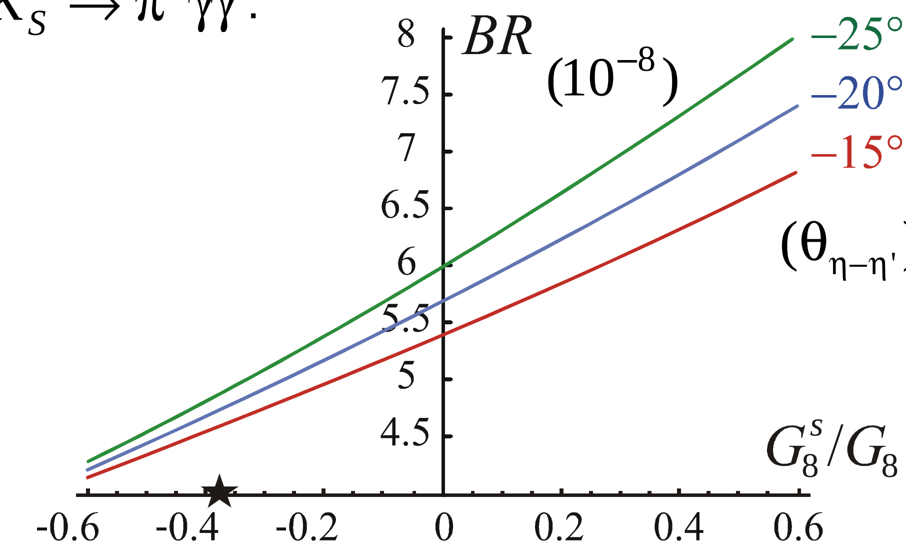
For comparison, if $G_8^S / G_8 \approx +1/3 \Rightarrow F_P \approx 130\%, F_{CC} \approx -30\%$

i.e., this corresponds to $C_{1,2}(\mu_{hadr.})$ of opposite signs compared to $C_{1,2}(1\,GeV)$

$K_L \to \ell^+\ell^-$

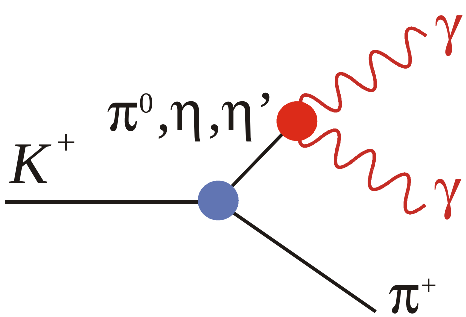
Experimentally, G_8^s could be fixed from $K_S \to \pi^0\gamma\gamma$:

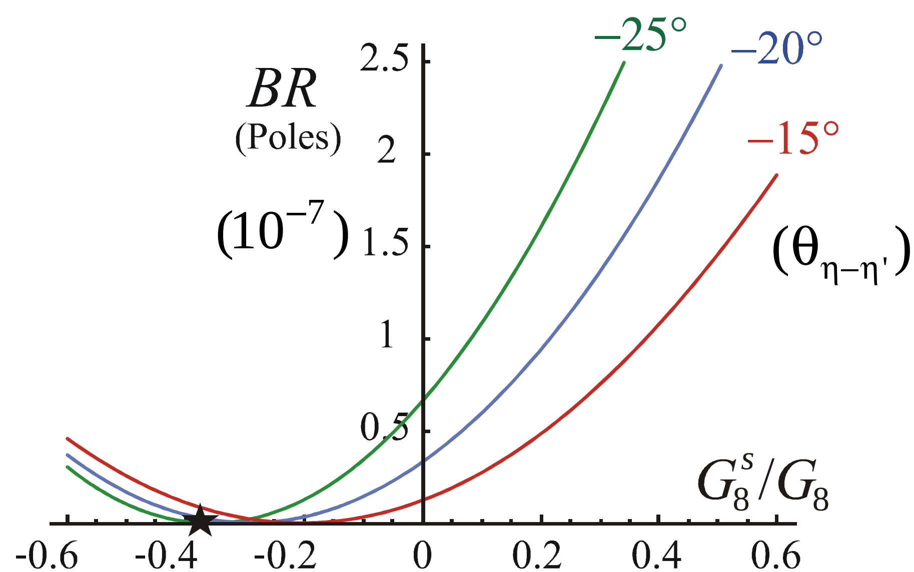




$$Br(K_S \to \pi^0\gamma\gamma)^{\text{exp}}_{z>0.2} = (4.9 \pm 1.8)\cdot 10^{-8}$$

or from pole contributions to $K^+ \to \pi^+\gamma\gamma$





(even more constraining at the low-energy end of the spectrum)

Conclusion

Theoretical control over the SM contributions:

- $K_L \to \pi^0 \nu\bar{\nu},\ K^+ \to \pi^+ \nu\bar{\nu}$ QCD effects are known to a high level of precision: NNLO for the dimension-six operators, with the smaller dimension-eight and LD contributions under control.

  Possible improvements: *Isospin breaking in the vector/scalar form-factors*
  Better estimate of charm-quark mass
  Lattice study for higher-dimensional operators

- $K_L \to \pi^0 e^+ e^-,\ K_L \to \pi^0 \mu^+ \mu^-$ Long-distance effects under control, but could be improved. NLO effects for the running (sufficient).

  Possible improvements: *Better measurements of* $K_S \to \pi^0 \ell^+ \ell^-$ *for* a_S
  Better measurements of $K_L \to \pi^0 \gamma\gamma$ *for* $\gamma\gamma\,(0^{++}, 2^{++})$

- $K_L \to \mu^+ \mu^-$ QCD effects known to NNLO (dimension six), but large uncertainty for the long-distance, two-photon piece.

  Possible improvements: *Better theoretical treatment of* Disp($\gamma\gamma$) (?)
  Better measurements of $K_S \to \pi^0 \gamma\gamma, K^+ \to \pi^+ \gamma\gamma$
  and $K_L \to \gamma^* \gamma^*$ *for* Sign(Disp($\gamma\gamma$))