Lattice calculations for B and K mixing

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outline

- introduction
- lattice systematics
- lattice determinations of B_K
- lattice determinations of heavy–light decay constants
- lattice determinations of B_{B_q}
- a lattice calculation of the $B_{(s)} \to D_{(s)}\ell\nu$ form factors
- outlooks

- at present the following experimental inputs allow **a** complete Unitarity Triangle Analysis (UTA) without the need of non–perturbative theoretical inputs:

α	from	$B \to \pi\pi, B \to \pi\rho, B \to \rho\rho$
γ	from	$B \to D^{(*)}K^{(*)}$
$2\beta + \gamma$	from	$B \to D^{(*)}\pi(\rho)$
$\cos(2\beta)$	from	$B_d^0 \to J/\psi K_S^{0*}$
β	from	$B \to D^0\pi^0$
$\sin(2\beta)$	from	$B_d^0 \to J/\psi K_S$

- on the other hand, the **full** UTA includes quantities that require non–perturbative theoretical inputs

$\frac{\|V_{ub}\|}{\|V_{cb}\|}$	requires	semil. form factors/HQET m. el.
ϵ_K	requires	B_K
ΔM_d	requires	$f_{B_d} B_{B_d}$
ΔM_s	requires	$f_{B_s} B_{B_s}$

- we are now in the position in which the hadronic matrix elements a part from the semileptonic form factors, can be extracted by the UTA with a quite accurate precision.

Q: do we still need lattice calculations? If yes, why?

- the UTA is able to extract the bag parameters and the decay constants *within the Standard Model*

A: yes we do. In order to test the Standard Model.

- in the **past** years the task of the lattice was to provide an *estimate* of the hadronic inputs to the UTA since we where talking about *unknown* quantities.

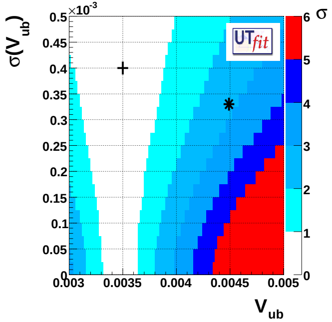
- **now** to the lattice it is asked to provide, let's say, ΔM_s *within the standard model* with an error of the same order of the one quoted by the CDF collaboration

- i.e. all the systematics have to be under control at a given level of accuracy:
  without any reasonable doubt!

- different groups are following different strategies to reach the same goal and in the end, hopefully, we will have the best we can ask: *the same result confirmed by different calculations*

- **presently** we have numbers still affected by largely different systematics ...

Q: is there any hierarchy between the different systematics? is quenching the worse systematic?



- the UT_{fit} collaboration has observed a "tension" between the exclusive and inclusive determinations of V_{ub}
- if the exclusive determination was not available we would have probably guessed new physics effects. . .
- . . . now we ask our self whether HQET and parton/hadron duality are under control in the inclusive determination

given a discrepancy between lattice and experiment

Q: would we claim new physics if the lattice result has been obtained by relying on effective theories and/or perturbation theory?

A: (non–renormalizable) effective theories

A: perturbative renormalization/matching

UNC: quenching

- quenching is definitively bad
    - the effect of the quenching is observable dependent and impossible to quantify without unquenched simulations in the same range of parameters
- at the same time quenched simulations have been able to predict $\sin 2\beta$

1995	$\sin(2\beta)$	$=$	0.650 ± 0.120	Ciuchini et al Z. Phys. C **68** (1995) 239
2000	$\sin(2\beta)$	$=$	0.698 ± 0.066	Ciuchini et al JHEP **0107** (2001) 013
exp	$\sin(2\beta)$	$=$	0.687 ± 0.032	

- the effectiveness of quenching may be due to a sort of "matching": one has to tune the quark masses and the lattice spacing by using experimental inputs

UNC: rooted staggered

- staggered fermions are introduced on the lattice by simulating the following quark action (actually in its improved version):

$$\bar{\chi} D_{\text{stag}} \chi = \sum_n \bar{\chi}_n \left[\sum_\mu \frac{\eta_{n,\mu}}{2} \left(U_{n,\mu} \chi_{n+\mu} - U^\dagger_{n-\mu,\mu} \chi_{n-\mu} \right) + m_0 \chi_n \right]$$

affected by doubling, i.e. it has $2^4 = 16$ one-component fermions

- rooting means that gauge configurations are generated according to the following partition function:

$$Z^{\text{root}}_{N_f=3} = \int DU e^{-S_g} \left\{ \det[D_{\text{stag}}(m_u)] \det[D_{\text{stag}}(m_d)] \det[D_{\text{stag}}(m_s)] \right\}^{1/4}$$

S. R. Sharpe@LATTICE 2006 [hep-lat/0610094]:

Q: **“Rooted staggered fermions: Good, bad or ugly?”**

A: **ugly**! in the sense that are affected by unphysical contributions at regulated stage that need a complicate analysis to be removed

other sources of systematics

see C Sachrajda talk
see T Onogi talk

- chiral pathologies (χE)
  - f_B, for example, is expected to diverge in the quenched chiral limit
  - use of NLO χPT formulas to extrapolate results gives complete control or large(reliable) errors
- cutoff effects (aE)
  - single and coarse lattice spacing
  - extrapolations with 3 or more lattice spacings gives complete control or large(reliable) errors
- finite volume effects (LE)
  - may be the dominant source of uncertainty in unquenched calculations of M_π, f_π, etc.

the bag parameter is defined as the matrix element of the following operator

$$O_1 = \bar{\psi}^i \gamma_\mu (1-\gamma_5) q^i \bar{\psi}^j \gamma_\mu (1-\gamma_5) q^j = O_{VV+AA} - O_{VA+AV}$$

$$\langle \bar{K}^0 | \hat{O}_1 | K^0 \rangle = \langle \bar{K}^0 | \hat{O}_{VV+AA} | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 \, B_K(\mu)$$

$$\langle \bar{B}_q^0 | \hat{O}_1 | B_q^0 \rangle = \langle \bar{B}_q^0 | \hat{O}_{VV+AA} | B_q^0 \rangle = \frac{8}{3} M_{B_q}^2 f_{B_q}^2 \, B_{B_q}(\mu)$$

the breaking of chiral symmetry on the lattice complicates considerably the renormalization pattern with respect to the continuum

- for Wilson fermions

G Martinelli Phys. Lett. B **141**, 395 (1984)
M Bochicchio et al Nucl. Phys. B **262**, 331 (1985)
A Donini et al Eur. Phys. J. C **10** (1999) 121

$$\hat{O}_{[VV+AA]} = Z_{11} \left[O_{[VV+AA]} + \Delta_{12} O_{[VV-AA]} + \Delta_{13} O_{[SS-PP]} + \Delta_{14} O_{[SS+PP]} + \Delta_{15} O_{[TT]} \right]$$

- for Staggered fermions additional $U(1)$ axial symmetry: mixing of 4 operators at one loop

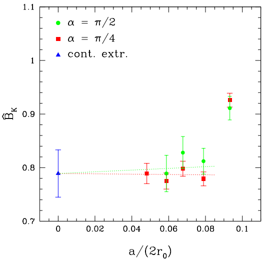
S Sharpe et al Nucl. Phys. B **417** (1994) 307

- for Ginsparg–Wilson fermions no mixing, the same renormalization pattern as in the continuum

The twisted mass QCD, in one of its variants, correspond to the following quark action

$$S_F = \int dx^4 \left\{ \bar{\psi} \left(\not{D} + m_l + i\mu_l\gamma_5\tau^3 \right) \psi + \bar{s} \left(\not{D} + m_s \right) s \right\}$$

ALPHA 06: P Dimopoulos et al. Nucl. Phys. B **749** (2006) 69



standard QCD is recovered at $\alpha = 0$ through

$$m_l^0 = m_l \cos\alpha + \mu_l \sin\alpha \qquad m_{u,d} = \sqrt{m_l^2 + \mu_l^2}$$

$$R_\alpha = e^{\frac{i\gamma_5\,\alpha\,\tau^3}{2}} \qquad \tan\alpha = \frac{\mu_l}{m_l}$$

$$\psi^0 = R_\alpha\ \psi \qquad \langle \mathcal{O}[\bar{\psi}, \psi] \rangle^0 = \langle \mathcal{O}[\bar{\psi} R_\alpha, R_\alpha \psi] \rangle$$

it has been observed

M Guagnelli et al JHEP **0603** (2006) 088

$$\langle M^0 | \hat{O}^0_{VV+AA} | \bar{M}^0 \rangle = -i \langle M^0 | \hat{O}_{VA+AV} | \bar{M}^0 \rangle$$

the parity–odd operator on the right renormalizes multiplicatively

UNC: (red) The calculation is quenched.

EFT: (green) Non–perturbative SF renormalization.

χE: (green) Interpolation of physical point plus estimate of $SU(3)$–breaking.

aE: (green) 5 lattice spacings.

LE: (green) Reliable estimate.

consider the Ward identities derived from

$$\delta u = \gamma_5 u \qquad \delta \bar{u} = \bar{u}\gamma_5$$
$$\delta d = -\gamma_5 d \qquad \delta \bar{d} = -\bar{d}\gamma_5$$

given the following interpolating operators

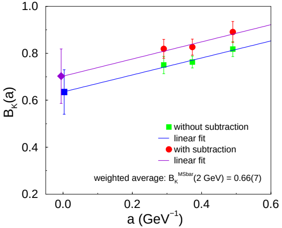
$$\begin{aligned}\Pi^0(x) &= \bar{d}(x)\gamma_5 d(x) - \bar{u}(x)\gamma_5 u(x)\\ K_P^0(t) &= \sum_{\vec{x}} \bar{d}(\vec{x},t)\gamma_5 s(\vec{x},t)\\ K_S^0(t) &= \sum_{\vec{x}} \bar{d}(\vec{x},t)s(\vec{x},t)\end{aligned}$$

one gets D Becirevic et al Phys. Lett. B **487** (2000) 74

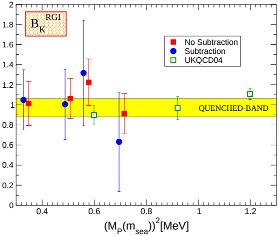
$$2\langle \hat{K}_P^0(t_1)\hat{Q}_1(0)\hat{K}_P^0(t_2)\rangle = 2m\sum_x \langle \Pi^0(x)\hat{K}_P^0(t_1)\hat{\mathcal{Q}}_1(0)\hat{K}_P^0(t_2)\rangle$$
$$-\langle \hat{K}_S^0(t_1)\hat{\mathcal{Q}}_1(0)\hat{K}_P^0(t_2)\rangle - \langle \hat{K}_P^0(t_1)\hat{\mathcal{Q}}_1(0)\hat{K}_S^0(t_2)\rangle + \mathcal{O}(a)$$

the parity–odd operator on the right renormalizes multiplicatively

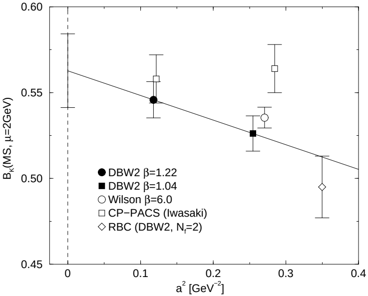
SPQ$_{\rm CD}$R 02: D Becirevic et al. Nucl. Phys. Proc. Suppl. **119** (2003) 359



SPQ$_{\rm CD}$R 05: F Mescia et al. PoS **LAT2005** (2006) 365



RBC 05: Y Aoki et al Phys. Rev. D **73** (2006) 094507



UNC: (orange) The calculation is quenched (heavy because of domain–wall). Reliable estimate of quenching errors by an alternative single simulation with the same action and coarser lattice spacing.

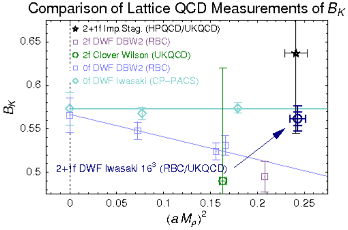
EFT: (green) Non–perturbative RI–MOM renormalization. No mixing due to residual symmetry breaking.

χE: (green) The calculation is done relying on isospin symmetry at $M_K(m_s/2,\, m_s/2)$ but good control and interpolation of the physical point.

aE: (orange) 2 lattice spacings.

LE: (red) No self estimate.

RBC-UKQCD 06: see S D Cohen talk



UNC: (green) The calculation is $N_f = 3$ domain–wall.

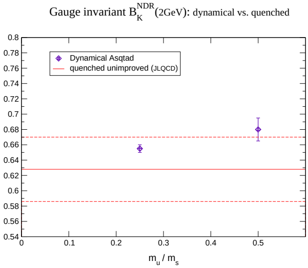
EFT: (green) Non perturbative RI-MOM renormalization. Neglected mixing due to the residual mass.

χE: (green) Under control thank to domain–wall.

aE: (red) 1 lattice spacing.

LE: (green) Reliable estimate. Second volume in progress (3 fm).

HPQCD-UKQCD 06: E Gamiz et al Phys. Rev. D **73** (2006) 114502



UNC: The calculation is $N_f = 3$ staggered.

EFT: Perturbative renormalization

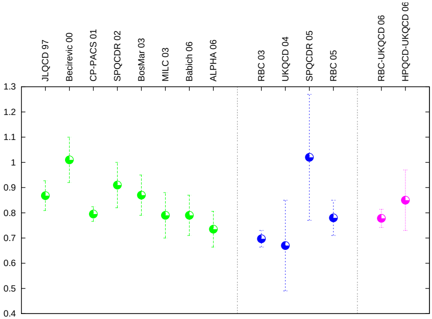
χE: Two sea quark masses. No estimate of $SU(3)$–breaking.

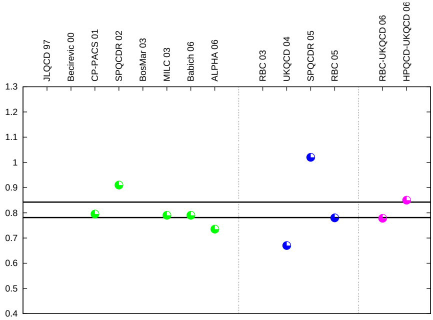
***a*E:** 1 lattice spacing. 2 sea quark masses

***L*E:** No self estimate.

the results do not show a significant dependence

- upon the number of flavours
- upon the renormalization procedure
- upon the finite size effects





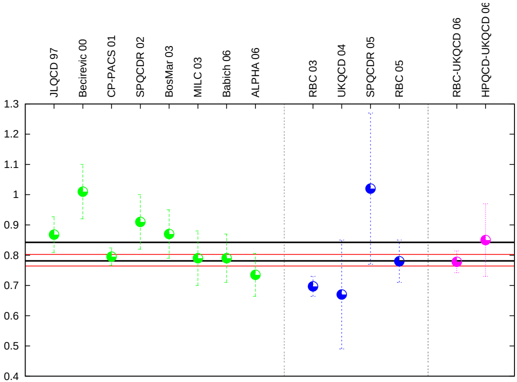
- a single calculation that it is able to control all the systematics is still missing. . .
  (RBC-UKQCD 06 is almost there)
- . . . but we have many different calculations that use different techniques, assumptions, approximations, etc. that all fall into the same ball park
- we can estimate the average by considering the "uncorrelated" calculations without the quoted errors

my average: $\hat{B}_K = 0.81(3)$

$$\frac{\Delta \hat{B}_K}{\hat{B}_K} = 4\%$$

unq. w. average: $\hat{B}_K = 0.78(2)$

$$\frac{\Delta \hat{B}_K}{\hat{B}_K} = 3\%$$



the red band shows the average of the unquenched data only.

- on currently affordable lattice sizes (at least in unquenched simulations) one has

$$am_b > 1 \qquad Lm_d > 1$$
$$am_b < 1 \qquad Lm_d < 1$$

- we are able to simulate "relativistic" beauty–light systems on
  - big volumes with big cutoff effects
  - small volumes with big finite volume effects

- so we have to devise smart strategies to cope with this *two scale problem*. we can divide the different approaches in
  - big volume strategies
  - finite volume strategies

HQET one can resort to the static approximation that can also be non–perturbatively renormalized

E Eichten et al Phys. Lett. B **234** (1990) 511
J Heitger et al JHEP **0402** (2004) 022
extensively discussed by Sachrajda

HQET EXT one can simulate the relativistic theory with heavy masses around the physical charm mass and extrapolate (or interpolate with the static) to the beauty mass relying on HQET predictions

Fermilab the fermilab approach consists in simulating the following action with $am_0 > 1$

$$S = \sum_n \bar{\psi}_n \left[m_0 + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - r_t \frac{aD_0^2}{2} - r_s \frac{a\vec{D}^2}{2} + c_B \frac{i\sigma_{ij}F_{ij}}{4} + c_E \frac{i\sigma_{0i}F_{0i}}{2} \right] \psi_n$$

i.e. the Symanzik effective action for quarks with $|a\vec{p}| \ll 1$ with *mass dependent* coefficients usually computed perturbatively

A X El-Khadra et al Phys. Rev. D **55** (1997) 3933
S Aoki et al Prog. Theor. Phys. **109** (2003) 383
N H Christ et al hep-lat/0608006

NRQCD another possibility consists in simulating one among the possible lattice discretizations of the NRQCD lagrangian at a given order in v^2 and α_s; for example

$$
\begin{aligned}
H &= -\frac{\Delta^2}{2m_b} + \delta H \\
\delta H &= -c_1 \frac{(\Delta^2)^2}{8m_b^3} + c_2 \frac{ig}{8m_b^2}(\vec{\Delta}\cdot\vec{E} - \vec{E}\cdot\vec{\Delta}) \\
&\quad - c_3 \frac{g}{8m_b^2}\vec{\sigma}\cdot(\vec{\Delta}\times\vec{\tilde{E}} - \vec{\tilde{E}}\times\vec{\Delta}) \\
&\quad - c_4 \frac{g}{8m_b^2}\vec{\sigma}\cdot\vec{\tilde{B}} + c_5 \frac{a^2\Delta^{(4)}}{24m_b} - c_6 \frac{a(\Delta^2)^2}{16m_b^2}
\end{aligned}
$$

A Gray et al Phys. Rev. D **72** (2005) 094507

the theory is non–renormalizable, can be matched to QCD only perturbatively, the continuum limit cannot be taken (heavy–light?)

SSM the Step Scaling Method has been introduced in order to deal with two–scale problems in lattice QCD.

M Guagnelli et al Phys. Lett. B **546** (2002) 237

it is based on a simple identity

$$f_B(m_b, m_d, \infty) = f_B(m_b, m_d, L_0) \underbrace{\frac{f_B(m_b, m_d, 2L_0)}{f_B(m_b, m_d, L_0)}}_{\sigma(m_b, m_d, L_0)} \underbrace{\frac{f_B(m_b, m_d, 4L_0)}{f_B(m_b, m_d, 2L_0)}}_{\sigma(m_b, m_d, 2L_0)} \cdots$$

the dependence of the step scaling functions upon the heavy quark mass can be predicted by using HQET both at numerator and denominator

$$\sigma(m_h, m_d, L) = \frac{f_B^0(m_d, 2L)\left(1 + \frac{f_B^1(m_d, 2L)}{m_h} + \ldots\right)}{f_B^0(m_d, L)\left(1 + \frac{f_B^1(m_d, L)}{m_h} + \ldots\right)} = \sigma^{\text{stat}}(m_d, L)\left(1 + \frac{f_B^1(m_d, 2L) - f_B^1(m_d, L)}{m_h} + \ldots\right)$$

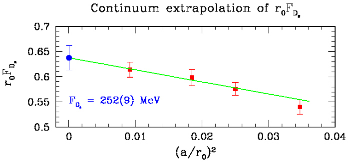
successfully applied also to the computation of the b–quark mass

$$\sigma(m_h, m_d, L) = \frac{M(m_h, m_d, 2L)}{M(m_h, m_d, L)} = \frac{m_h + \bar{\Lambda}(m_d, 2L) + \ldots}{m_h + \bar{\Lambda}(m_d, L) + \ldots} = 1 + \frac{\bar{\Lambda}(m_d, 2L) - \bar{\Lambda}(m_d, L)}{m_h} + \ldots$$

$m_b^{\overline{MS}}(m_b) = 4.33(10)$ MeV

M Guagnelli et al Nucl. Phys. B **675** (2003) 309

ALPHA 03: A Juttner et al Phys. Lett. B **560** (2003) 59



UNC: 🔴 The calculation is quenched.

EFT: 🟢 Fully non perturbative f_{D_s}.

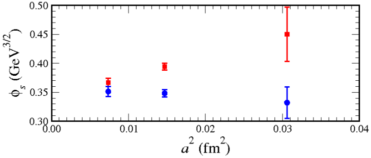
χE: 🟢 The strange quark is under control.

aE: 🟢 4 lattice spacings.

LE: 🟢 Estimate.

FNAL-MILC-HPQCD 06: C Aubin et al Phys. Rev. Lett. **95** (2005) 122002

FNAL-MILC-HPQCD 06: see Van de Water talk



UNC: 🟠 The calculation is $N_f = 3$ staggered.

EFT: 🟠 Fermilab approach perturbatively renormalized. They compute a perturbative factor to correct non perturbative Z_V mass dependent.

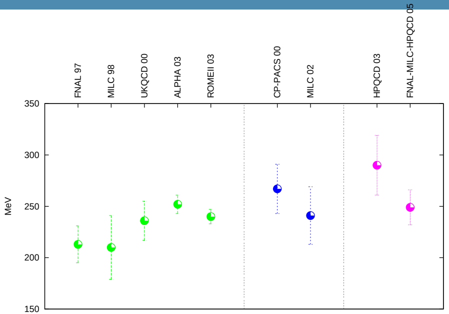
χE: 🟢 The strange quark is under control.

aE: 🟢 3 lattice spacings.

LE: 🟢 Estimated through χ-PT.

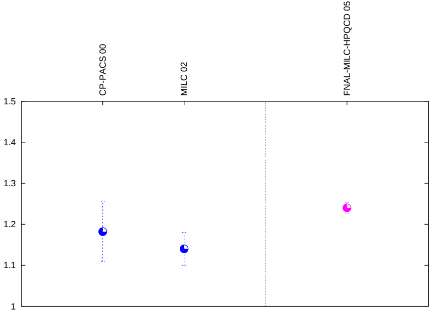
lattice calculations of f_{D_q}, summary

f_{D_s}:



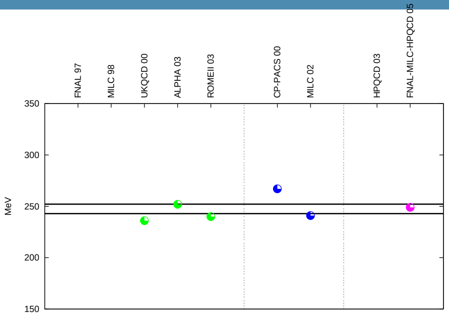
- the quenched results are very precise and not significantly different from unquenched ones
- at the same time unquenched numbers are systematically higher than the quenched ones

f_{D_s}/f_D:



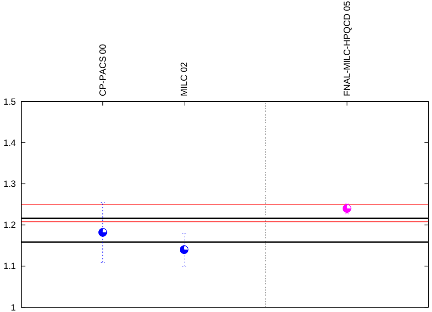
f_{D_s}:

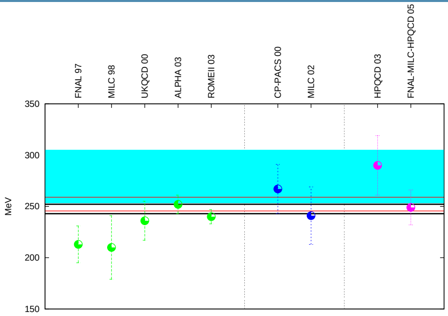
- again, a single calculation that it is able to control all the systematics is still missing...
- we can estimate the average by considering the "uncorrelated" calculations without the quoted errors

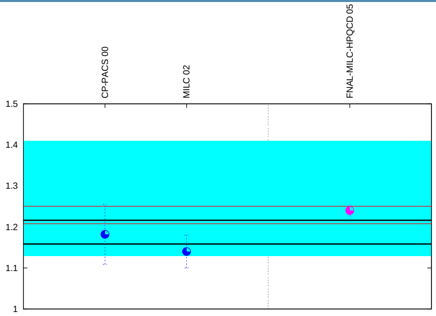


f_{D_s}/f_D:

- the number of uncorrelated results is too small: we can just consider a weighted average of the unquenched results (red band)
- as a "conservative" estimate we can also consider the linear regression without the errors (black band)







my average: $f_{D_s} = 248(5)$ MeV

$$\frac{\Delta f_{D_s}}{f_{D_s}} = 2\%$$

unq. w. average: $f_{D_s} = 252(7)$ MeV

$$\frac{\Delta f_{D_s}}{f_{D_s}} = 3\%$$

BaBar: $f_{D_s} = 283(17)(7)(14)$ MeV

B Aubert et al hep-ex/0607094

my average: $\frac{f_{D_s}}{f_D} = 1.19(3)$

$$\frac{\Delta R_D}{R_D} = 2\%$$

unq. w. average: $\frac{f_{D_s}}{f_D} = 1.23(2)$

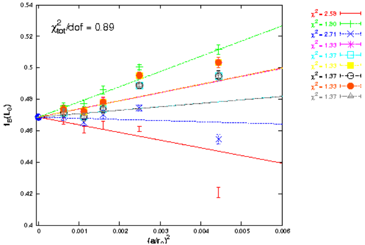
$$\frac{\Delta R_D}{R_D} = 2\%$$

BaBar+CLEO: $\frac{f_{D_s}}{f_D} = 1.27(14)$

M Artuso et al Phys. Rev. Lett. **95** (2005) 251801

ROMEII 03: G M de Divitiis et al Nucl. Phys. B **672** (2003) 372

ROMEII-ALPHA 06: D Guazzini et al PoS **LAT2006** (2006) 084



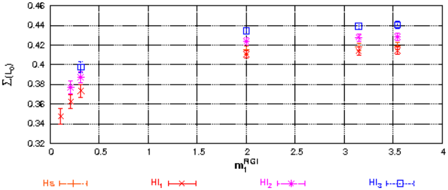
UNC: The calculation is quenched.

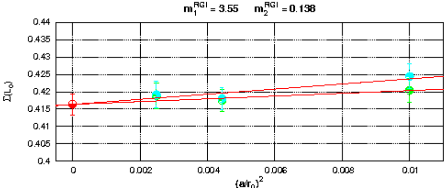
EFT: Fully non perturbative through SSM.

χE: The strange quark is under control.

aE: 4 lattice spacings.

LE: Naturally estimated.





lattice calculations of f_{B_s}, unquenched highlights

MILC 02: C Bernard et al Phys. Rev. D **66** (2002) 094501

UNC: The calculation is $N_f = 2$ staggered.

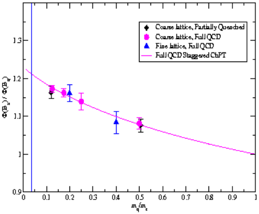
EFT: Fermilab perturbatively renormalized.

χE: The strange quark is under control.

aE: >3 lattice spacings.

LE: Estimate.

HPQCD 05: A Gray et al Phys. Rev. Lett. **95** (2005) 212001



UNC: The calculation is $N_f = 3$ staggered.

EFT: NRQCD perturbatively matched.

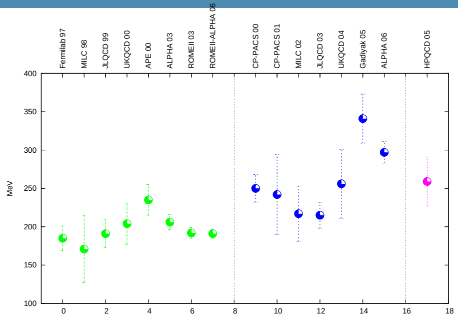
χE: The strange quark is under control.

aE: 2 lattice spacings.

LE: No self estimate.

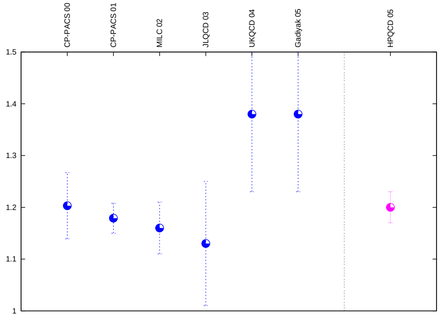
lattice calculations of f_{B_q},   summary

$\mathbf{f_{B_s}}$:



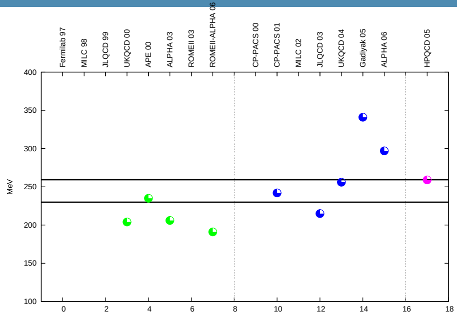
- the quenched results are very precise but significantly smaller than the unquenched ones
- this may signal a strong dependence of this observable upon the number of flavours

$\mathbf{f_{B_s}/f_B}$:



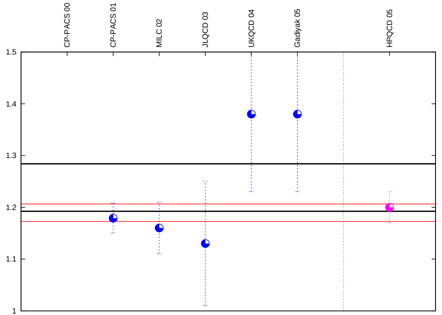
- the static numbers by Gadiyac et al 06, UKQCD 04 are identical and one sigma higher than the others

f_{B_s}:



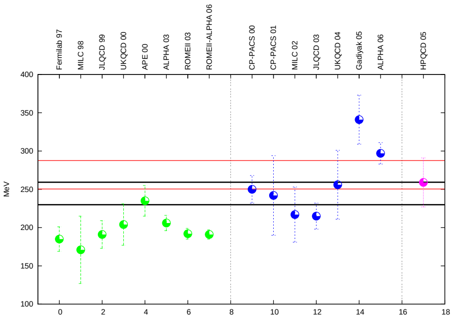
- again, a single calculation that it is able to control all the systematics is still missing...
- the un-weighted regression prefers the unquenched numbers...

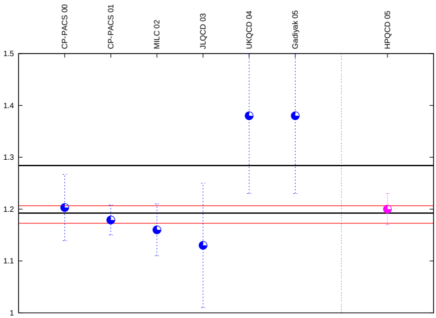
f_{B_s}/f_B:



- the un-weighted regression can be biased by the static results that have large errors (black band)

lattice calculations of f_{B_q}, summary





my average:

$$f_{B_s} = 245(15)\ \text{MeV}$$

$$\frac{\Delta f_{B_s}}{f_{B_s}} = 6\%$$

unq. w. average:

$$f_{B_s} = 269(19)\ \text{MeV}$$

$$\frac{\Delta f_{B_s}}{f_{B_s}} = 7\%$$

my average:

$$\frac{f_{B_s}}{f_B} = 1.24(5)$$

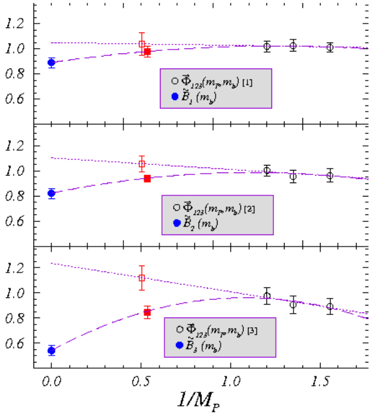
$$\frac{\Delta R_B}{R_B} = 4\%$$

unq. w. average:

$$\frac{f_{B_s}}{f_B} = 1.19(2)$$

$$\frac{\Delta R_B}{R_B} = 1.5\%$$

SPQCDR 01: D Becirevic et al JHEP **0204** (2002) 025



UNC: The calculation is quenched.

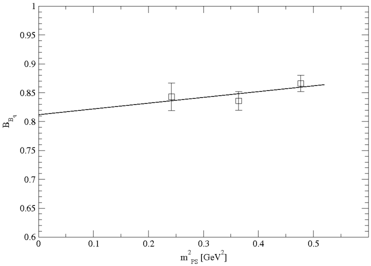
EFT: The calculation is at charm extrapolated at bottom with static point at NNLO. Non perturbative renormalization.

χE: The B_{B_q} observable does not show a sensible dependence upon the light quark mass.

***a*E:** 1 lattice spacing.

***L*E:** No self estimate.

V Gadiyak et al Phys. Rev. D **72** (2005) 114504



UNC: The calculation is $N_f = 2$ domain-wall.

EFT: Static approximation perturbatively renormalized.

χ**E:** The light extrapolation is under control thank to GW.

a**E:** 1 lattice spacing.

L**E:** Not self estimate.

HPQCD 06: E Dalgic et al hep-lat/0610104

the authors actually quote the combination

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 281(21)\ \text{MeV}$$

UNC: The calculation is $N_f = 3$ staggered.

EFT: The calculation is NRQCD, perturbative matching.

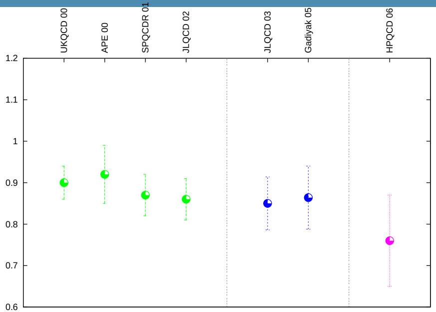
χ**E:** The B_{B_q} observable does not show a sensible dependence upon the light quark mass.

a**E:** 2 lattice spacings.

L**E:** No self estimate.

lattice calculations of B_{B_q}, summary

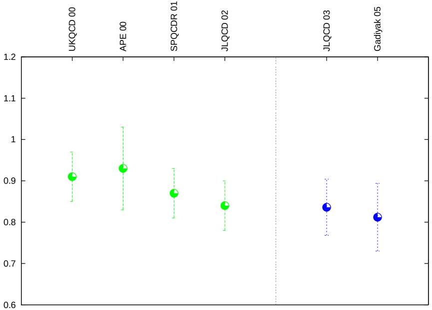
$B_{B_s}(m_b)$:



the observable does not seem to depend upon

- the number of flavours
- the renormalization systematics
- the heavy quark “technology”

$B_B(m_b)$:

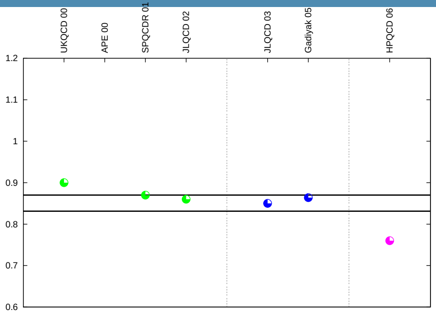


- there isn’t a sizable dependence even on the quark mass
- all the “dependencies” are factorized in the vacuum saturation approximation: decay constants.

lattice calculations of B_{B_q}, summary

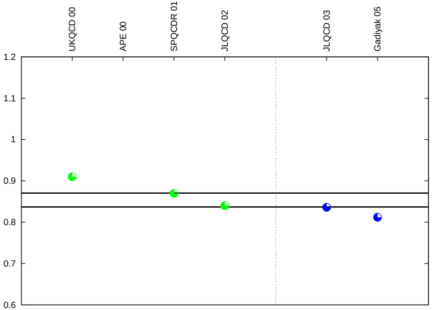
$B_{B_s}(m_b)$:

- again, a single calculation that it is able to control all the systematics is still missing. . .

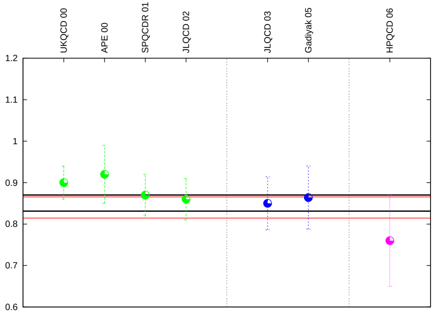


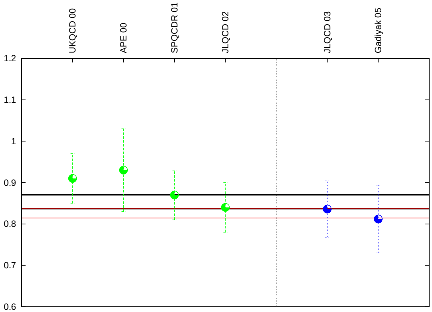
$B_B(m_b)$:

- we can afford a scatter fit also in this case. . .



lattice calculations of B_{B_q}, summary





my average: $B_{B_s}(m_b) = 0.85(2)$

$$\frac{\Delta B_{B_s}}{B_{B_s}} = 2\%$$

unq. w. average: $B_{B_s}(m_b) = 0.84(3)$

$$\frac{\Delta B_{B_s}}{B_{B_s}} = 3\%$$

my average: $B_B(m_b) = 0.85(2)$

$$\frac{\Delta B_B}{B_B} = 2\%$$

unq. w. average: $B_{B_s}(m_b) = 0.83(1)$

$$\frac{\Delta B_B}{B_B} = 2\%$$

the differential decay rate for the process$B \to D\ell\nu$ is given by

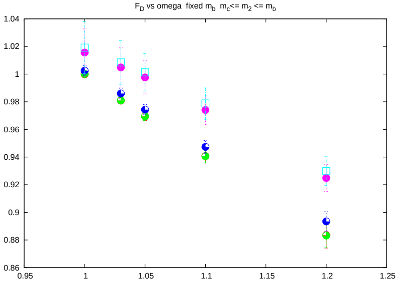
$$\frac{d\Gamma(B \to D\ell\nu)}{d\omega} = (\text{known factors})|V_{cb}|^2(\omega^2 - 1)^{\frac{3}{2}} F_D^2(\omega)$$
$$\omega = \frac{p_B \cdot p_D}{M_B M_D} = v_B \cdot v_D$$

we have applied the step scaling method to the calculation of $F_D(\omega)$

G M de Divitiis,E Molinaro,R Petronzio,N T

$$F_D(m_b, \infty\ |m_c, m_l, \omega) = \quad F_D(m_b, 0.4\ \text{fm}) \quad \underbrace{\frac{F_D(m_b, 0.8\ \text{fm})}{F_D(m_b, 0.4\ \text{fm})}}_{\sigma_{F_D}(m_b, 0.4\ \text{fm})} \quad \cdots$$

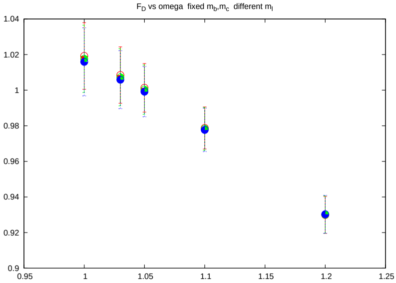
on the small volume, $L_0 = 0.4$ fm, we have $m_b = m_b^{phys}$:



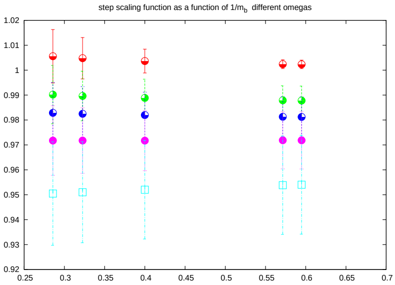
- here we see $F_D(m_b, m_1, m_l, \omega)$ at fixed m_b as a function of ω for different values of the charm quark mass in the range $m_c \le m_1 \le m_b$

- here we see $F_D(m_b, m_c, m_l, \omega)$ at fixed m_b and m_c as a function of ω for different values of the light quark mass in the range $m_s/5 \le m_l \le m_s$

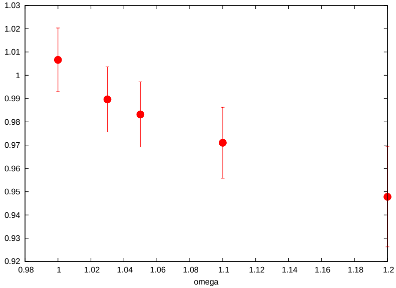
- there is absolute insensitivity of the form factor upon the light quark mass!



for the first step we have, $L_0 \longrightarrow 2L_0 = 0.8$ fm, we have $m_h \leq m_b/2$:

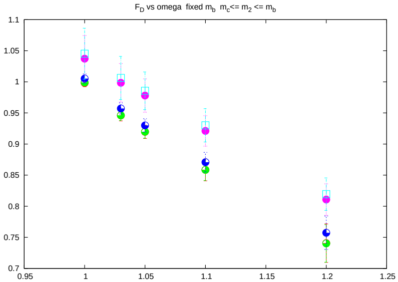


- here we see $\sigma_{F_D}(m_h, m_c, m_s, \omega)$ at fixed m_c as a function of $1/m_h$ for different values of ω
- the step scaling functions are absolutely flat within the errors

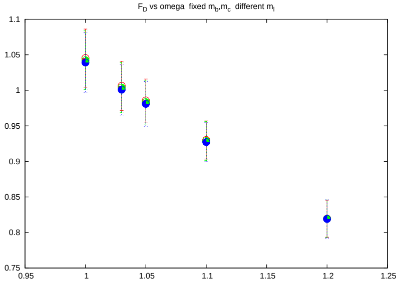


- here we see $\sigma_{F_D}(m_b, m_c, m_s, \omega)$ at fixed m_c and m_b (after the extrapolation) as a function of ω
- the step scaling functions are very close to 1 signaling very small finite volume effects at 0.4 fm!

on the "big" volume, $L = 0.8$ fm, we have:



- here we see $F_D(m_b, m_1, m_l, \omega)$ at fixed m_b as a function of ω for different values of the charm quark mass in the range $m_c \leq m_1 \leq m_b$
- for $m_c \simeq 2m_b/3$ the form factors enters the Isgur-Wise limit



- here we see $F_D(m_b, m_c, m_l, \omega)$ at fixed m_b and m_c as a function of ω for different values of the light quark mass in the range $m_s \leq m_l \leq 2m_s$
- the step scaling functions did not induced a dependence of the form factor upon the light quark mass!
- in the end we get the form factors with a relative precision of about 3%: $F_D(1.05) = 0.986(30)$

outlooks

- we (at least part of us!) got the message: complete control of the systematics!
- presently we can get a "good idea" about the systematics by combining the many different calculations
- tomorrow:
  - the way to B_K: wait for continuum limit from the RBC-UKQCD collaboration
  - the other way to B_K: unquenched Wilson fermions + "tmQCD strategy" and/or "4–point Green function strategy"

    L Del Debbio et al hep-lat/0610059
  - the way to f_B and B_B: + step scaling method (+ HQET non-perturbatively renormalized)