Photon Energy Cut Effects in $B \to X_s \gamma$ at NNLO

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Outline

- The photon energy cut $B \longrightarrow X_s \gamma$
  - Appearance of low scales
  - Factorization and resummation
- Event fraction at NNLO

TB, M. Neubert, hep-ph/0512208, hep-ph/0603140, hep-ph/0610067

- Photon energy moments at NNLO

M. Neubert, hep-ph/0506245

Branching ratio vs. Moments

- $B \to X_s \gamma$ branching ratio:
  - FCNC process, suppressed in SM
  - Sensitive to New Physics
- Moments of the decay spectrum

$$\langle E_\gamma \rangle \qquad\qquad \sigma_E^2 = \langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2$$

  - Determination of m_b and heavy quark parameter μ_π
  - Insensitive to New Physics

Prediction of the rate and moments: OPE

- The good "news"

"inclusive *B*-meson decay rate"

= "perturbative calculation of *b*-quark decay rate"

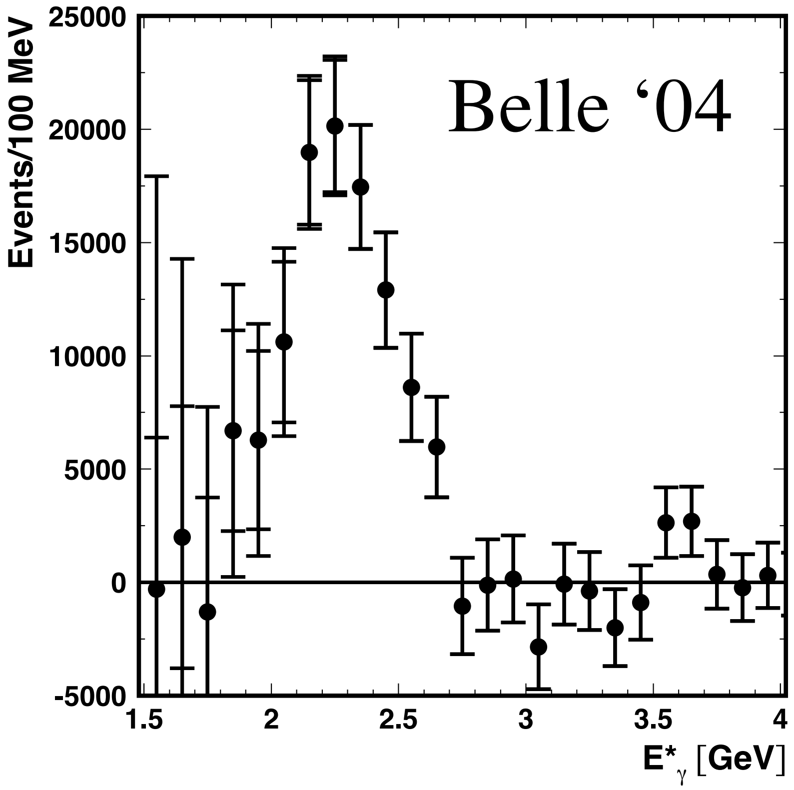
$+ O(\Lambda^2/m_b^2)$ ← $\propto \mu_\pi, \mu_G$

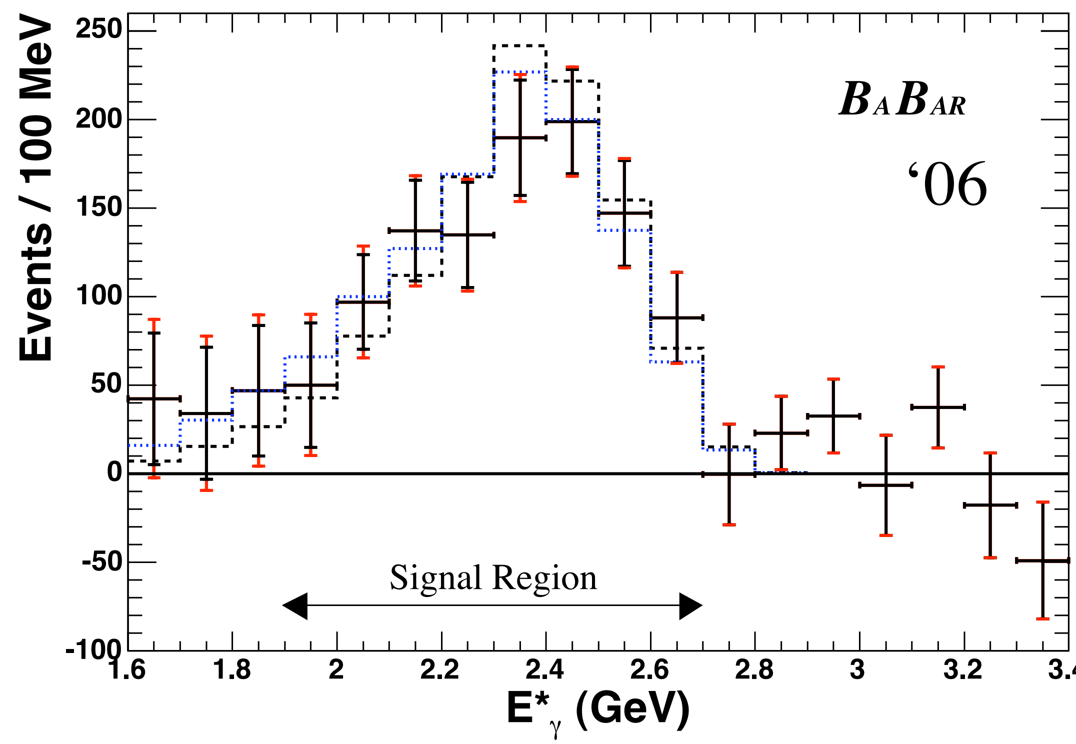
- The rate and moments of the photon energy spectrum can be calculated accurately using perturbation theory.
- However, there are two complications.

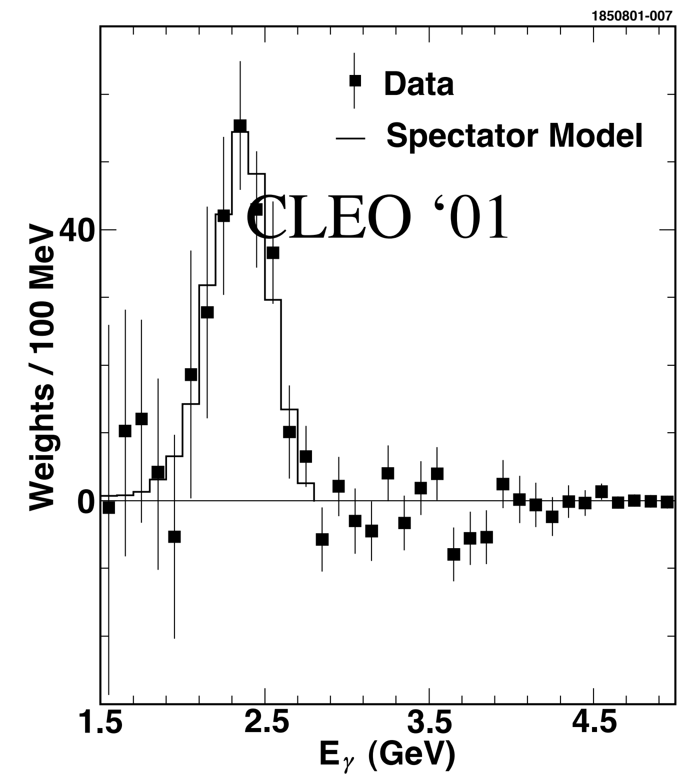
Complications

1. Non-OPE corrections for operators other than Q_7
   - $1/m_b$ power corrections
   - twist expansion: non-local, functional hadronic input → M. Neubert's talk
2. For cut rate with $E_\gamma > E_0$ not an expansion in $1/m_b$, but in $1/\Delta$ and $\alpha_s(\Delta)$, where $\Delta = m_b - 2E_0$.
   - OPE breaks down for $E_0 > 1.8$ GeV. Shape function.

Photon energy cut $E_\gamma > E_0$







- Experimentally very energetic photon is necessary to suppress background. Impose $E_\gamma > E_0$.
  - Belle has E_0=1.8 GeV, BaBar E_0=1.9 GeV
  - Note: $E_\gamma < m_B/2 \approx 2.6$ GeV

Scales in the presence of cut

- With a cut $E_\gamma > E_0$, problem contains three relevant scales
  - Hard scale: m_b
  - Soft scale: $\Delta = m_b - 2E_0$
  - Jet scale: $M_X \sim (m_b \Delta)^{1/2}$
- The physics associated with the three scales can be factorized:

$$\Gamma_{E_\gamma > E_0} = \underbrace{H}_{\text{hard function}} \cdot \underbrace{J}_{\text{jet function}} \otimes \underbrace{S}_{\text{soft function/ shape function}}$$

Shape-function vs. OPE region

- For E_0>1.8 GeV, Δ<1GeV: OPE breaks down. Need non-perturbative shape function.

Intermediate region $\Lambda \ll \Delta \ll m_b$:

SCET, Multi-Scale OPE (Neubert '04),
expansion in $1/\Delta$, $\alpha_s(\Delta)$, $\Delta = m_b - 2E_0$

- For E_0<1.0 GeV, standard OPE works.

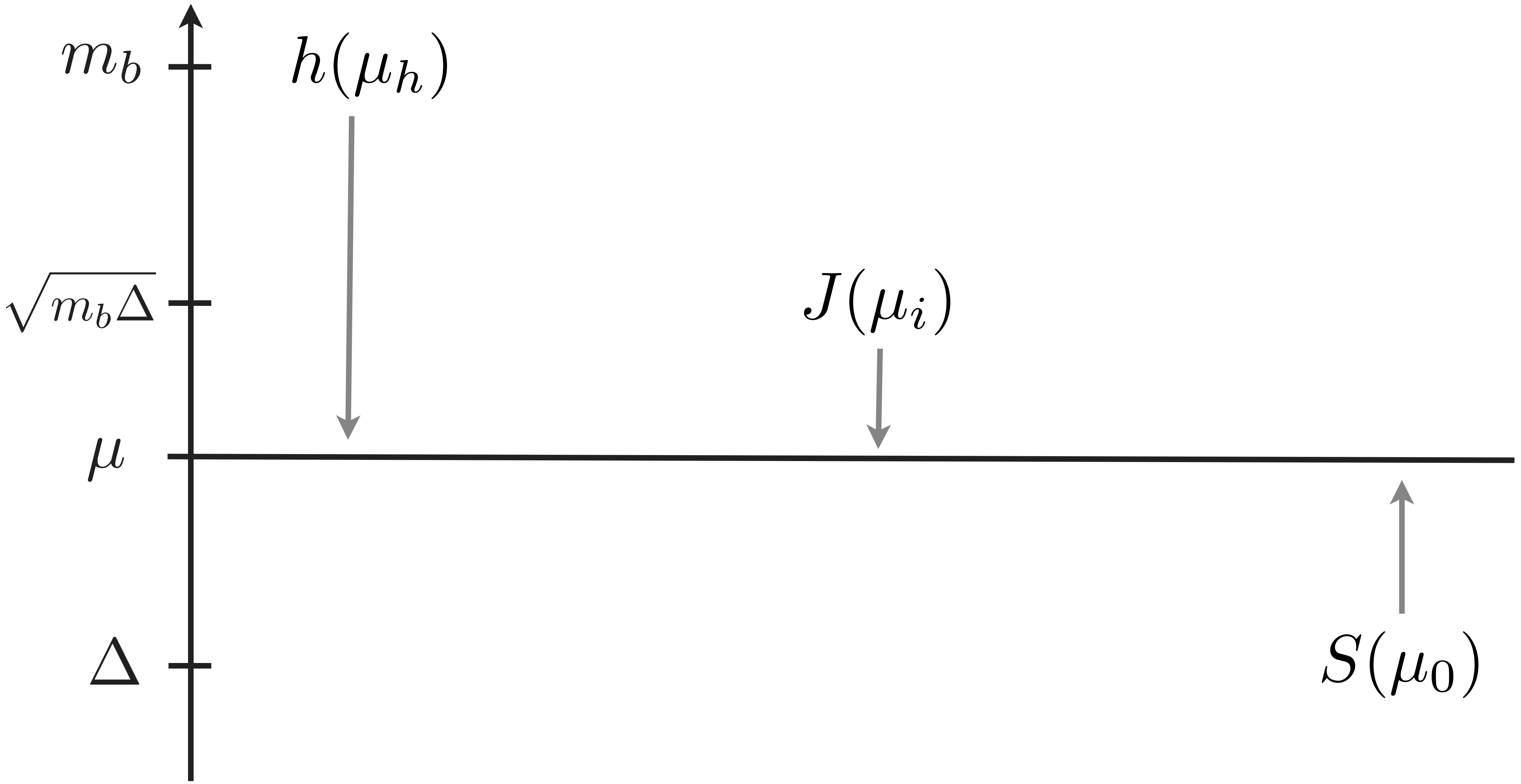
Event fraction

- Define event fraction $F(E_0) = \Gamma_{E_\gamma > E_0} / \Gamma_{\mathrm{tot}}$

$$F(E_0) = h(m_b,\mu) \int_0^\Delta dP \int_0^P d\omega\, J(m_b(P-\omega),\mu)\, S(\omega,\mu)$$

  - Partial BR is obtained by combining with result total rate of Misiak et al.
  - Moments can be obtained from integrals over $F(E_0)$.
- Have evaluated the three parts to NNLO
- Use RG evolution in SCET to separate the contributions from different scales.

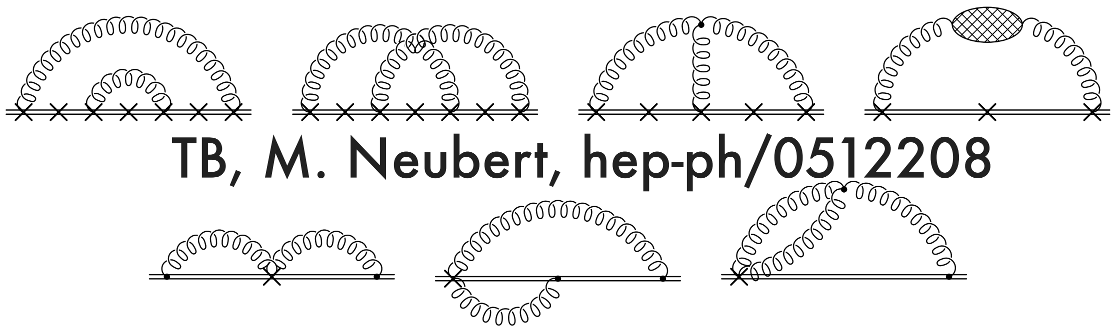
RG evolution in SCET



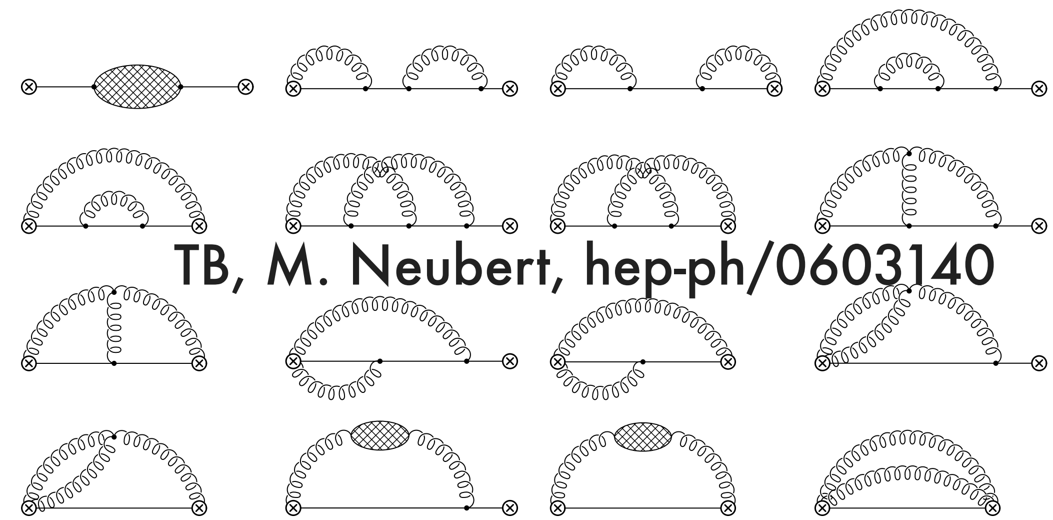
- Evaluate each part at its characteristic scale, evolve to (arbitrary) common reference scale μ.
  - No large (Sudakov) logarithms of scale ratios

Two-loop calculations of jet- and soft function

Soft function



Jet-function



- Soft function is the partonic shape function.
- Jet-function is propagator in axial light-cone gauge.
- Jet and soft functions are distributions. Manage to avoid dealing with distribution valued Feynman diagrams.
- In hep-ph/0610067, we infer the hard function *h* from Melnikov & Mitov ‘05.

Solution of RG evolution: jet-function

$$\frac{dJ(p^2,\mu)}{d\ln\mu} = -\left[2\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{p^2}{\mu^2} + 2\gamma^J(\alpha_s)\right] J(p^2,\mu)$$
$$- 2\Gamma_{\rm cusp}(\alpha_s)\int_0^{p^2} dp'^2\,\frac{J(p'^2,\mu)-J(p^2,\mu)}{p^2-p'^2}$$

Sudakov factor

$$\boxed{J(p^2,\mu) = \exp\left[-4S(\mu_i,\mu) + 2a_{\gamma^J}(\mu_i,\mu)\right] \times \widetilde{j}(\partial_\eta,\mu_i)\,\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}\,\frac{1}{p^2}\left(\frac{p^2}{\mu_i^2}\right)^\eta ,}$$

$$\eta = 2\int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\,\Gamma_{\rm cusp}[\alpha_s(\mu)]$$
$$= 2a_\Gamma(\mu_i,\mu).$$

- Associated jet-function $\widetilde{j}$ is Laplace transform of $J(p^2,\mu_i)$.
- RG evolution of shape function $S(\omega, \mu_i)$ has exactly the same form.

“Wonderful formula”

$$F(E_0) = U(\mu_h, \mu_i, \mu_0; \mu) \left(\frac{m_b}{\mu_h}\right)^{-2a_\Gamma(\mu_h,\mu)} \left(\frac{m_b\Delta}{\mu_i^2}\right)^{2a_\Gamma(\mu_i,\mu)} \left(\frac{\Delta}{\mu_0}\right)^{-2a_\Gamma(\mu_0,\mu)}$$
$$\times h\left(\frac{m_b}{\mu_h}\right) \tilde{j}\left(\ln\frac{m_b\Delta}{\mu_i^2} + \partial_\eta\right) \tilde{s}\left(\ln\frac{\Delta}{\mu_0} + \partial_\eta\right) \frac{e^{-\gamma_E \eta}}{\Gamma(1+\eta)} \Bigg[1 - \underbrace{\frac{\eta(1-\eta)}{6}\frac{\mu_\pi^2}{\Delta^2}}_{\text{leading } \Lambda/\Delta \text{ power correction}} + \dots\Bigg] + \underbrace{\delta F(E_0)}_{\Delta/m_b \text{ power corrections}},$$

$$\eta = 2\int_{\mu_i}^{\mu_0} \frac{d\mu}{\mu}\, \Gamma_{\text{cusp}}(\alpha_s(\mu)) \approx 0.2$$

leading Λ/Δ power correction

Δ/m_b power corrections

- All scales separated, no “large” perturbative logarithms.
- Include Δ/m_b corrections in fixed order PT.
- Leading power correction is $\eta(\Lambda_{\text{QCD}}/\Delta)^3 \approx 2\%$
- Standard fixed order OPE result for $\mu_h = \mu_i = \mu_0 = \mu$.

Combination with fixed order result

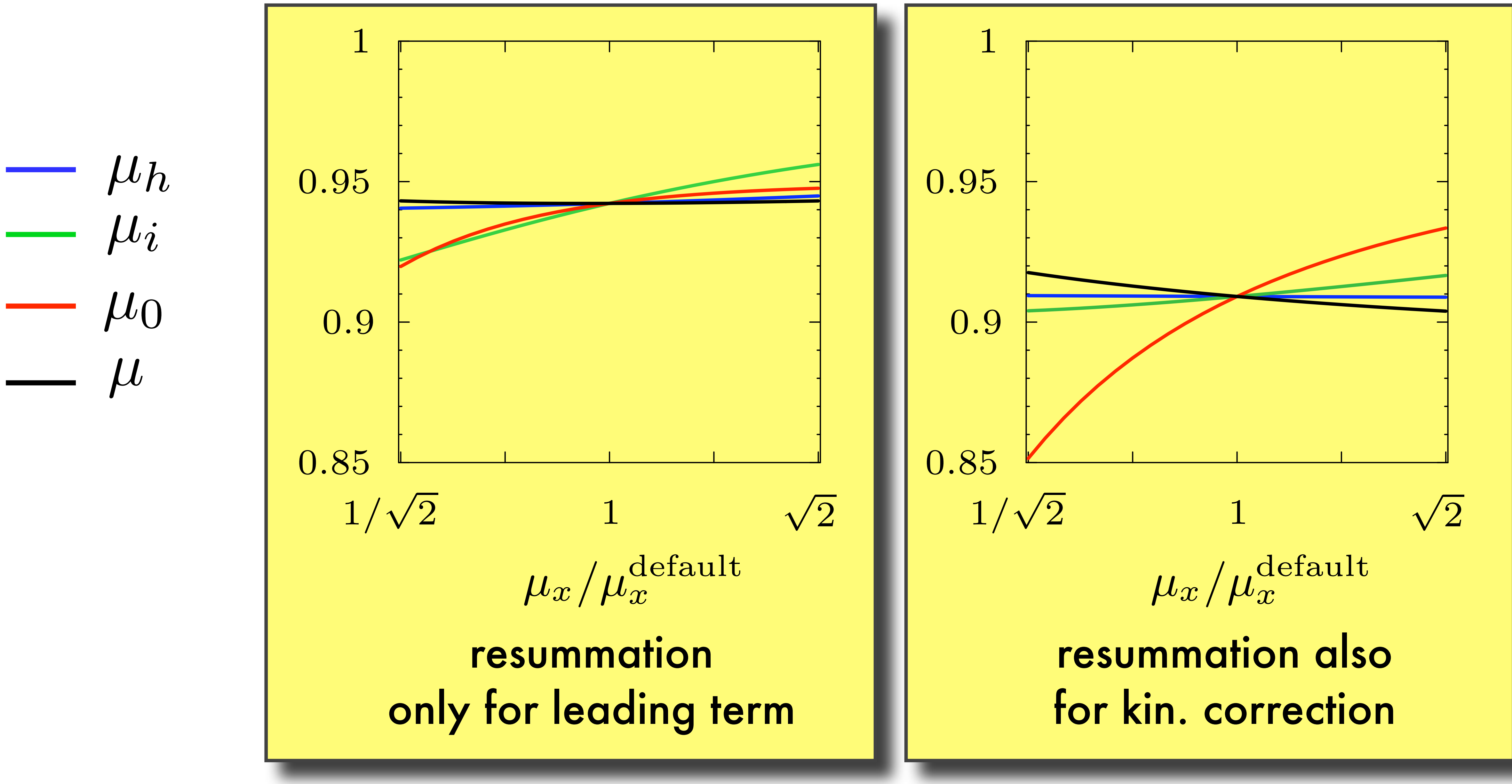
$$T = F(1.6\mathrm{GeV})/F(1.0\mathrm{GeV})$$

$$\mathrm{Br}(\bar{B} \to X_s\gamma)\big|_{E_\gamma > 1.6\mathrm{GeV}} = T \times \underbrace{\mathrm{Br}(\bar{B} \to X_s\gamma)\big|_{E_\gamma > 1.0\mathrm{GeV}}}_{\Delta \approx 2.5\mathrm{GeV},\ \text{fixed order OK}}$$

- We combine T with fixed order result of Misiak et al. at E_0=1.0GeV.
- Correlation of uncertainties small
  - Parameter dependence of T small.
  - μ_h dependence of T is small.
- We combine individual uncertainties quadratically.

Scale variation of *T*

$$T = F(1.6\text{GeV})/F(1.0\text{GeV})$$



$$T = 0.93\,^{+0.03}_{-0.05}{}_{\text{pert}} \pm 0.02_{\text{hadr}} \pm 0.02_{\text{pars}}$$

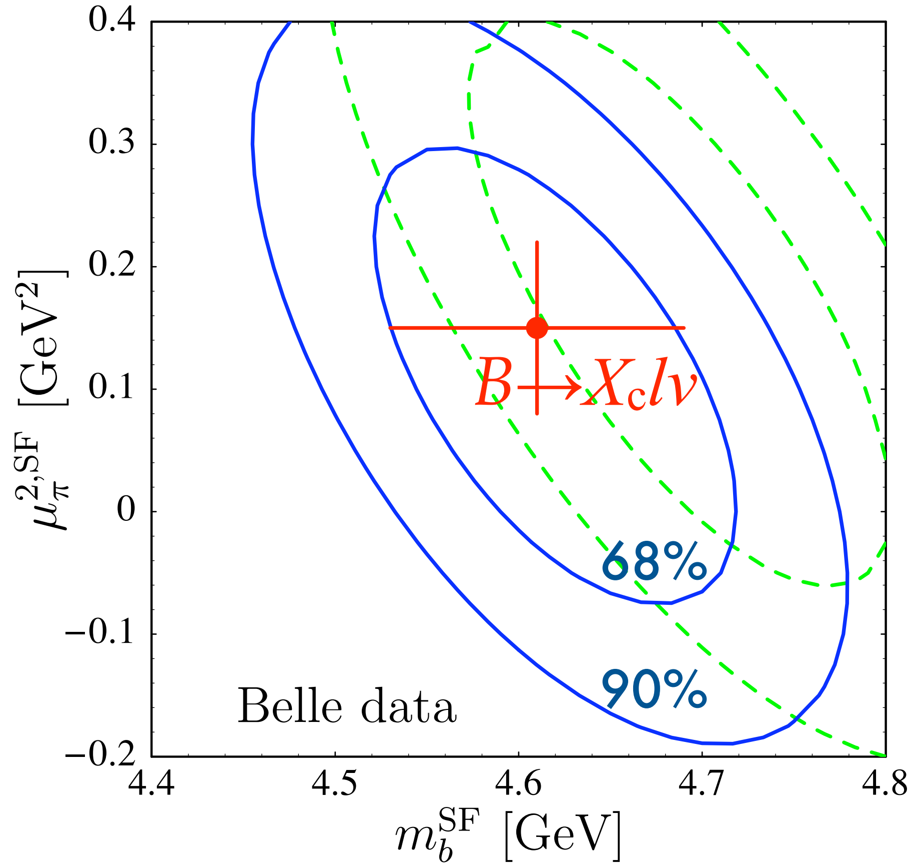
Note: fixed order result is *T*=0.96

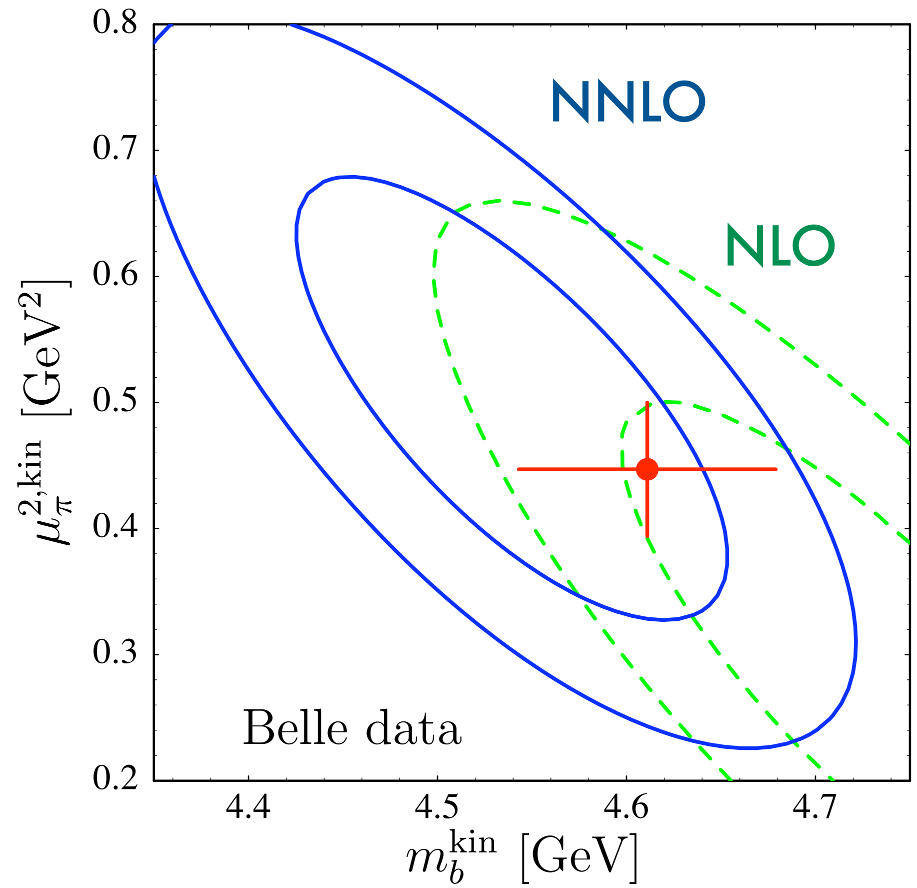
Beyond E_0=1.6GeV

- HFAG extrapolates all expt. results to common value of E_0=1.6GeV
  - 1.6 GeV is lower than what is expt. achieved.
  - Use model shape function with two parameters (m_b and μ_π).
  - (Too?) small extrapolation uncertainty,
    - e.g. 1% for extrapolation from E_0=1.9GeV to 1.6GeV
    - Note: Babar, hep-ex/0607071, obtains 20% different value when extrapolating with two different schemes!!
- Analysis can be improved
  - Try to use MSOPE result up to E_0=1.8 GeV.
  - If shape-function is necessary: estimate uncertainty from unknown shape. Include known two-loop results?

Moments

M. Neubert, hep-ph/0506245





- MSOPE result for m_b and μ_π from moments consistent with $B \to X_c l v$ moment analysis.
- Knowledge of anomalous dimensions sufficient. Two-loop constants drop out.

Summary

- Combination of our NNLO evaluation of cut-effects with result of Misiak et al. for total rate leads to

$$\mathrm{Br}(\bar{B} \to X_s\gamma) = (2.98 \pm 0.26) \cdot 10^{-4} \quad \text{for } E_\gamma > 1.6\mathrm{GeV}$$

  - ${}^{+4}_{-6}\,\%$ perturbative, 4% parametric, 5% power corrections, 3% interpolation in m_c.
  - 1.4σ below exp. value. 1-2σ below NLO value. (Gambino Misiak '01 found BR=$(3.6\pm0.3)\times10^{-4}$ at NLO.)

- To do: improve extrapolation to experimental E_0
  - Use MSOPE result at experimental value of E_0?
  - Vary shape function model, include 2-loop effects.

Extra slides

Theory vs. Experiment

- Experimental average (HFAG)

$$\mathrm{Br}(\bar{B} \to X_s\gamma) = (3.55 \pm 0.24\,^{+0.09}_{-0.10} \pm 0.03) \cdot 10^{-4}$$

  - for cut $E_\gamma > E_0 = 1.6\mathrm{GeV}$
  - stat.+syst., extrapolation to low E_0, $b \to \gamma d$ subtr.

- Theory @ NNLO (hep-ph/0610067 with hep-ph/0609232)

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