An estimate of the $B \to K^{*}\gamma$ decay form factor on lattice QCD

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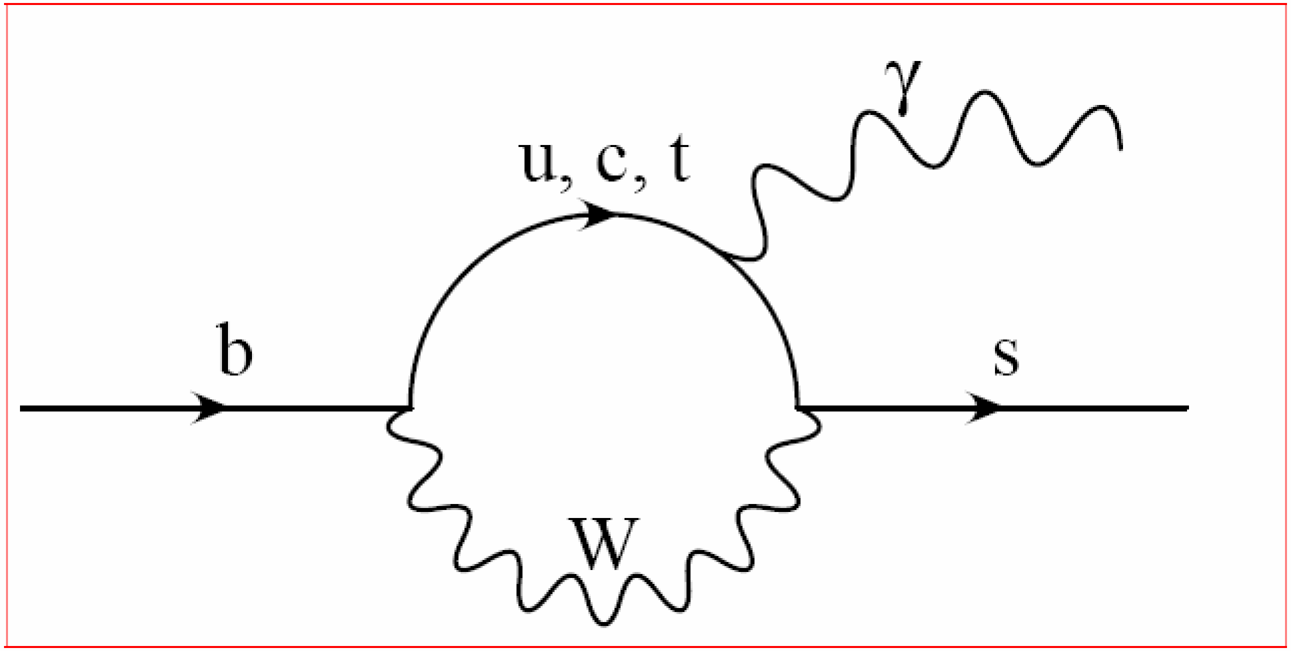
based on the work done with V. Lubicz and D. Becirevic

OUTLINE

- Introduction and Motivation:
- Novel Strategy on the Lattice and Results for $T_1(0)$
- Conclusions

Introduction

FCNC $b \to s$ processes are responsible for the decay $B \to K^* \gamma$



SM contribution occurs at 1-loop

- *window to heavy particles exchanged in the loops* $\leftrightarrow$ "New Physics"
- *exclusive decay* $\leftrightarrow$ "QCD non pert. effects: Lattice or QCD Sum Rules

In details …

$$\langle K^*\gamma | H_{eff} | B\rangle \simeq \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left(C_2 \langle \bar{s}\gamma_L^\mu c \cdot \bar{c}\gamma_L^\mu b \rangle + C_8 m_b \langle \bar{s}\sigma_R^{\mu\nu} b \cdot G^{\mu\nu} \rangle + C_7 m_b \langle \bar{s}\sigma_R^{\mu\nu} b \cdot F^{\mu\nu} \rangle \right)$$

$$B(B^0 \to K^{*0}\gamma) = \tau_B \frac{G_F^2 \alpha m_B^3 m_b^2}{32\pi^4} \left(1 - \frac{m_{K^*}^2}{m_B^2}\right)^3 |V_{cs}^* V_{cb} \cdot a_7|^2 \, T_1(0)^2$$

$\delta B/B \sim 10\%$

Belle-Babar-Cleo

$$a_7 = C_7 + O(a_s, 1/m_b)$$

under factorisation hypothesis!!

Bosch-Buchalla, Beneke-Feldmann-Seidel, Ali-Lunghi-Parkhomenko, Becker-Hill-Neubert

$T_1(0)$, tensor form factor at $q^2=0$

$T_1(q^2)$ are defined as:

$$\langle K^*(p', e_\lambda) | \bar{s}\sigma_{\mu\nu} b | B(p)\rangle = c^{(1)}_{\mu\nu} T_1(q^2) + c^{(2)}_{\mu\nu} T_2(q^2) + c^{(3)}_{\mu\nu} T_3(q^2)$$

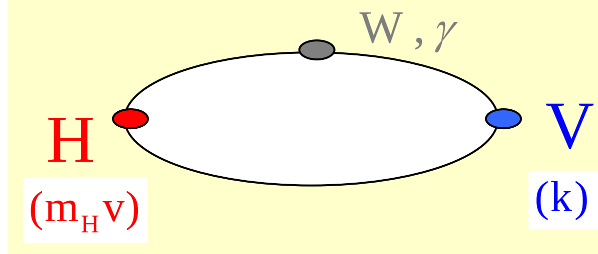
$c^{(1,2,3)}$-known functions of the kinematical variables $(p, p', e_\lambda, m_{K^*}, m_B)$

$T_{123}(q^2)$ – form factors relevant to $B \to K^* \ell\ell$ –

in $B \to K^*\gamma$, *photon is on-shell:*

$q^2=0$, $T_1(0)=T_2(0)$ and $T_3(0)$ irrelevant $(c^{(3)}_{\mu\nu}=0)$

Form Factors *on the lattice*



$$q^2 = m_H^2 + m_V^2 - 2 m_B v.k$$

Difficulty: Accessible Lattices are not fine enough to accommodate $m_b \sim 4.2$GeV

doable setup: $a^{-1} \leq 4$ **GeV** $\Rightarrow$ $m_H \sim 2\text{-}3$GeV, $m_V \sim m_{K^*}$

How to reach m_B?

a) HQET scaling laws: $m_H \to \infty$, **v.k fixed** [k~0] → ***large*** q^2

$\Rightarrow$ $B \to \pi \ell \nu$ f.fs in the experimental range $16\text{GeV}^2 < q^2 < 26\text{GeV}^2$.

Lattice strategies:

1. QCD with propagating heavy q. $a m_b \ll 1$ & $0 \leq q^2 \leq (m_H - m_V)^2$
2. eff. actions on the lattice (FNAL-NRQCD: static limit + $1/a m_b$ terms) $\Rightarrow$ $q^2 \cong q^2_{max}$

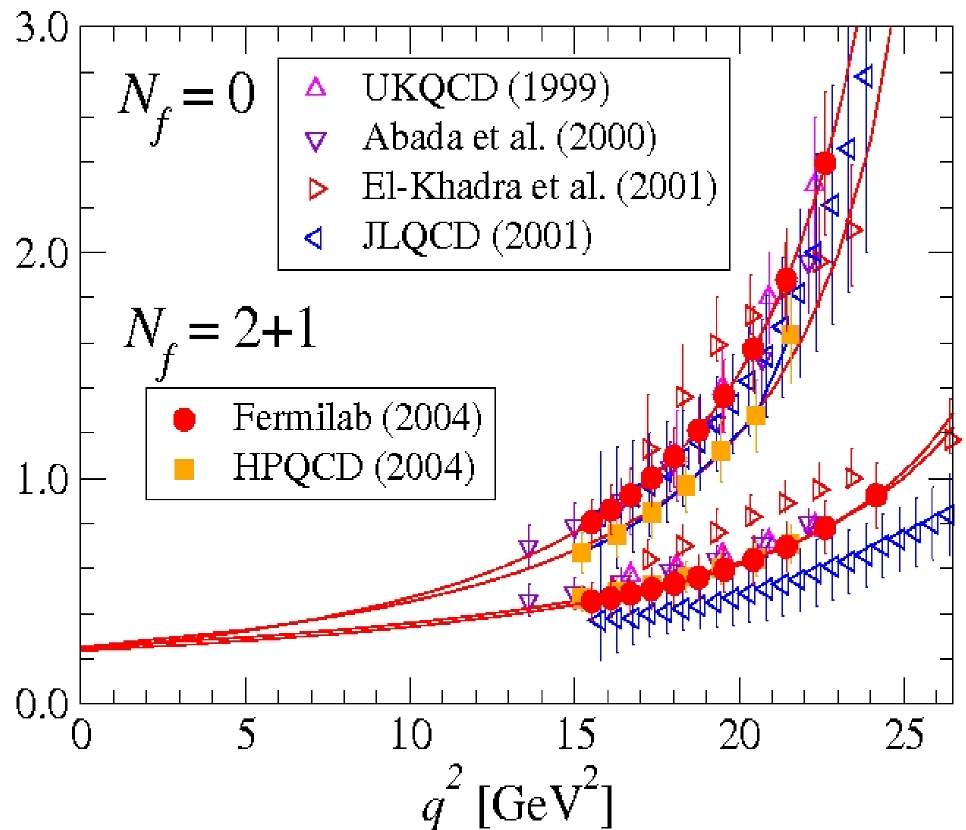
Lesson from B→ $\pi\ell\nu$: method 1

a) HQET scaling laws: $m_H\to\infty$, v.k fixed⇒ ***large q^2***

⇒ B→ $\pi\ell\nu$ f.fs in the experimental range $16GeV^2 < q^2 < 26GeV^2$.

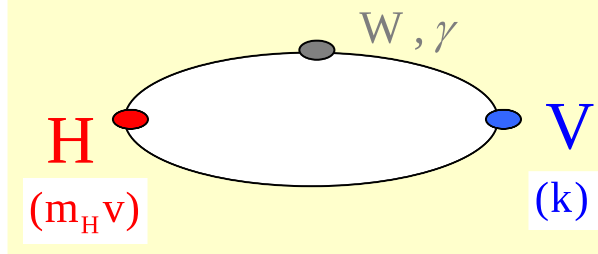
- Overall agreement between $N_F=0$ and first $N_F=2+1$ results, within present accuracy (*better precision is needed!*);
- Results from different approaches and groups agree well:

B→ $\pi\ \ell\ \nu$: f_+, f_0



See Jonathan's talk?

Form Factors *on the lattice*



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Lattice strategies:

1. QCD with propagating heavy q. $am_b \ll 1$ & $0 \leq q^2 \leq (m_H - m_V)^2$
2. eff. actions on the lattice (FNAL-NRQCD: static limit + $1/am_b$ terms) $\Rightarrow$ $q^2 \cong q^2_{max}$

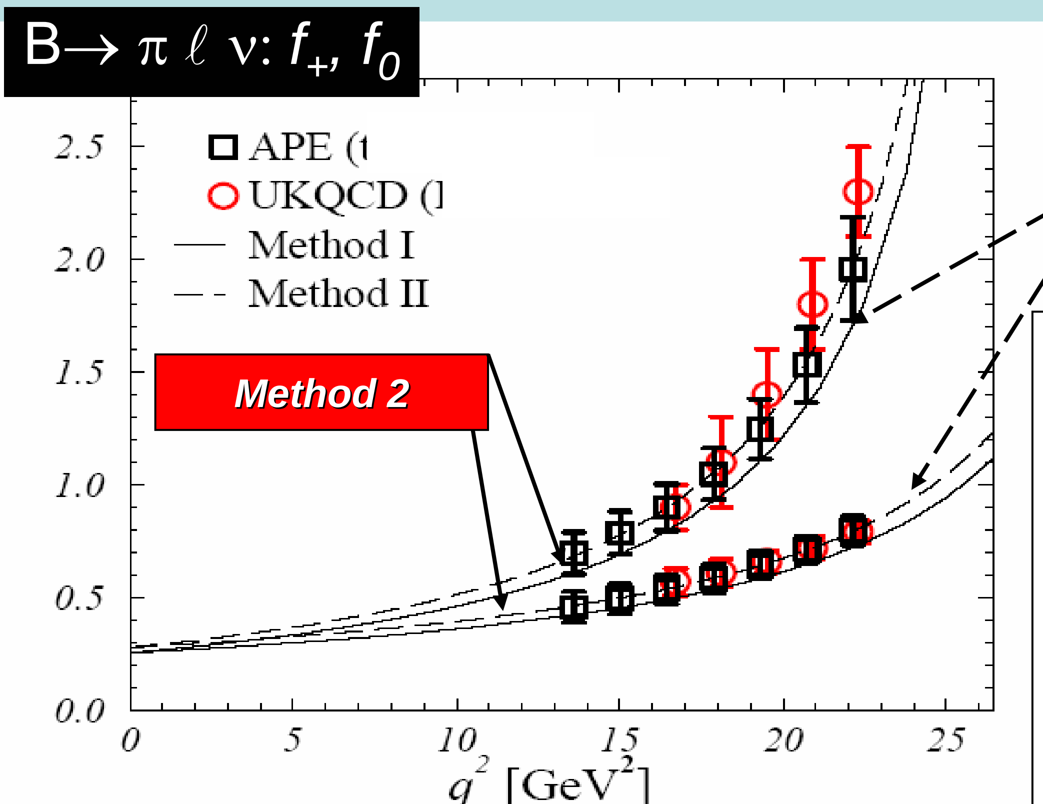
b) SCET scaling laws: $m_H \to \infty$, $E_V \to m_H/2$ $[k \sim m_H/2]$ $\Rightarrow$ $q^2 \cong 0$

$\Rightarrow$ $B \to K^* \gamma$ f.fs at $q^2 = 0$.

Approaches by eff. actions are compromised $\Rightarrow$ *large extrapolation on* q^2

Lesson from B→ $\pi\ell\nu$: method 2

B→ $\pi\ell\nu$ f.fs was also studied from
highly energetic light meson (low q^2), where LEET/SCET scaling law works.



The two approaches turn out to be fully compatible for B→ $\pi\ell\nu$, with

- "method 1" favoured;
- "method 2" as good cross-check⇒ *model dependence on q^2*

Now for *B→ $K^*\gamma$ f.fs at q^2=0,* situation is reverse, with "method 1" plagued by model dependence.

In details our strategy for B→ $K^*\gamma$ f.fs

STRATEGY FOR $T_1(0)$
D.Becirevic, F.M.,V. Lubicz hep-ph-0611295

Lattice setup:

QCD action: 1.2 GeV<m_H<3 GeV, a^{-1}~4 GeV

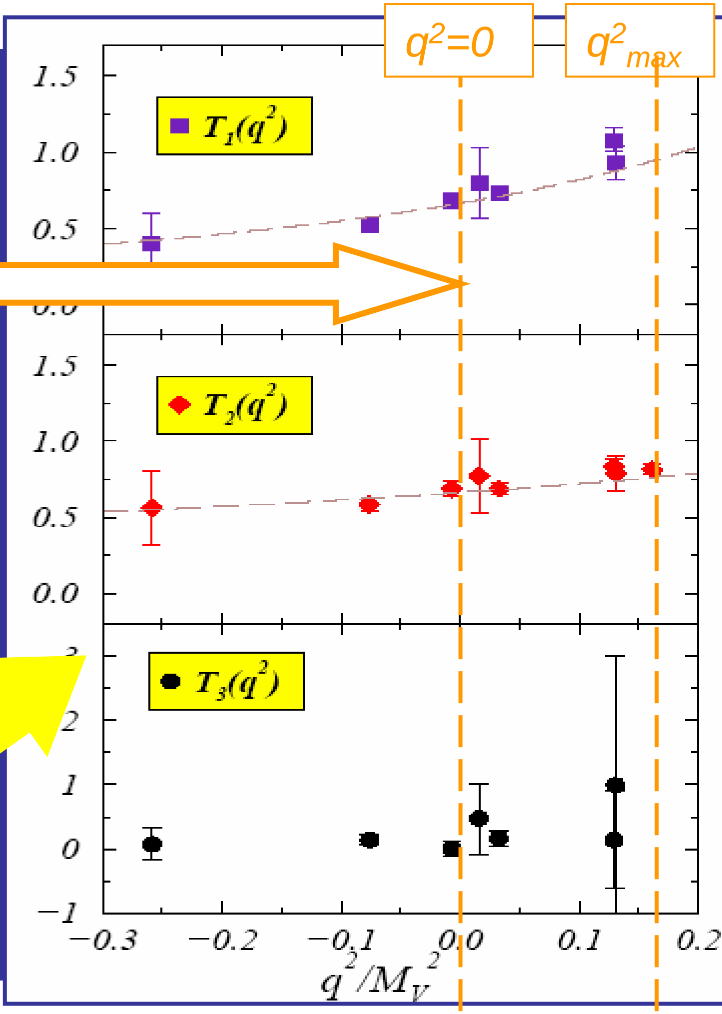
$\Rightarrow$ *$T_1^{H\rightarrow V^*}$ at simulated masses are around $q^2=0$*

$$\langle V|\bar{q}\sigma_{\mu\nu}Q|H\rangle = e^{\alpha}\,\varepsilon_{\mu\nu\alpha\beta}P^{\beta}\,T_1(q^2) + e^{\alpha}\,\varepsilon_{\mu\nu\alpha\beta}q^{\beta}\,T_2(q^2) + e^{\alpha}\,p_{\alpha}/(M_H-M_V)^2\,\varepsilon_{\mu\nu\beta\gamma}q^{\gamma}p^{\beta}\,T_3(q^2)$$

$$T_i(q^2) = T_i^{\vec{p}\vec{q}}(q^2, M_H^2, M_V^2)$$

$$T_1(q^2) = \frac{T_1(0)}{(1-q^2/M_V^2)(1-\alpha q^2/M_V^2)}$$

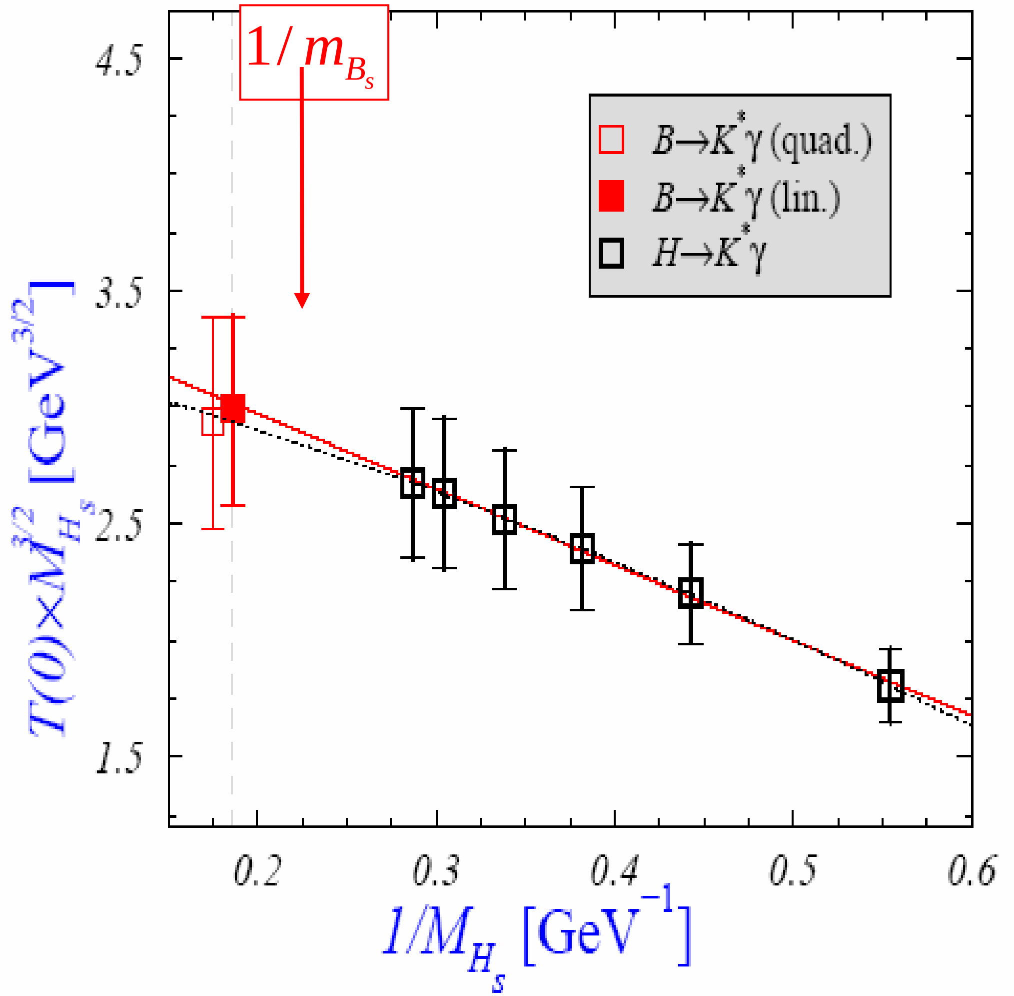
$$T_2(q^2) = \frac{T_1(0)}{1-\beta q^2/M_V^2}$$



Results

Method 2

$$T(0, M_H) M_H^{3/2} = a_0 + a_1/M_H + a_2/M_H^2$$



$$T_1^{B \to K^*}(0) = 0.25(4)$$

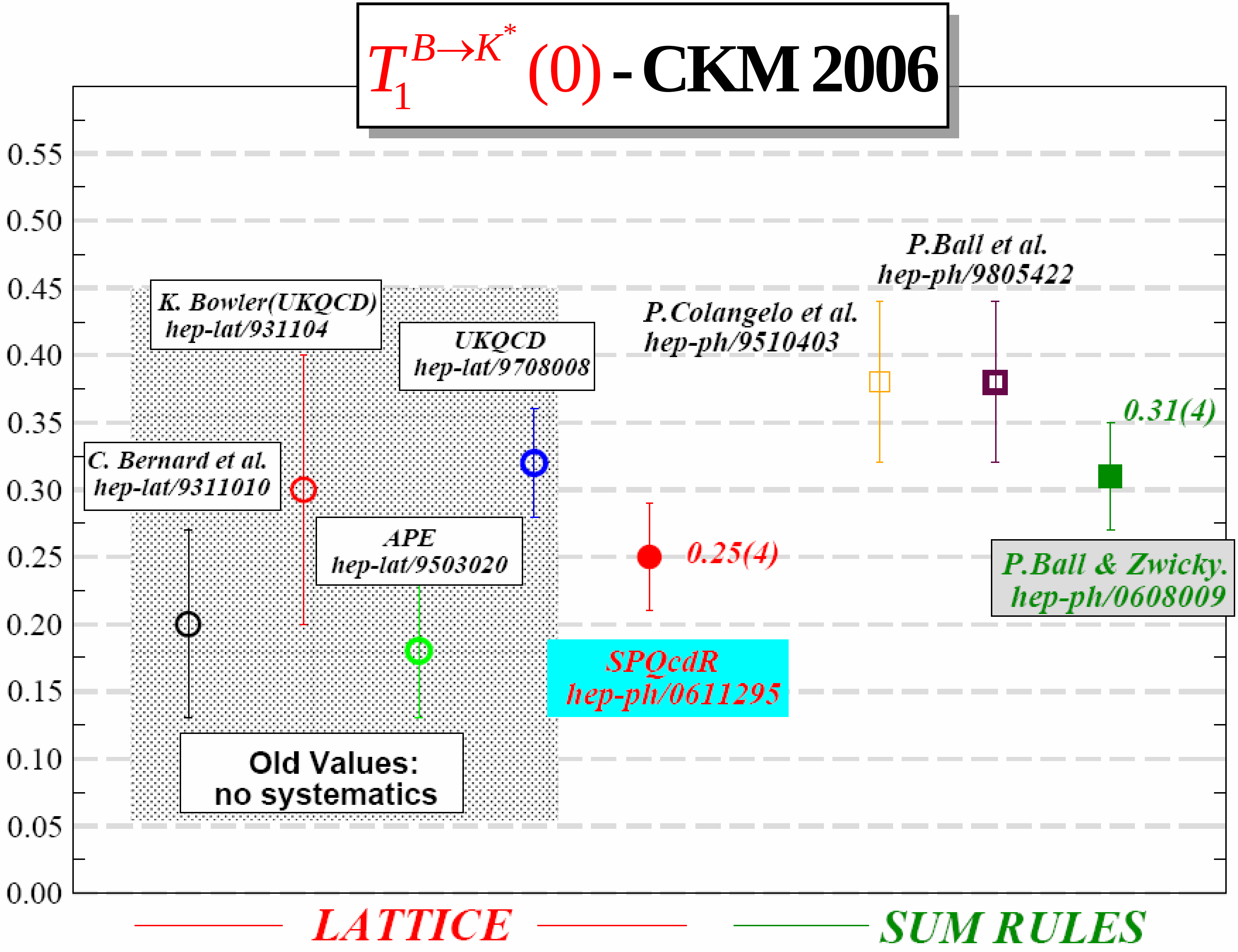
$$\frac{T_1^{B \to K^*}(0)}{T_1^{B \to \rho}(0)} = 1.2(1)$$

Systematics includes:

- ``rough'' control on discretisation errors: (2 lat. spac. a^{-1}=3 & 4 GeV)
- quad. & lin extr. in heavy q.

Systematics left:

- unquenched effects



$T_1^{B\to K^*}(0)$ - CKM 2006
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
K. Bowler(UKQCD) hep-lat/931104
UKQCD hep-lat/9708008
C. Bernard et al. hep-lat/9311010
APE hep-lat/9503020
Old Values: no systematics
SPQcdR hep-ph/0611295
0.25(4)
P.Colangelo et al. hep-ph/9510403
P.Ball et al. hep-ph/9805422
0.31(4)
P.Ball & Zwicky. hep-ph/0608009
LATTICE
SUM RULES

Lattice'06

$$T_1^{B\to K^*}(0) = 0.25(4)$$

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = (3.4 \pm 1.0)\times 10^{-5}$$

QCDSR'06

$$T_1^{B\to K^*}(0) = 0.31(4)$$

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = (5.0 \pm 2.0)\times 10^{-5}$$

assuming QCD factorisation: $\mathcal{B} \propto |\boldsymbol{a_7 T_1}|^2$

$\boldsymbol{a_7} = C_7 + O(\alpha_s, 1/m_b) \gg C_7$

(from typeI, type-II hard scattering: no annihilation cor.)

$$\mathcal{B}^{\exp}(B^0 \to K^{*0}\gamma) = (4.01 \pm 0.20)\times 10^{-5}$$ **HFAG06**

$B(B_d \to \rho\gamma)/B(B_d \to K^*\gamma)$ case

$$\frac{B(B_d^0 \to \rho^0\gamma)}{B(B_d^0 \to K^*\gamma)} = \left(\frac{1-m_\rho^2}{1-m_{K^*}^2}\right)^3 \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{T_1^{B\to\rho}(0)}{T_1^{B\to K^*}(0)}\right)^2 (1+\Delta R)$$

- $B(B\to\rho\gamma)/B(B\to K^*\gamma)$ cannot be competitive with $\Delta m_d/\Delta m_s$:

1. ***SU(3) breaking on $T_1(0)$ less accurate than on f_{Bs}/f_B:***

Lattice	$T_1^{B\to K^*}(0)/T_1^{B\to\rho}(0) = 1.2(1)$	QCDSR'06	$T_1^{B\to K^*}(0)/T_1^{B\to\rho}(0) = 1.17(9)$

(with respect to f_B, new complications as f.f.)

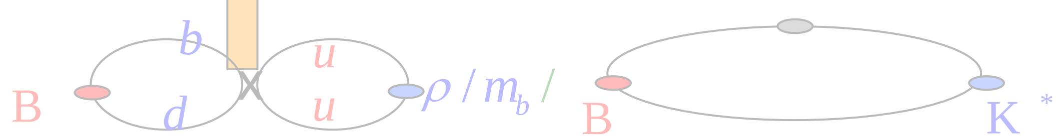
2. ***going beyond factorization for O_2, O_8 matrix elements***

$$m_b V_{ts}^* V_{tb} \langle K^*\gamma | H_{eff}^t | B_d^0 \rangle \simeq \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} m_b \left(C_2 \langle \bar{s}\gamma_L^\mu c \cdot \bar{c}\gamma_L^\mu b \rangle / m_b + C_8 \langle \bar{s}\sigma_R^{\mu\nu} b \cdot G^{\mu\nu} \rangle + C_7 \langle \bar{s}\sigma_R^{\mu\nu} b \cdot F^{\mu\nu} \rangle \right)$$

3. ***weak annihilation contributions from $B(B\to\rho\gamma)$***

$$\langle \rho\gamma | H_{eff} | B_d^0 \rangle \simeq m_b V_{ud}^* V_{ub} \left[C_2 \left(\langle \bar{d}\gamma_L^\mu u \cdot \bar{u}\gamma_L^\mu b \rangle - \langle \bar{d}\gamma_L^\mu c \cdot \bar{c}\gamma_L^\mu b \rangle \right) + C_1(\ldots) \right] / m_b + m_b V_{td}^* V_{tb} \langle \rho\gamma | H_{eff}^t | B_d^0 \rangle$$

$\Delta R =$



$B(B_d\to\rho\gamma)/B(B_d\to K^*\gamma)$ case

$$\frac{B(B_d^0\to\rho^0\gamma)}{B(B_d^0\to K^*\gamma)}=\left(\frac{1-m_\rho^2}{1-m_{K^*}^2}\right)^3\left|\frac{V_{td}}{V_{ts}}\right|^2\left(\frac{T_1^{B\to\rho}(0)}{T_1^{B\to K^*}(0)}\right)^2(1+\Delta R)$$

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(with respect to f_B, new complications as f.f.)

2. going beyond factorization for O_2, O_8 matrix elements

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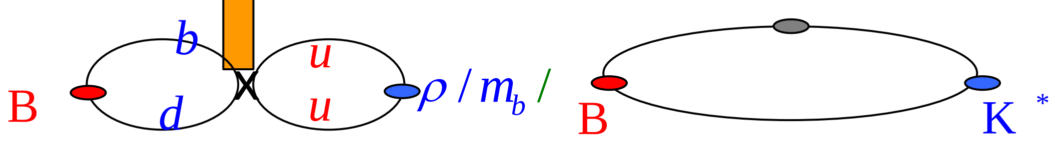
$\Delta R =$



$B(B_d \to \rho\gamma) / B(B_d \to K^*\gamma)$ case

$$\frac{B(B_d^0 \to \rho^0\gamma)}{B(B_d^0 \to K^*\gamma)} = \left(\frac{1-m_\rho^2}{1-m_{K^*}^2}\right)^3 \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{T_1^{B\to\rho}(0)}{T_1^{B\to K^*}(0)}\right)^2 (1+\Delta R)$$

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(with respect to f_B, new complications as f.f.)

2.

Conclusions:

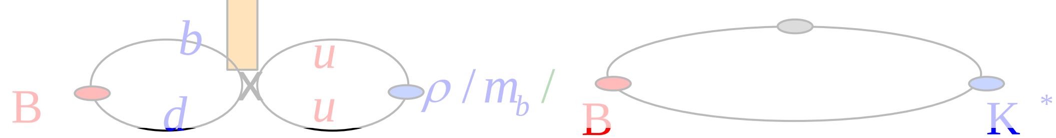
as long as you cannot achieve these points,

$B(B \to \rho\gamma) / B(B \to K^*\gamma)$ is a cross-check

3. weak annihilation contributions from $B(B \to \rho\gamma)$

$$\langle \rho\gamma | H_{eff} | B_d^0 \rangle \simeq m_b V_{ud}^* V_{ub} \left[C_2 \left(\langle \bar{d}\gamma_L^\mu u \cdot \bar{u}\gamma_L^\mu b \rangle - \langle \bar{d}\gamma_L^\mu c \cdot \bar{c}\gamma_L^\mu b \rangle \right) + C_1(\ldots) \right] / m_b + m_b V_{td}^* V_{tb} \langle \rho\gamma | H_{eff}^t | B_d^0 \rangle$$

$\Delta R =$



BACKUP

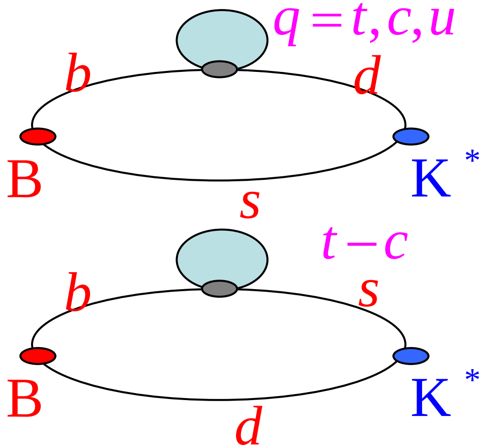
$B(B_s \to K^*\gamma)/B(B_d \to K^*\gamma)$ better shape! (LHcb/Babar-Belle

$$\frac{B(B_s^0 \to K^{*0}\gamma)}{B(B_d^0 \to K^{*0}\gamma)} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{T_1^{Bs\to K^*}(0)}{T_1^{B\to K^*}(0)}\right)^2 \left(1+\text{chiral-loops}/m_b\right)$$

no annihilation t.

$V_{qd}^* V_{qb} \times$

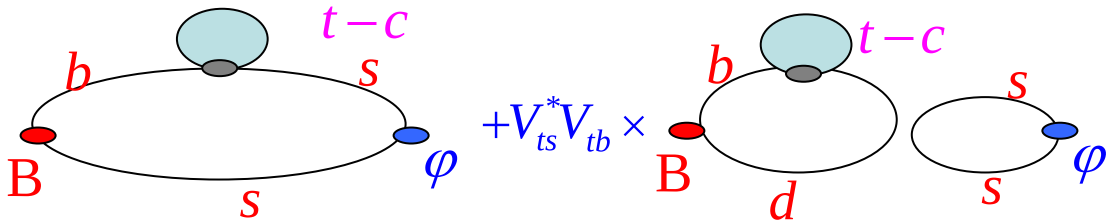
$V_{ts}^* V_{tb} \times$



$B(B_s \to K^*\gamma)/B(B_s \to \phi\,\gamma)$ disconneted dia.: potential problem

$$\frac{B(B_s^0 \to K^{*0}\gamma)}{B(B_s^0 \to \varphi\gamma)} = \left(\frac{1-m_{K^*}^2}{1-m_\varphi^2}\right)^3 \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{T_1^{B\to K^*}(0)}{T_1^{B\to\varphi}(0)}\right)^2 \left(1+ +\text{chiral-loops}/m_b\right)$$

$V_{ts}^* V_{tb} \times$ ... $+V_{ts}^* V_{tb} \times$



$$T_1^{B\to\varphi}(0)?$$