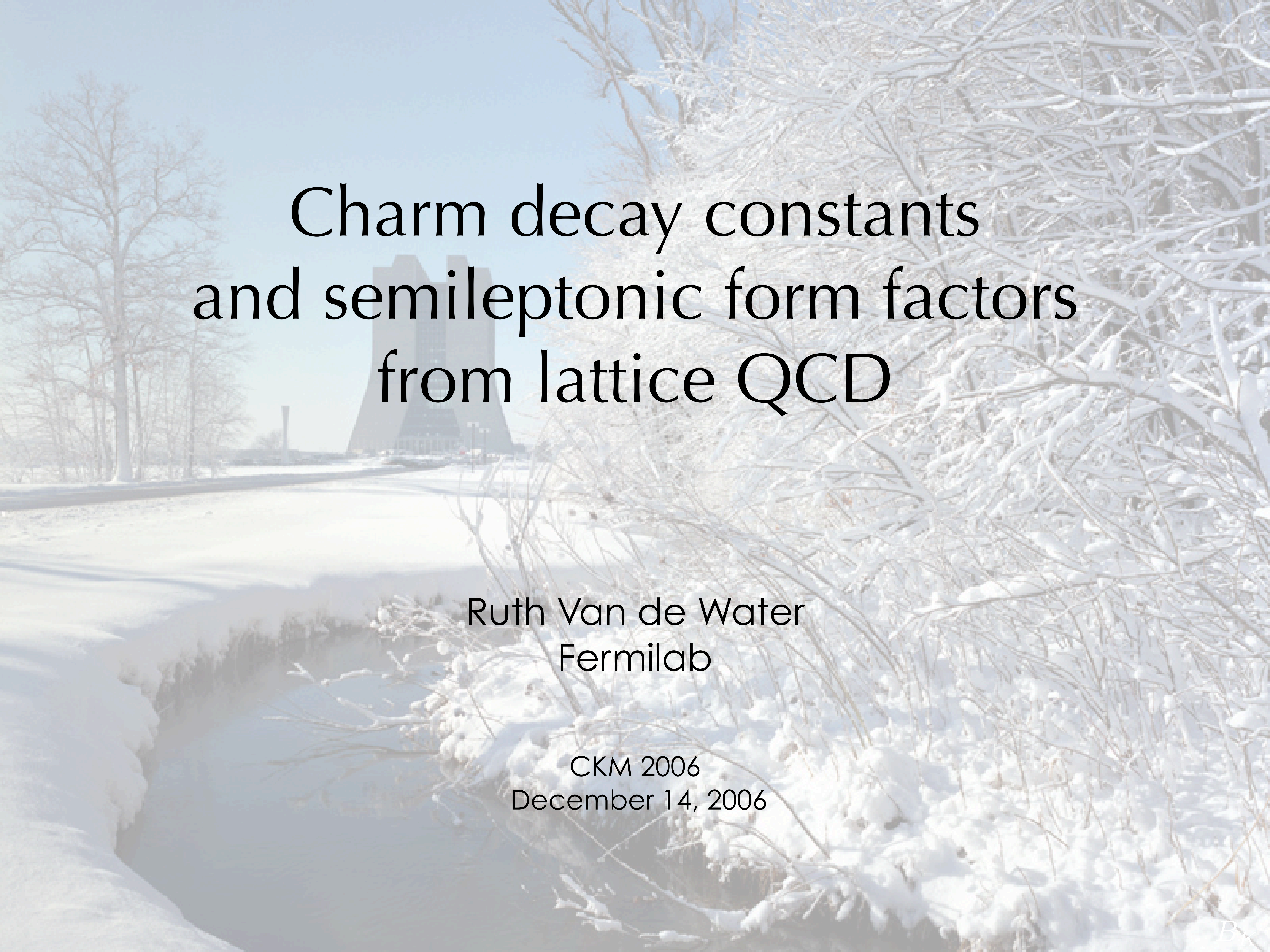


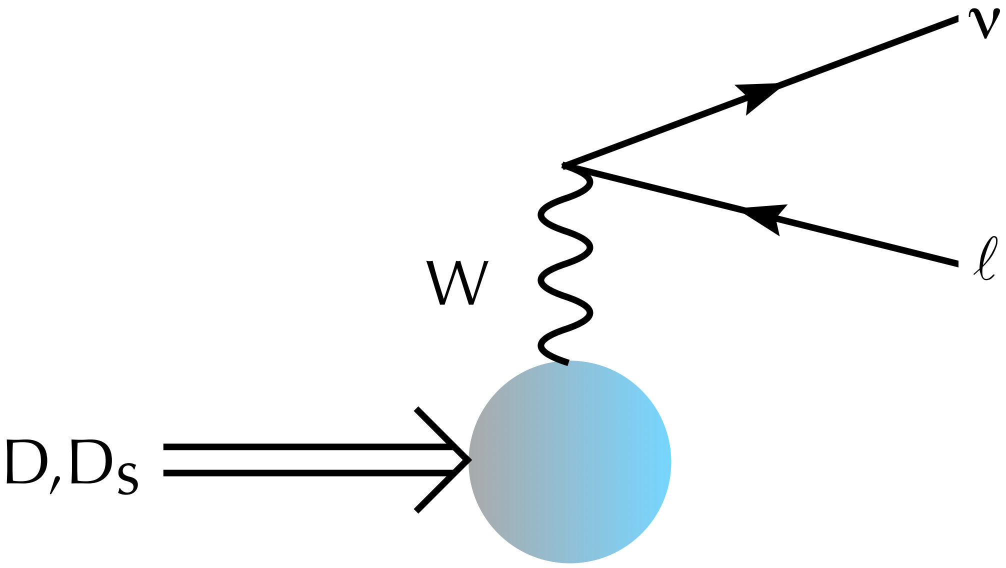
Charm decay constants and semileptonic form factors from lattice QCD

Ruth Van de Water
Fermilab

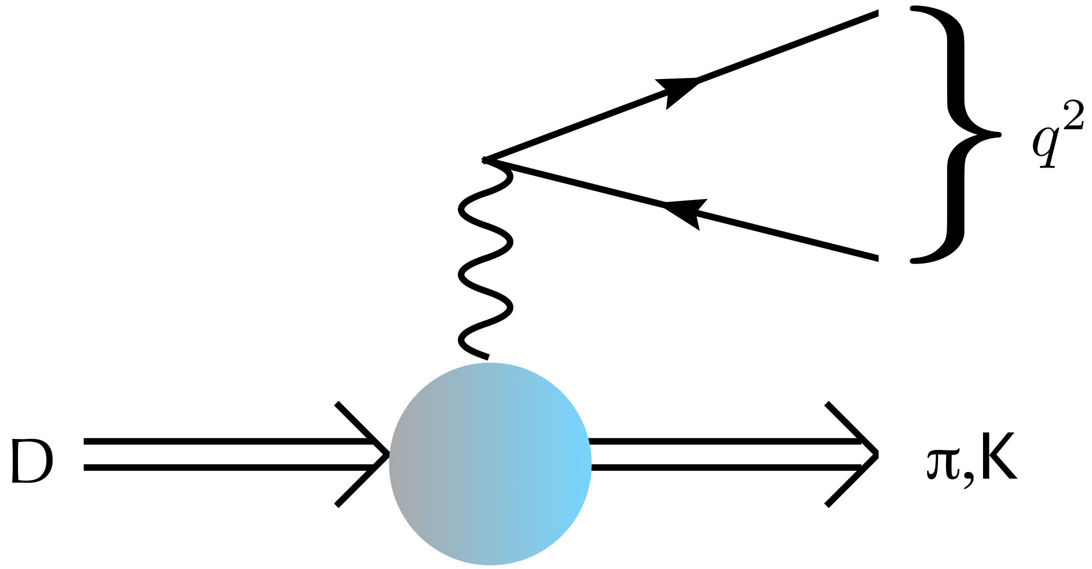
CKM 2006
December 14, 2006

Decays of charmed mesons

LEPTONIC DECAYS



SEMILEPTONIC DECAYS



$$\text{B.R.}(D \to \ell\nu) = (\text{known factor}) \times f_D^2 |V_{cd}|^2$$

$$\text{B.R.}(D_s \to \ell\nu) = (\text{known factor}) \times f_{D_s}^2 |V_{cs}|^2$$

$\Big\{$ f_{D,D_s} are the D-meson decay constants:

$$i f_{P_{ci}} p_\mu = \langle 0 | \bar{c} \gamma_\mu \gamma_5 q_i | P_{ci} \rangle$$

$f_+(q^2)$ are the D-meson form factors: $\Big\}$

$$\text{B.R.}(D \to \pi\ell\nu) = |V_{cd}|^2 \int_0^{q^2_{max}} dq^2 \, f_+^{D\to\pi}(q^2)^2 \times (\text{known factor})$$

$$\text{B.R.}(D \to K\ell\nu) = |V_{cs}|^2 \int_0^{q^2_{max}} dq^2 \, f_+^{D\to K}(q^2)^2 \times (\text{known factor})$$

- ✦ In both cases, experiments measure a **hadronic M.E. times a CKM element**

Why calculate f_{D,D_s} and $D \to \pi, K$ on the lattice?

(1) Can combine experimental measurements of branching fractions with lattice calculations of decay constants & form factors to extract $|V_{cd}|$, $|V_{cs}|$

(2) Can combine experimental measurements of branching fractions with values of $|V_{cd}|$, $|V_{cs}|$ from elsewhere to experimentally determine decay constants or form factors, then compare with lattice QCD calculations

- Approach #2 provides a **test of lattice QCD** methods, *e.g.*:
  - Dynamical (sea) quark effects
  - Light quark formalism
  - Heavy quark formalism
  - Chiral extrapolations
- Correct lattice QCD results for D-mesons *give confidence in similar lattice calculations with B-mesons*

Current lattice measurements of D decays

CAVEAT: This talk will be restricted to three-flavor unquenched lattice calculations

- Currently two groups calculating heavy-light meson quantities with three dynamical quark flavors: Fermilab/MILC & HPQCD
- Both use the publicly available “2+1 flavor” **MILC configurations** [Phys.Rev.D70:114501,2004] which have three flavors of improved staggered quarks:
  - Two degenerate light quarks and one heavy quark $(\approx m_s)$
  - Light quark mass ranges from $m_s/10 \leq m_l \leq m_s$
- Groups use **different heavy quark discretizations**:
  - Fermilab/MILC uses Fermilab quarks
  - HPQCD uses nonrelativistic (NRQCD) heavy quarks

Systematics in lattice calculations

- Lattice calculations typically quote the following sources of error:

  1. Monte carlo statistics & fitting
  2. Tuning lattice spacing, a, and quark masses
  3. **Matching lattice gauge theory to continuum QCD**
     - (Sometimes split up into relativistic errors, discretization errors, perturbation theory, ...)
  4. Extrapolation to continuum
  5. **Chiral extrapolation to physical up, down quark masses**

- Errors #3 and #5 are dominant sources of uncertainty in current heavy-light lattice calculations -- will discuss them in turn

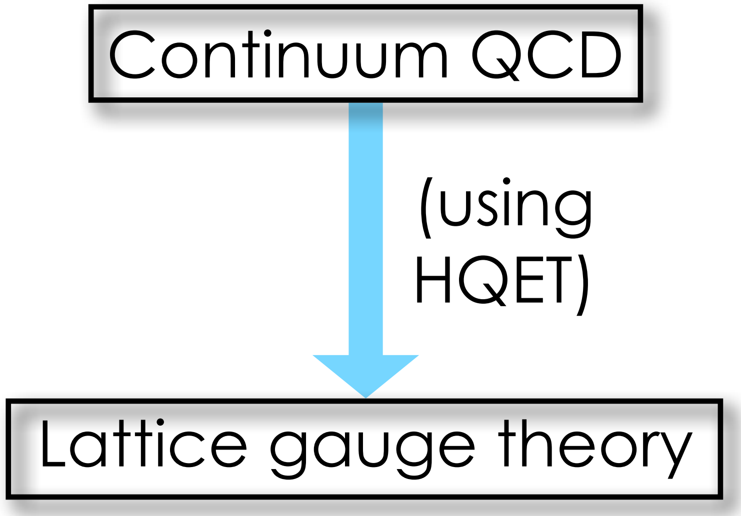
Heavy quarks on the lattice

PROBLEM: Generic lattice quark action will have discretization errors $\propto (am_Q)^n$

SOLUTION: Use knowledge of the heavy quark/nonrelativistic quark limits of QCD to systematically eliminate HQ discretization errors order-by-order

FERMILAB METHOD

[Phys.Rev.D55:3933-3957,1997]



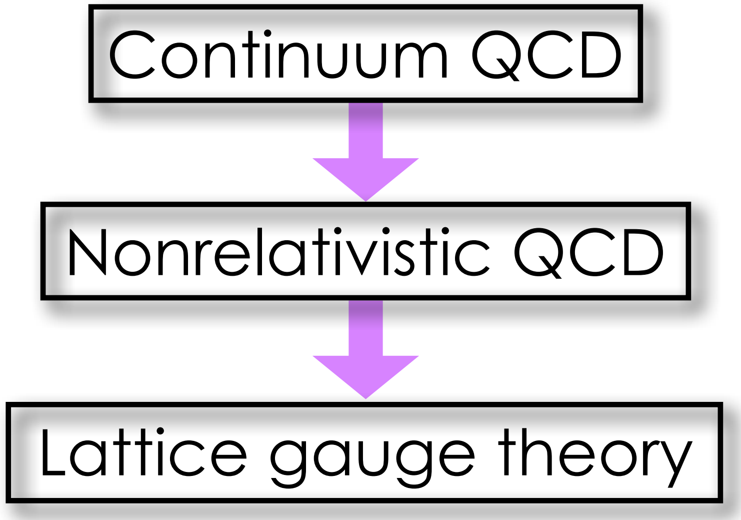
Continuum QCD

(using HQET)

Lattice gauge theory

LATTICE NRQCD

[Phys.Rev.D46:4052-4067,1992]



Continuum QCD

Nonrelativistic QCD

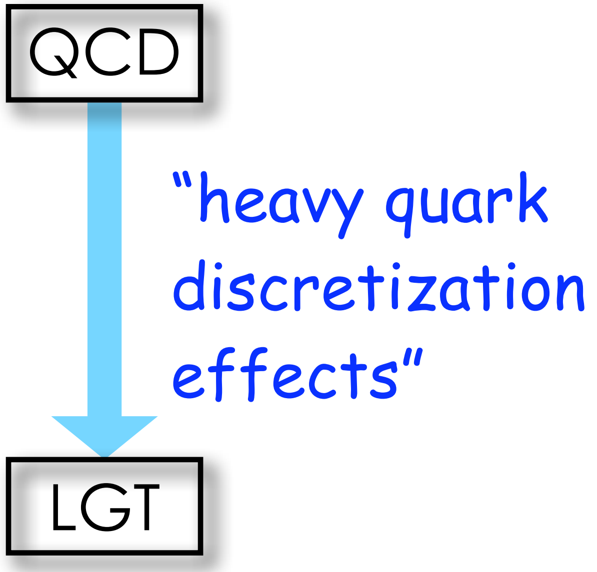
Lattice gauge theory

- Both methods require tuning parameters of lattice action
- For heavy-light decays, must also match lattice *currents* to continuum
- Typically calculate matching coefficients in lattice perturbation theory [Phys.Rev.D48:2250-2264,1993]

Matching errors

- ✦ In principle, can remove errors of any order in heavy quark mass, but, in practice, becomes increasingly difficult at each higher order
- ✦ $\Longrightarrow$ Must estimate size of errors due to inexact matching

FERMILAB METHOD



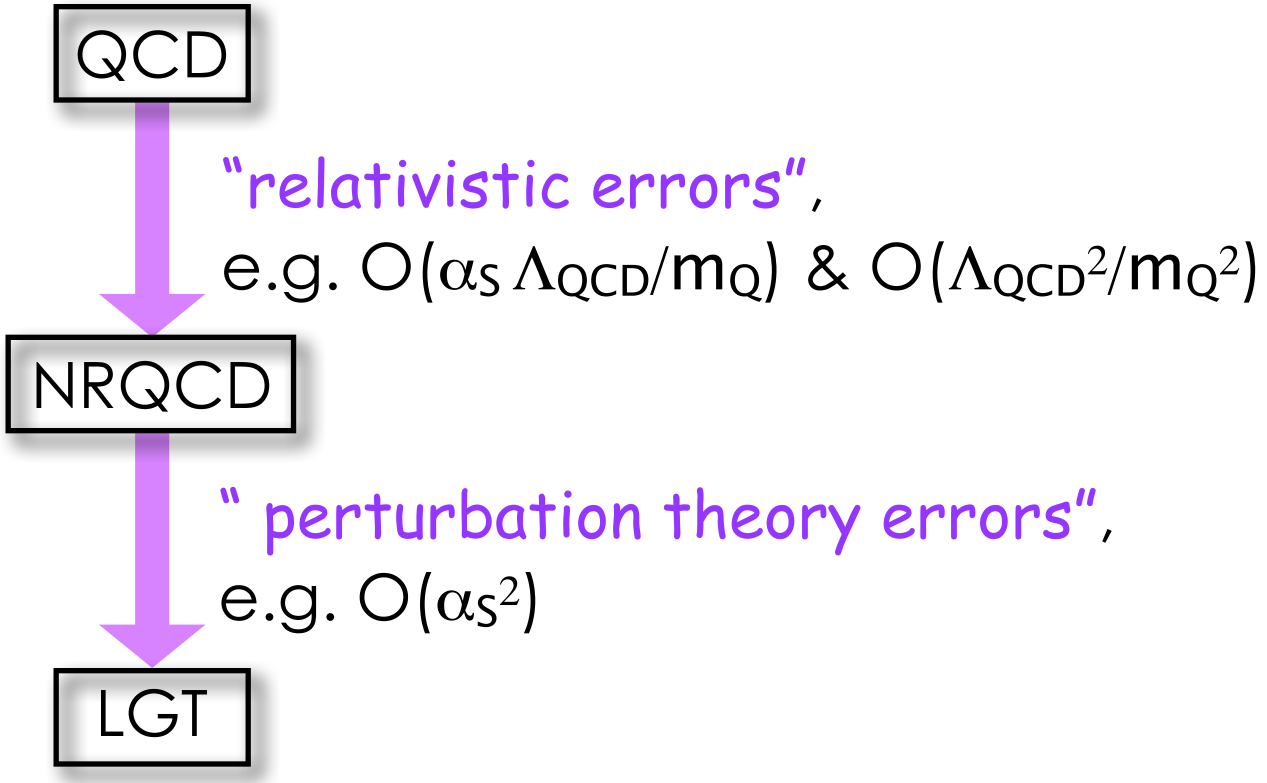
QCD

"heavy quark discretization effects"

LGT

- ❖ Combine all errors associated with discretizing action
- ❖ Estimate errors using knowledge of short-distance coefficients and power-counting

LATTICE NRQCD



QCD

"relativistic errors",
e.g. O($\alpha_S \Lambda_{QCD}/m_Q$) & O($\Lambda_{QCD}^2/m_Q^2$)

NRQCD

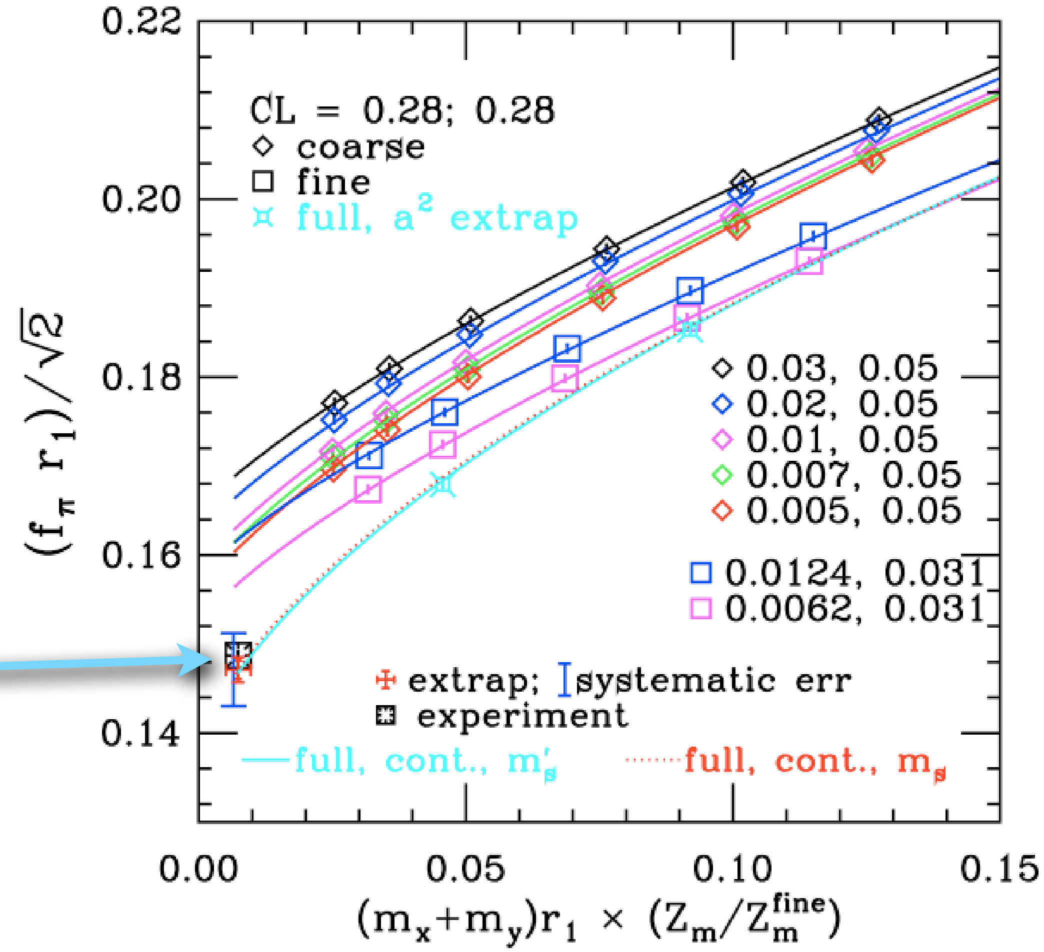
" perturbation theory errors",
e.g. O(α_S^2)

LGT

- ❖ Estimate errors using power-counting

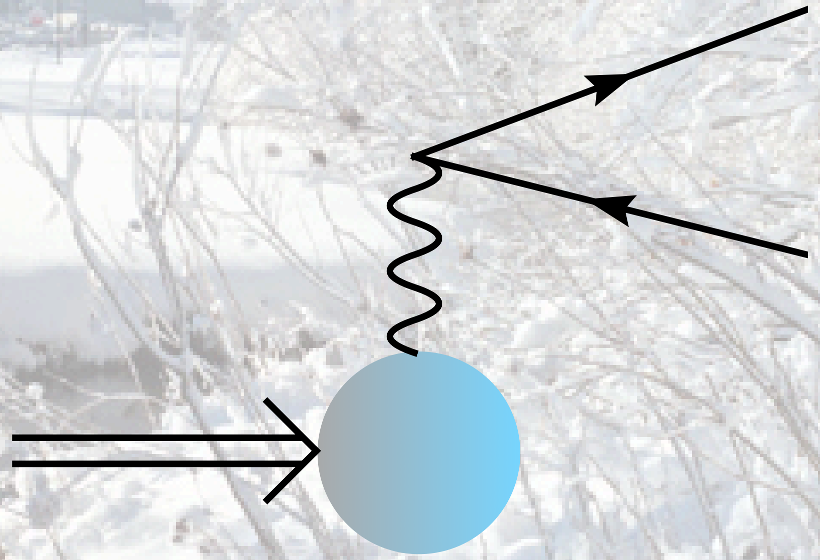
Chiral extrapolation of lattice data

- Must extrapolate lattice results to physical values of up, down quark mass
- For MILC 2+1 flavor lattices, must use **staggered chiral perturbation theory** [Lee & Sharpe, Aubin & Bernard, Sharpe & RV]
  - Accounts for next-to-leading order light quark mass dependence
  - Also accounts for light quark discretization effects through $O(\alpha_S^2 a^2 \Lambda_{QCD}^2)$
  - Extremely successful for light-light meson quantities such as f_π



- **Comment:** Staggered results agree with experimental values after chiral extrapolation in large part because the simulated quark masses are light and the lattice results are already close to the correct answer

Lattice results for f_{D,D_s}



HPQCD calculation of f_{D_s}

$$\boxed{f_{D_s} = 290 \pm 20 \pm 29 \pm 29 \pm 6 \text{ MeV}}$$

statistics & fitting

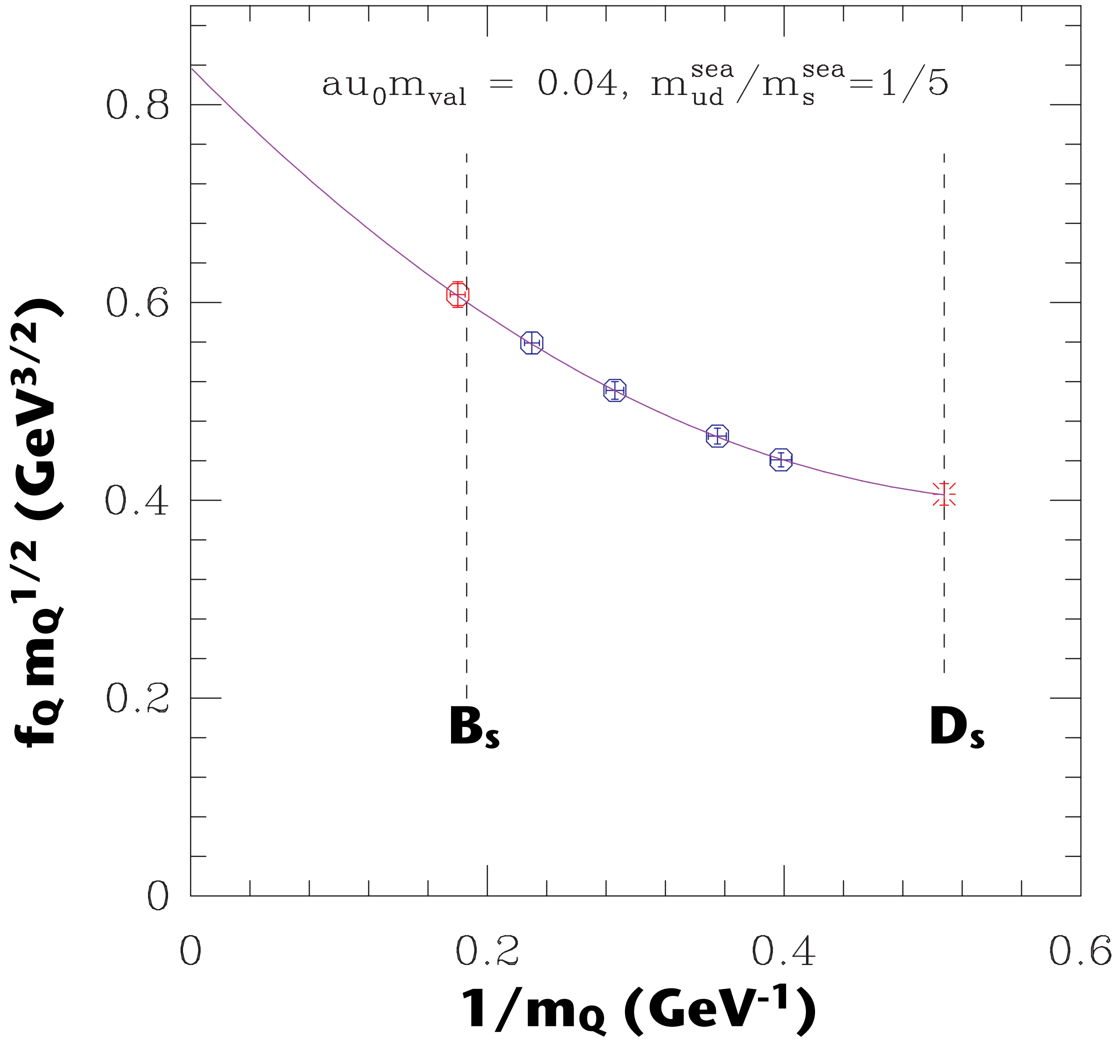
perturbation theory

relativistic corrections

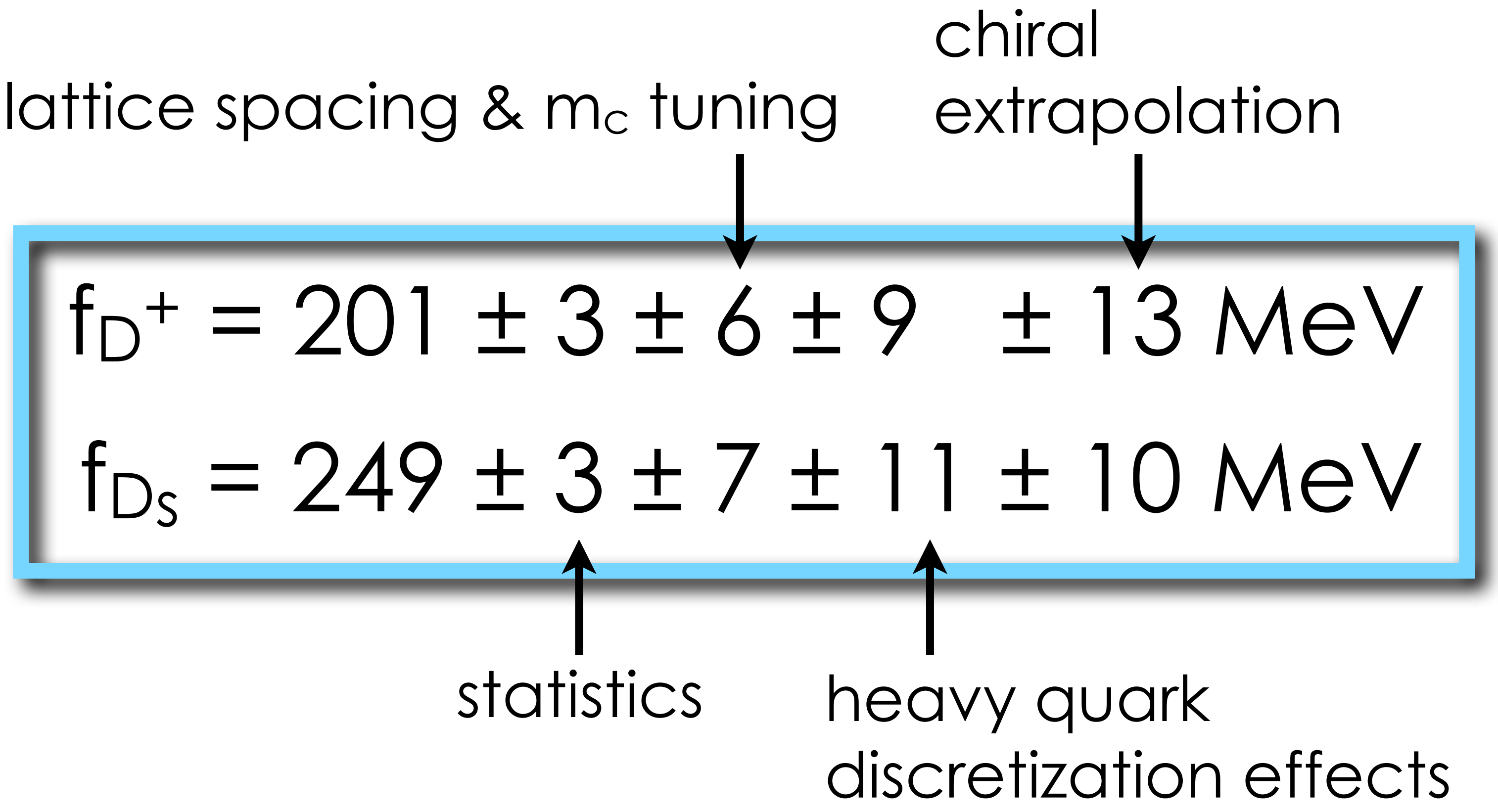
generic discretization effects

- ✦ Agrees with experiment:
  $f_{D_s} = 279 \pm 17 \pm 20$ MeV [BaBar]
- ✦ Statistical error dominated by **extrapolation of m_Q to charm** quark mass
- ✦ Perturbation theory error from **1-loop lattice-to-continuum operator matching**

[Phys.Rev.Lett.92:162001,2004]



Fermilab/MILC calculation of f_{D,D_s}



lattice spacing & m_c tuning

chiral extrapolation

$$f_{D^+} = 201 \pm 3 \pm 6 \pm 9 \quad \pm 13 \text{ MeV}$$

$$f_{D_s} = 249 \pm 3 \pm 7 \pm 11 \pm 10 \text{ MeV}$$

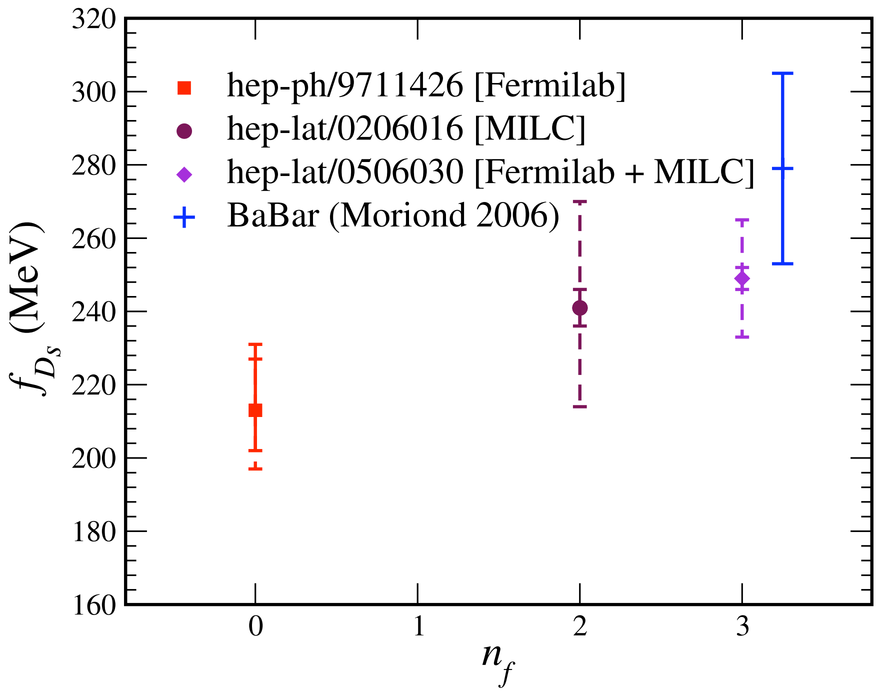
statistics

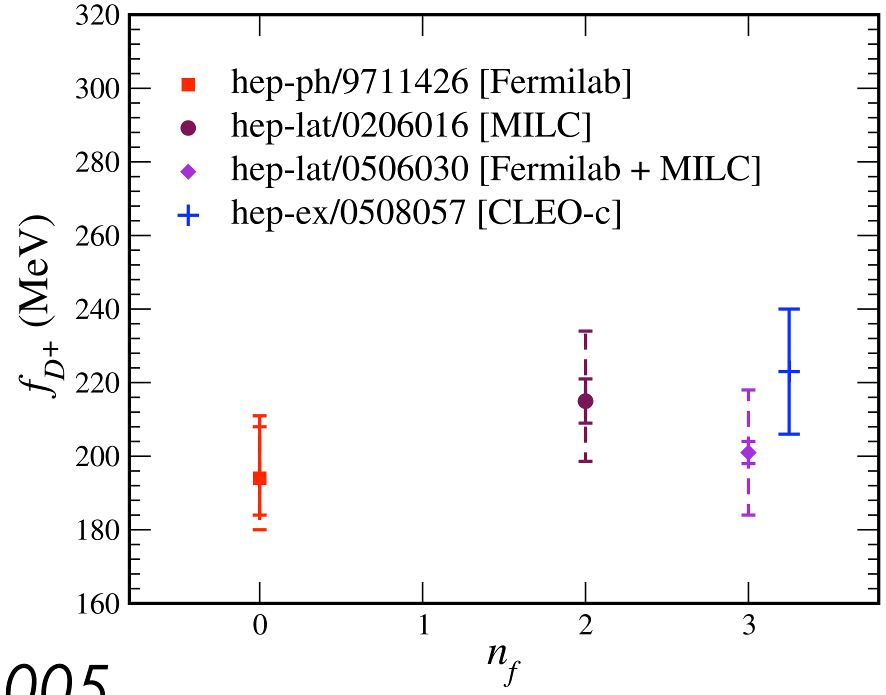
heavy quark discretization effects

[Phys.Rev.Lett.95:122002,2005]

- Simulate directly **at charm quark mass**
- Current matching **partly nonperturbative**
- f_{D^+}, f_{D_s} calculations preceded Cleo-c measurements ⇒ lattice *predictions*

Results finalized since CKM 2005





Potential sources of improvement

FOR HPQCD:

- 2-loop perturbative (or nonperturbative) matching
- **Highly-improved staggered quark (HISQ) action** to simulate directly at charm (*in progress* -- hep-lat/0610092)

FOR FERMILAB/MILC:

- 2-loop matching of heavy-light current ρ-factor
- Nonperturbative determination of clover coefficient in heavy-quark action (*e.g.* see Lin & Christ)
- Improved heavy-quark action (*in progress* -- Kronfeld & Oktay)

IN GENERAL:

- Lighter quark masses and finer lattice spacings
- Heavy-light calculations with **different light quark action**, e.g domain-wall (RBC) or overlap fermions (JLQCD)

Extension to f_B

- Successful predictions of f_D, f_{D_s} **lend confidence in lattice methods**
- The ratio of decay constants, in which several lattice uncertainties cancel, is particularly compelling:

$$R_{d/s}^{\text{lat.}} = 0.786 \pm 0.043$$
$$R_{d/s}^{\text{exp.}} = 0.779 \pm 0.093$$

[lat: Phys.Rev.Lett.95:122002,2005; exp: Cleo-c/BaBar]

- HPQCD f_B better than f_D because can simulate directly at b quark mass

$$f_B = 216(9)(19)(4\ )(6\ )\text{MeV}$$
$$f_{B_s}/f_B = 1.20(3)(1)$$

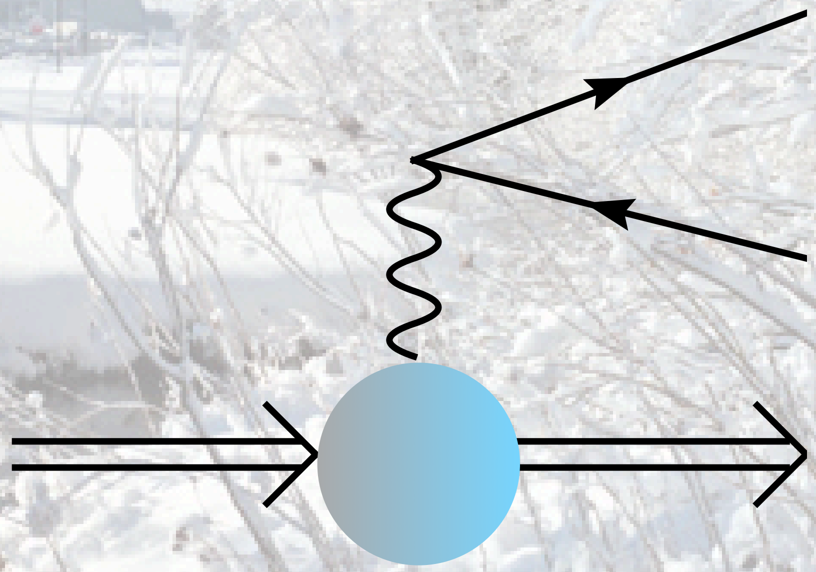
HPQCD:
Phys.Rev.Lett.95:212001,2005

- Fermilab/MILC f_B comparable to f_D, and heavy quark discretization errors somewhat smaller

Fermilab/MILC:
Simone, Lattice '06
(Preliminary)

$$f_{B_s}/f_{D_s} = 0.99(2)(6)$$
$$f_B/f_D = 0.95(3)(6)$$

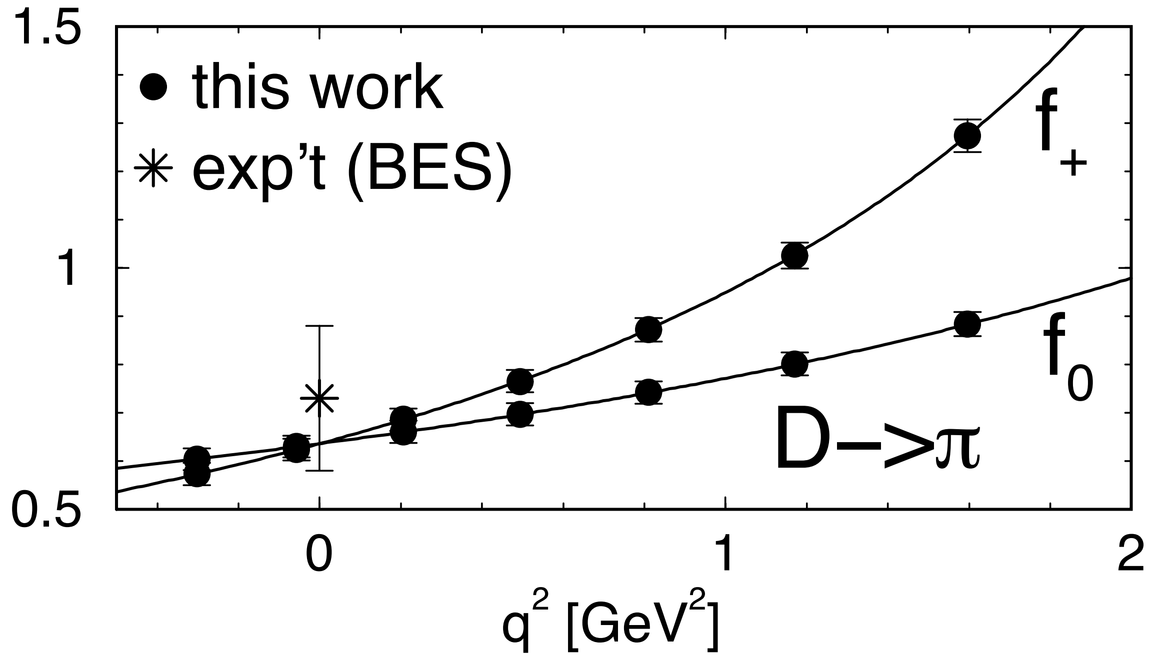
Lattice results for D→π,K



Fermilab/MILC calculation of D→π

$$f_+^{D\to\pi}(0) = 0.64\overset{\text{stat.}}{\overset{\downarrow}{(3)}}\overset{\text{sys.}}{\overset{\downarrow}{(6)}}$$

$$|V_{cd}| = 0.239\underset{\text{stat.}}{\underset{\uparrow}{(10)}}\underset{\text{sys.}}{\underset{\uparrow}{(24)}}\underset{\text{exp.}}{\underset{\uparrow}{(20)}}$$



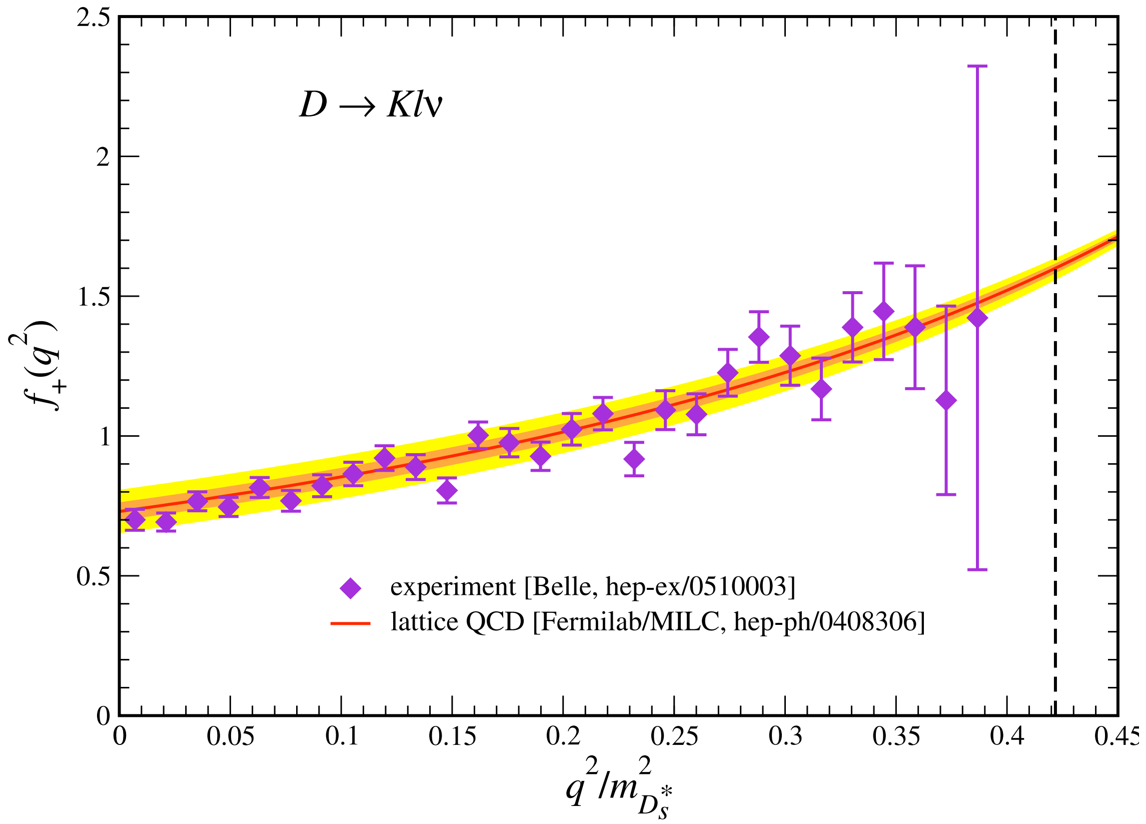
(Statistical errors only)

[Phys.Rev.Lett.94:011601,2005]

- Given $|V_{cd}|$, result for f(0) consistent with experiment
- Conversely, 14% measurement of $|V_{cd}|$ -- **error dominated by discretization effects:**
  - 5% from lattice momenta
  - 7% from heavy quark discretization

Fermilab/MILC calculation of D→K

- Form factor shape and normalization consistent with experiment
- Calculations preceded Focus, Belle, BaBar measurements
  ⇒ lattice *prediction*



sys.
stat.

$$f_+^{D\to K}(0) = 0.73(3)(7)$$

$$|V_{cs}| = 0.969(39)(94)(24)$$

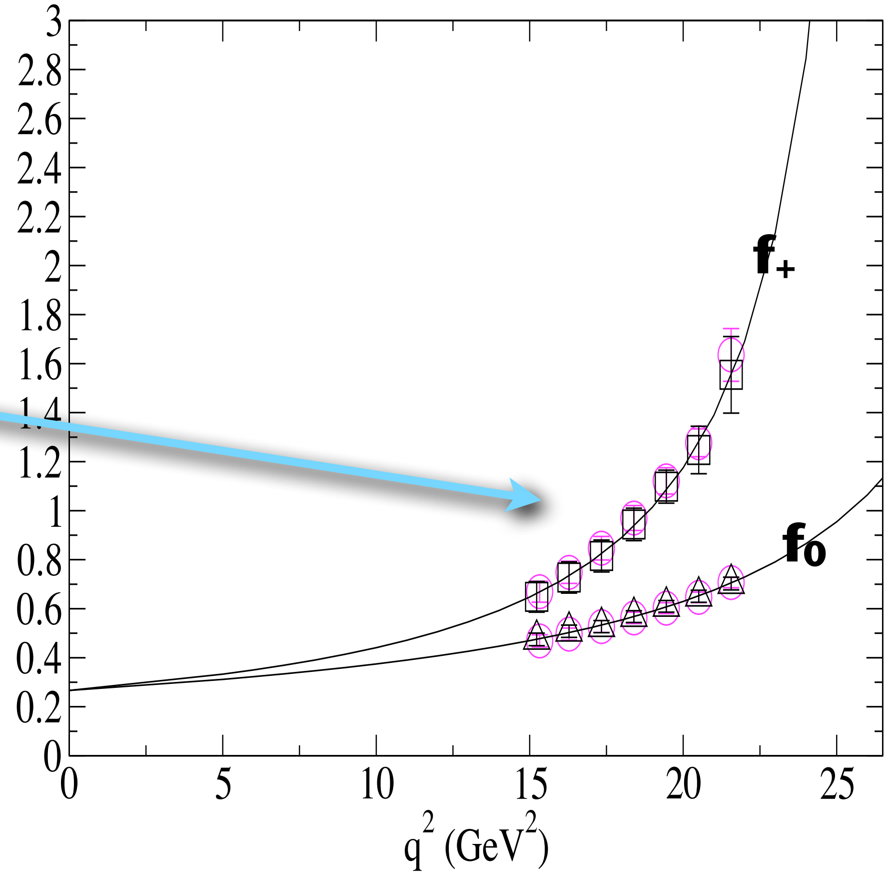
stat. sys. exp.

[Phys.Rev.Lett.94:011601,2005]

- 11% measurement of $|V_{cs}|$ -- **error dominated by discretization effects:**
  - 5% from lattice momenta
  - 7% from heavy quark discretization

Extension to $B\to\pi\ell\nu$

- Two essential differences in Fermilab/MILC error budgets for D and B semileptonic form factors:
  - Discretization error decreases from D- to B-decays: 9% → 7%
  - Extrapolation error from fit to q^2 dependence increases: 2% → 11% for f(0)
- Dominant error in D-decays is heavy-quark discretization
- Dominant error in B-decay is **q^2 extrapolation**
- While methods translate from D to B semileptonic decays, errors do not; each calculation **needs improvement in different areas**



[HPQCD: Phys.Rev.D73:074502,2006]

Potential sources of improvement

FOR FERMILAB/MILC:

- Same as for decay constants -- higher-order matching and improved action

IN GENERAL:

- Lighter quark masses and finer lattice spacings
- Additional lattice calculations

FOR B→π:

- Generate data at additional q^2 points -- two promising methods:
  - **Moving NRQCD:** generate lattice data at low q^2 (high pion momentum) while keeping statistical errors under control [Foley & Lepage; Davies, Lepage, & Wong]
  - **Twisted Boundary Conditions:** generate additional high q^2 data points with pion momenta at noninteger values of $2\pi/L$ (L = spatial lattice size) [Bedaque; Sachrajda & Villadoro]

Summary and outlook

- Leptonic and semileptonic D-decays allow ~10% determinations of CKM matrix elements $|V_{cd}|$, $|V_{cs}|$
- Also provide **important test of lattice QCD methods**
- In particular, lattice QCD had made **successful predictions** for:
    - Leptonic decay constants f_D, f_{D_s}
    - Shape of D→K form factor
- Give confidence in similar lattice calculations of B-meson quantities
- Ongoing effort to improve heavy-quark actions
- Ongoing effort to increase/improve lattice data at nonzero q^2
    - Possibly essential for less than 10% determination of $|V_{ub}|$ exclusive
- Progress is being made, but more work is necessary...