Enhanced Nonlocal Power Corrections to the $B \to X_s \gamma$ Decay Rate

Matthias Neubert

Cornell University, Ithaca, New York

&

Johannes Gutenberg University, Mainz

Based on: S.J. Lee, MN, G. Paz, hep-ph/0609224

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Introduction

- Precision studies of inclusive $B \to X_s \gamma$ decay are a cornerstone of quark-flavor physics
- Theoretical analysis of total inclusive rates based on local operator product expansion, by which total rates are expressed in terms of local HQET matrix elements; power corrections enter first at order $(\Lambda_{QCD}/m_b)^2$
- Leading term now computed (almost) at NNLO in RG-improved perturbation theory [$\to$ talks by P. Gambino, T. Becher]

Introduction

- OPE breaks down for differential decay distributions near phase-space boundaries
- Twist expansion involving forward B-meson matrix elements of nonlocal operators (shape functions) required to systematically account for nonperturbative effects [$\to$ talk by G. Paz]

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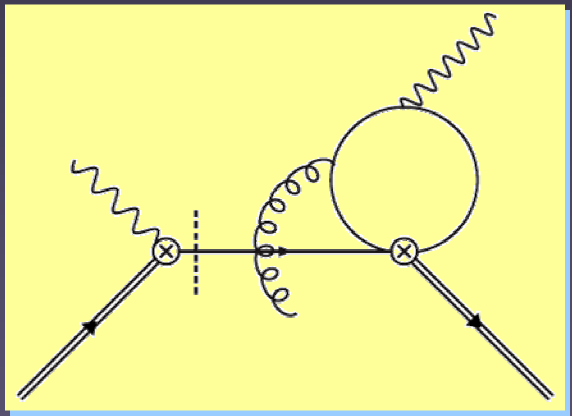
Introduction

- Well-known that OPE faces limitations in $B \to X_s \gamma$ decay, e.g.:
  - Charm-penguin contribution leads to λ_2/m_c^2 correction when charm is treated as a heavy quark [Voloshin]



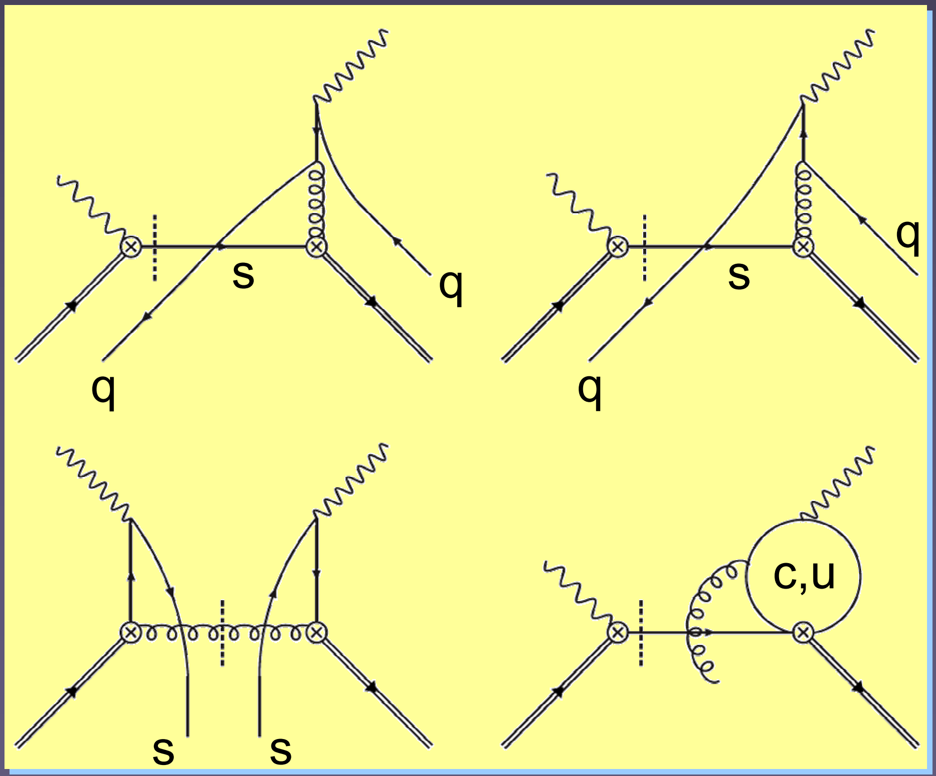
  - Effect described by a nonlocal subleading shape function when counting $m_c^2 \gg m_b \Lambda_{QCD}$ is adopted

Nonlocal Power Corrections

- Power corrections to high-energy part of $B \to X_s \gamma$ photon spectrum can be parameterized systematically in terms of subleading shape functions
- Some of them (those considered so far) reduce to local operators when photon spectrum is integrated over energy
- *But:* Several others do not! [Lee, MN, Paz (in prep.)]

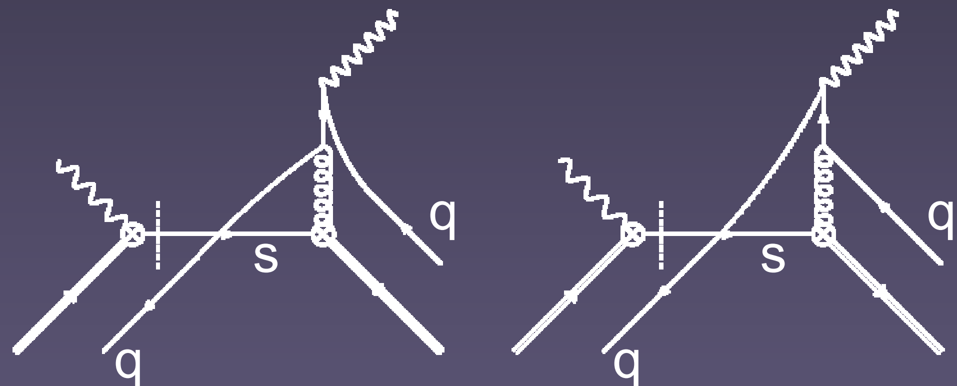
Nonlocal Power Corrections

- Example of contributions to discontinuity of hadronic tensor:



Nonlocal Power Corrections

- First two graphs give rise to 4-quark operators containing light u,d quarks besides s quarks



$\rightarrow$ expected to give dominant effect for B^0, B^+

- Due to interference with $Q_{7\gamma}$, effect is centered at large photon energy (cannot be eliminated by a cut)

Nonlocal Power Corrections

- Contribution to total rate parameterized by matrix elements of tri-local operators:

$$\Delta\Gamma = -\Gamma_{77}\,\frac{C_{8g}}{C_{7\gamma}}\,\frac{4\pi\alpha_s}{N_c m_b}\int_{-\infty}^{0} ds\int_{-\infty}^{0} dt \times \langle\bar{B}|\,C_F\,(O_1+O_2)-(T_1+T_2)\,|\bar{B}\rangle$$

where:

$$O_1 = \sum_q e_q\,\bar{h}_v(0)\,\Gamma_R\,q(t\bar{n})\;\bar{q}(s\bar{n})\,\Gamma_R\,h_v(0)\,,$$

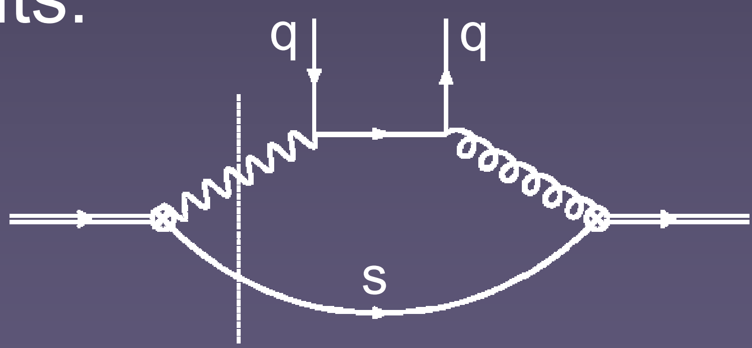
$$O_2 = \sum_q \frac{e_q}{2}\,\bar{h}_v(0)\,\Gamma_R\gamma_{\perp\alpha}\,q(t\bar{n})\;\bar{q}(s\bar{n})\,\gamma_\perp^\alpha\Gamma_R\,h_v(0)\,,$$

$$T_1 = \sum_q e_q\,\bar{h}_v(0)\,\Gamma_R\,t_a q(t\bar{n})\;\bar{q}(s\bar{n})\,\Gamma_R\,t_a h_v(0)\,,$$

$$T_2 = \sum_q \frac{e_q}{2}\,\bar{h}_v(0)\,\Gamma_R\gamma_{\perp\alpha}\,t_a q(t\bar{n})\;\bar{q}(s\bar{n})\,\gamma_\perp^\alpha\Gamma_R\,t_a h_v(0)$$

Relation with Optical Theorem

- Normally, expect 4-quark operators to contribute at order $(\Lambda_{QCD}/m_b)^3$ to the total decay rate - why different here?
- Reason is that in $B \to X_s \gamma$ one does not sum over all cuts:



- Process is not really inclusive (partonic sub-structure of photon)

Model Estimates

- Reliable field-theoretic estimates of these effects are very difficult to obtain (lattice QCD unable to handle light-cone operators)
- Naïve dimensional analysis suggests:

$$\Delta\Gamma/\Gamma_{77} \sim (C_{8g}/C_{7\gamma})\,\pi\alpha_s(\Lambda/m_b)$$

- Could be a 5% correction to the rate!

Model Estimates

- Our approach:
  Apply vacuum insertion approximation
  - motivated by large-N_c counting rules
  - well tested for local 4-quark operators in analysis of B-hadron lifetimes
  - not uncontroversial for nonlocal operators (not clear to me why …)

Model Estimates

- Matrix elements of operators O_2,T_1,T_2 vanish in VIA due to color and/or Dirac structure
- Matrix element of O_1 gets related to leading B-meson light-cone distribution amplitude in position space:

$$\langle \bar{B} | O_1 | \bar{B} \rangle_{\mathrm{VIA}} = e_q \frac{f_B^2 m_B}{4} \tilde{\phi}_+^B(s) \, [\tilde{\phi}_+^B(t)]^*$$

Model Estimates

- Integration over light-cone gives inverse moment of LCDA (parameter λ_B):

$$-i\int_{-\infty}^{0} ds\,\tilde{\phi}_+^B(s) = \int_0^\infty \frac{d\omega}{\omega}\,\phi_+^B(\omega) = \frac{1}{\lambda_B}$$

- Result:

$$\frac{\Delta\Gamma_{\mathrm{VIA}}}{\Gamma_{77}} = -\frac{e_q C_{8g}}{C_{7\gamma}}\,\frac{\pi\alpha_s}{2}\left(1-\frac{1}{N_c^2}\right)\frac{f_B^2 m_B}{\lambda_B^2 m_b}$$
$$\approx -0.26 e_q\left(\frac{f_B}{\lambda_B}\right)^2$$

Model Estimates

- Effect potentially large
- Leading contribution to flavor-dependent rate asymmetry:

$$\frac{\Gamma(B^- \to X_s\gamma) - \Gamma(\bar{B}^0 \to X_s\gamma)}{\Gamma(\bar{B} \to X_s\gamma)} \approx -0.05 \left(\frac{\lambda_B}{0.5\,\mathrm{GeV}}\right)^{-2}$$

- Expect effect between -2% and -20% (sign determined)
- Contribution to flavor-averaged decay rate is 6 times smaller (few % effect at most in VIA)

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Caveats!

- These estimates are very model dependent (even though reasonable)
- They do not provide conservative error estimates!
- We believe uncertainty on total $B \to X_s \gamma$ branching ratio should not exceed 5%, but this is difficult to prove

Conclusions

- There is no local OPE for the total $B \to X_s \gamma$ decay rate!
- Nonlocal power corrections (subleading shape functions) starting at order Λ_{QCD}/m_b exist even for the total decay rate
- Irreducible theory error, of similar magnitude as perturbative uncertainty at NNLO
- Measurement of flavor-dependent asymmetry could help to corroborate our model estimates