Learning from $b \to s\ell^+\ell^-$ effectively

Zoltan Ligeti

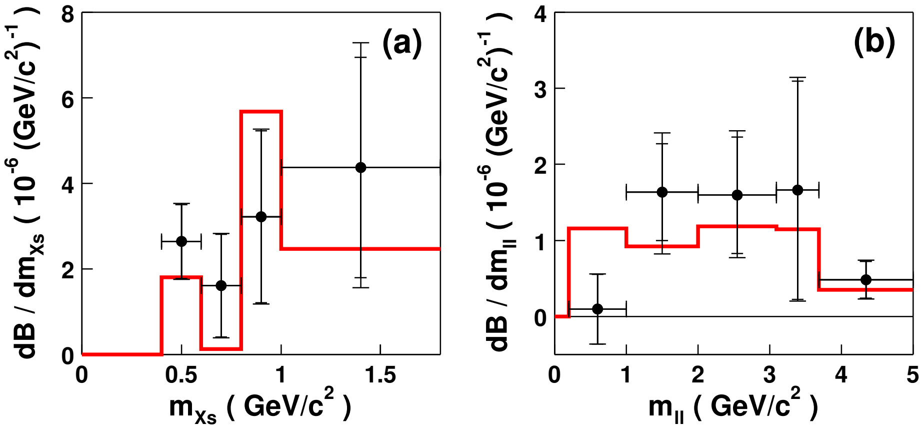
CKM workshop, Nagoya, Dec 13, 2006

- A theorist's survey of where we stand
- Search for optimal observables to constrain short distance physics
  Inclusive and exclusive decays
- $B \to X_s\ell^+\ell^-$ in the small q^2 region
  Cuts on q^2 & $m_X \Rightarrow$ theoretical issues similar to $B \to X_u\ell\bar{\nu}$ (cleanly SF region)
  Rate with cuts: results, universality, implications

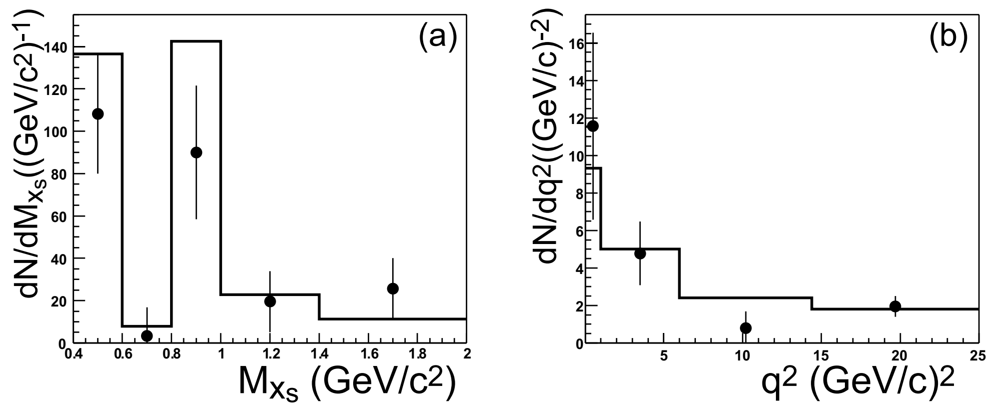
Details: K. Lee, ZL, I. Stewart, F. Tackmann, hep-ph/0512191; hep-ph/0612156

Inclusive $B \to X_s \ell^+ \ell^-$ measurements

BABAR, $89\,\mathrm{m}\,\Upsilon$



Belle, $152\,\mathrm{m}\,\Upsilon$



- Cut out J/ψ and ψ' regions, and impose an additional cut $m_X < 1.8\,\mathrm{GeV}$ or $2\,\mathrm{GeV}$ to suppress huge $b \to c\ell^-\bar\nu \to s\ell^-\ell^+\nu\bar\nu$ background

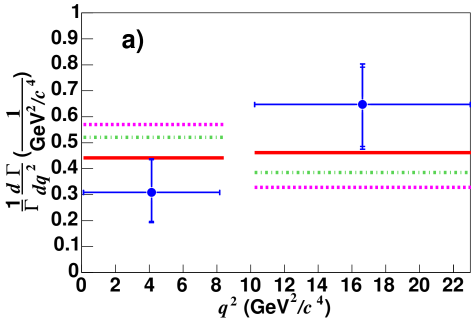
  Current measurements not really inclusive — sum exclusive modes (about $\sim 50\%$)

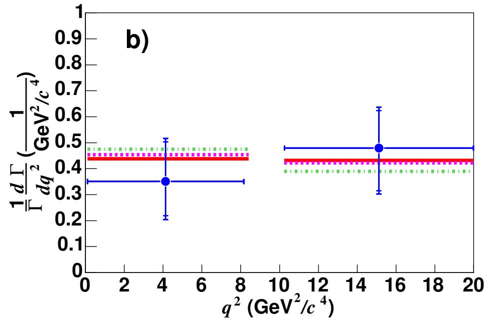
- World average: $\mathcal{B}(B \to X_s \ell^+ \ell^-) = (4.5 \pm 1.0) \times 10^{-6}$

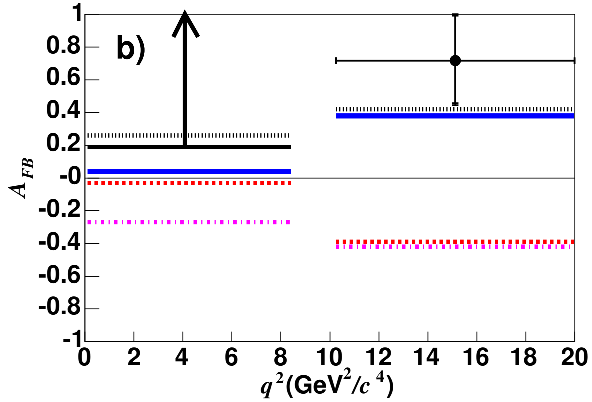
- Can improve a lot — a key measurement that uses only a fraction of the data!

Exclusive $B \to K^{(*)}\ell^+\ell^-$ measurements

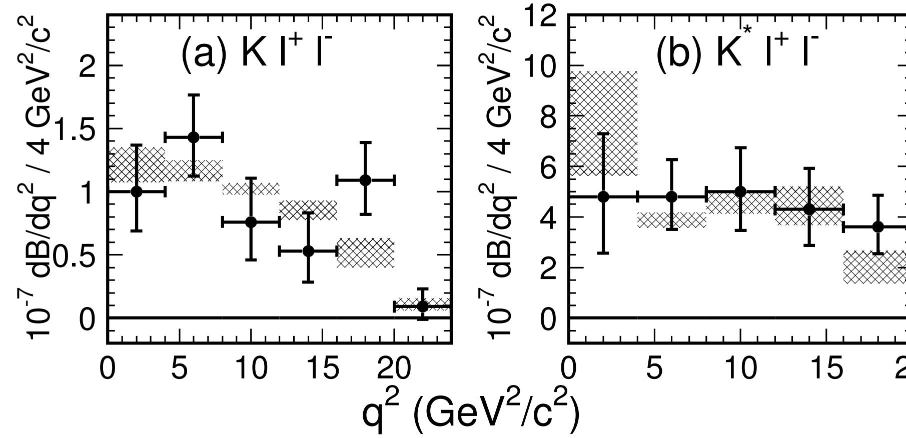
BABAR, $229\,\text{m}\ \Upsilon$

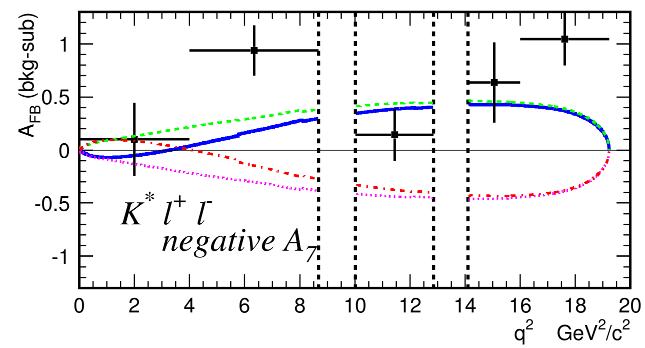






Belle, $253\,\text{m}$ and $386\,\text{m}\ \Upsilon$





- WA: $\mathcal{B}(B \to K\ell^+\ell^-) = (0.44 \pm 0.05) \times 10^{-6}$, $\mathcal{B}(B \to K^*\ell^+\ell^-) = (1.17 \pm 0.16) \times 10^{-6}$
- LHCb expects (5yr, $10\,\text{fb}^{-1}$) in SM: $\sigma(q^2_{A_{\rm FB}=0}) \approx 0.5\text{GeV}^2 \Rightarrow \sigma(C_7^{\rm eff}/C_9^{\rm eff}) \sim 13\%$

Perturbative $b \to s\ell^+\ell^-$ calculations

- Complementary to $B \to X_s\gamma$
- Subtleties in power counting (as in $K \to \pi e^+e^-$):

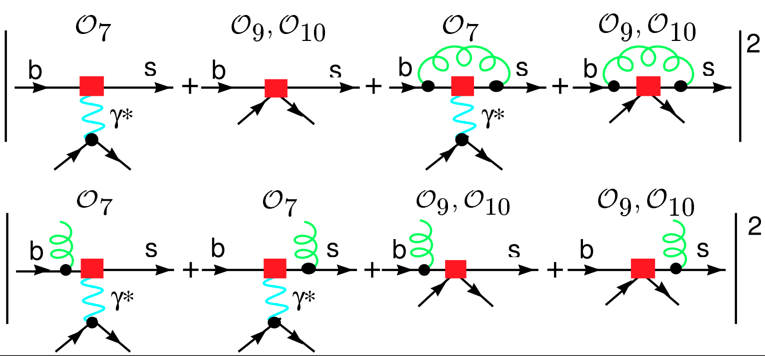
$$C_9(m_b) \sim C_9(m_W) + (\ldots)\frac{C_2(m_W)}{\alpha_s(m_W)}\left\{1 - \left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)}\right]^{(\cdots)}\right\}$$

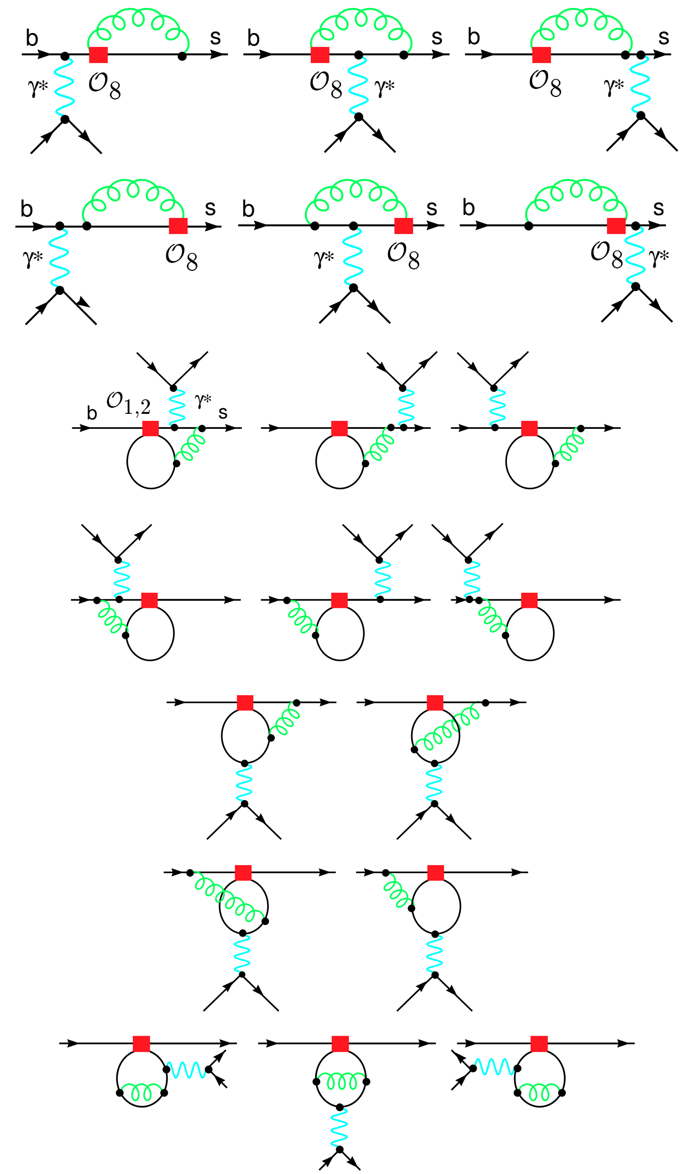
Scale & scheme dependence cancellation tricky

- NNLL: 2-loop matching, 2- and 3-loop running
  2-loop matrix elements

$$\mathcal{B}(B \to X_s\ell^+\ell^-)\Big|_{1<q^2<6\,\mathrm{GeV}^2} = (1.63 \pm 0.20) \times 10^{-6}$$

[Many authors: Bobeth, Misiak, Urban, Munz, Gambino, Gorbahn, Haisch, Asatryan, Asatrian, Bieri, Hovhannisyan, Greub, Walker, Ghinculov, Hurth, Isidori, Yao, etc.]





Status of $B \to X_s \ell^+ \ell^-$

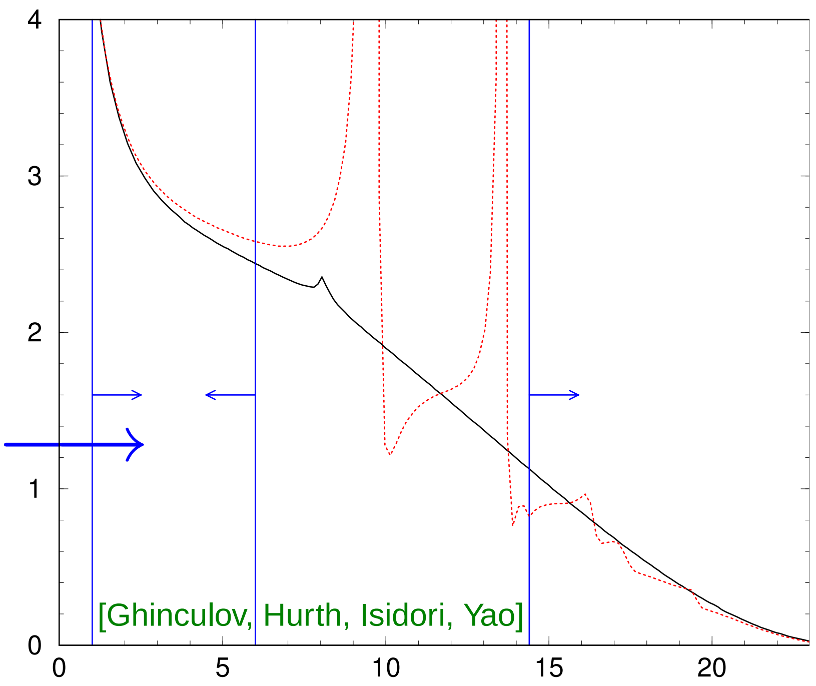
- Rate depends (mostly) on

$$O_7 = \overline{m}_b\, \bar{s}\sigma_{\mu\nu} e F^{\mu\nu} P_R b,$$

$$O_9 = e^2 (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell),$$

$$O_{10} = e^2 (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

Theory most precise for $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$ ⟶



- NNLL $b \to s\ell^+\ell^-$ perturbative calculations

  Nonperturbative corrections to q^2 spectrum

  [Falk, Luke, Savage; Ali, Hiller, Handoko, Morozumi; Buchalla, Isidori, Rey]

- Small q^2 region also has the larger rate $\Rightarrow$ smaller experimental errors

Aside: long distance effects

- A worry (at least, for me) that will be ignored in this talk

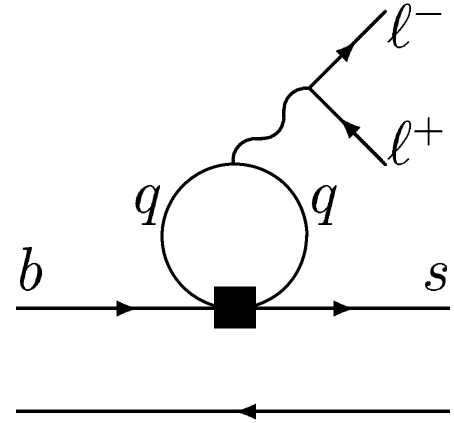
$\mathcal{B}(B \to \psi X_s) \sim 4 \times 10^{-3}$

$\downarrow$

$\mathcal{B}(\psi \to \ell^+ \ell^-) \sim 6 \times 10^{-2}$

Combined BR: $\mathcal{B}(B \to X_s \ell^+ \ell^-) \sim 2 \times 10^{-4}$

This is ~ 30 times the short distance contribution!



- Averaged over a large region of q^2, the $c\bar{c}$ loop expected to be dual to $\psi + \psi' + \ldots$
  This is what happens in $e^+e^- \to$ hadrons, in τ decay, etc., but NOT here

- Is it consistent to "cut out" the ψ and ψ' regions and then compare data with the short distance calculation? (Maybe..., but understanding is unsatisfactory)

Extracting short distance information effectively

Angular decomposition

- With limited data, what partial rates give most information on short dist. physics?

θ: angle between $\vec{p}_{\ell^+}$ and $\vec{p}_{\bar{B}^0,\, B^-}$ [$\vec{p}_{\ell^-}$ and $\vec{p}_{B^0,\, B^+}$] in $\ell^+\ell^-$ center of mass frame

- Previous analyses considered $\mathrm{d}\Gamma/\mathrm{d}s$ and $\mathrm{d}A_{\mathrm{FB}}/\mathrm{d}s$ ($s = q^2/m_b^2,\ z = \cos\theta$)

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}s\,\mathrm{d}z} \sim 3\Gamma_0(1-s)^2\Big\{(1+z^2)\Big[\Big(\mathcal{C}_9 + \frac{2}{s}\mathcal{C}_7\Big)^2 + \mathcal{C}_{10}^2\Big] \qquad H_T \quad [\Gamma = H_T + H_L]$$

$$- 4\, z\, s\, \mathcal{C}_{10}\Big(\mathcal{C}_9 + \frac{2}{s}\mathcal{C}_7\Big) \qquad H_A \quad [\equiv (4/3)A_{\mathrm{FB}}]$$

$$+ (1 - z^2)\Big[(\mathcal{C}_9 + 2\mathcal{C}_7)^2 + \mathcal{C}_{10}^2\Big]\Big\} \qquad H_L \quad [\text{no } \mathcal{C}_7/s \text{ pole}]$$

Same structure for inclusive $B \to X_s\ell^+\ell^-$ and exclusive $B \to K^*\ell^+\ell^-$

$B \to K^*\ell^+\ell^-$: Two further angles (ignore possibility of measuring $\ell^\pm$ polarization)

The $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(1/m_{c,b})$ corrections are different

- The three terms are sensitive to different combinations of Wilson coefficients

Our implementation of NNLL

- Introduce a scheme to separate new physics sensitive terms from four-quark operator contributions (for which the SM is assumed)

- Define $\mathcal{C}_{7,9}$ as μ- and q^2-independent constants, which are real in the SM

$$C_{7,9}^{\rm incl}(q^2) = \mathcal{C}_{7,9} + \underbrace{F_{7,9}(q^2)}_{\alpha_s} + \underbrace{G_{7,9}(q^2)}_{1/m_c^2}$$

- Λ^2/m_c^2 corrections can be included in $C_{7,9}^{\rm incl}$

$$G_9(q^2) = \frac{10}{1-2s}\, G_7(q^2) = -\frac{5}{6}\,\frac{\lambda_2}{m_c^2}\, C_2\, \frac{\mathcal{F}[q^2/(4m_c^2)]}{1-q^2/(4m_c^2)}$$

Blows up as $(4m_c^2 - q^2)^{-1/2}$ as $q^2 \to 4m_c^2$

- Use m_b^{1S} to improve perturbation expansions; do not normalize to $\Gamma(B \to X\ell\bar\nu)$

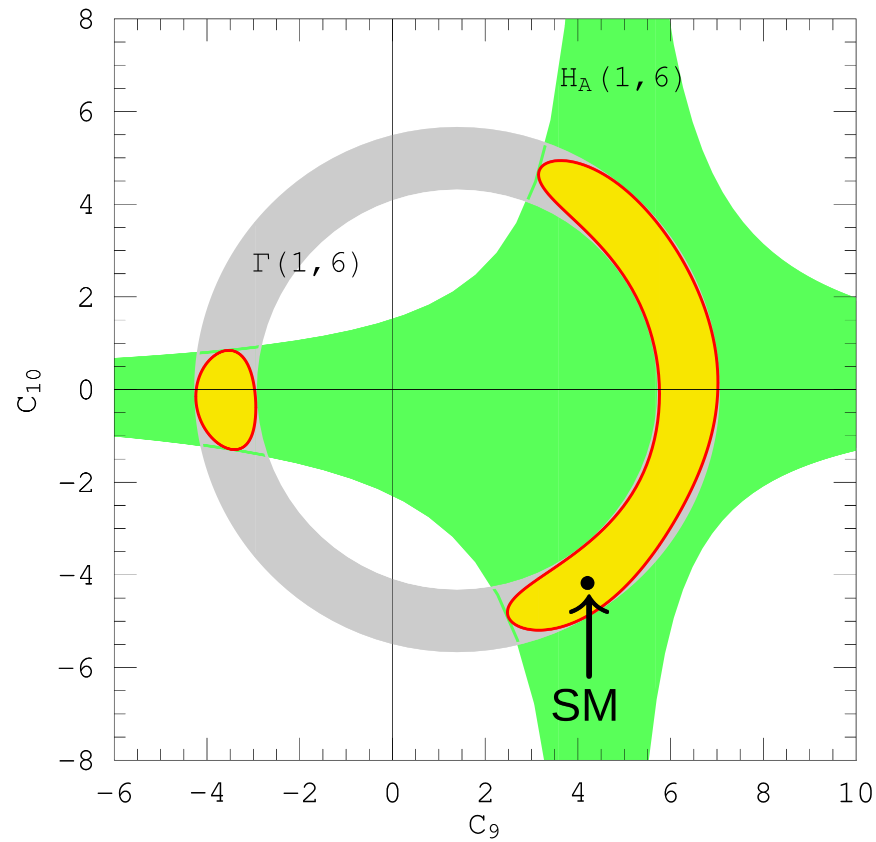
Keep $\overline{m}_b(\mu) C_7(\mu)$ together and unexpanded — no reason to expand $\overline{m}_b(\mu)$

An illustrative toy analysis

- Inclusive, with "guessed" errors for $\sim 1\,\text{ab}^{-1}$

  Define: $H_i(q_1^2, q_2^2) = \int_{q_1^2}^{q_2^2} \mathrm{d}q^2 H_i(q^2)$

- Small q^2-dependence $\Rightarrow$ splitting Γ in two regions not useful (splitting $H_A \equiv A_{\rm FB}$ is!)



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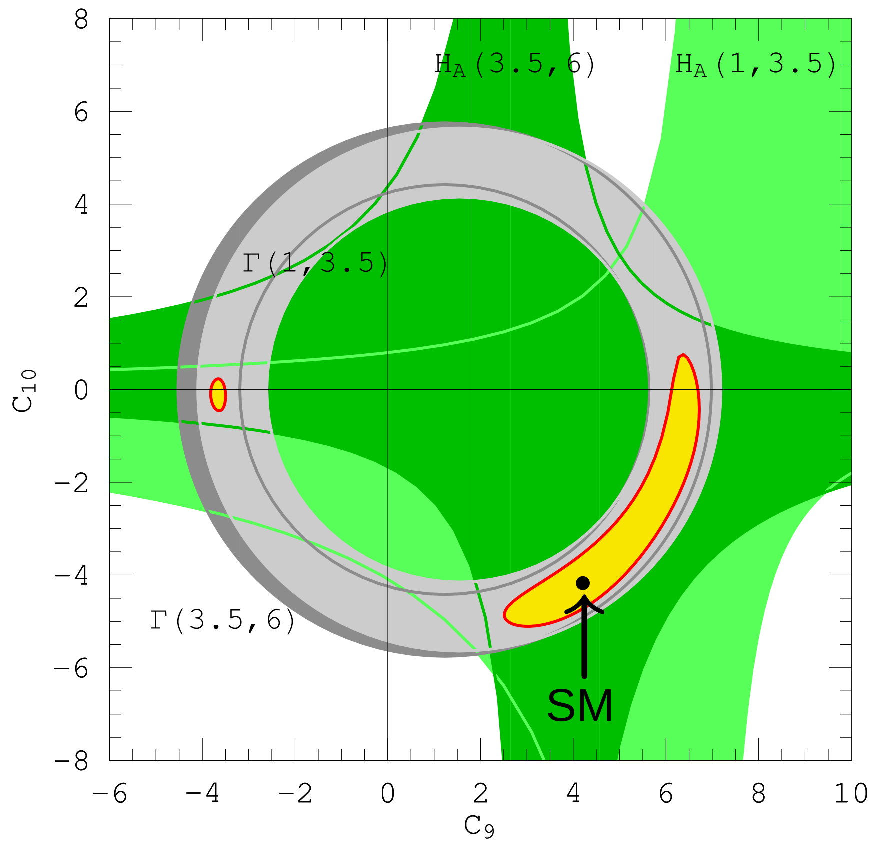

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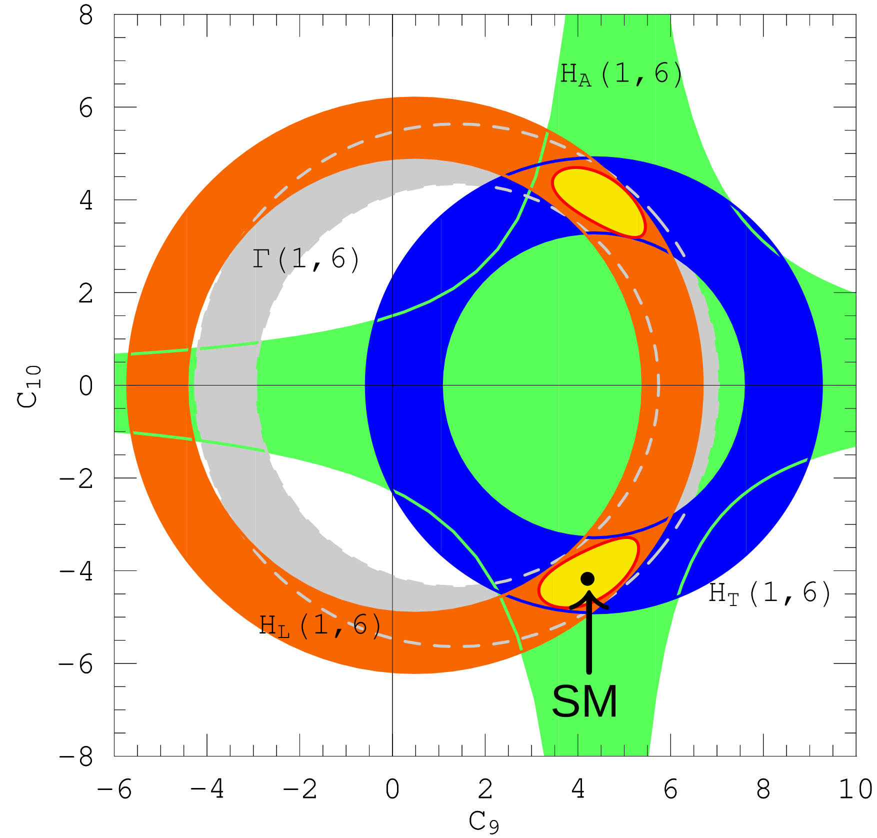
- $H_L \propto q^2$-independent combination of $\mathcal{C}_i$'s
  $\Rightarrow$ integrate over as large region as possible



- Separating H_T and H_L is very powerful

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Berkeley Lab

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- H_T and H_A: different q^2 regions sensitive to different combinations of $\mathcal{C}_i$’s

  Separating $H_A(1, 3.5)$ vs $H_A(3.5, 6)$ and/or $H_T(1, 3.5)$ vs $H_T(3.5, 6)$ appears promising

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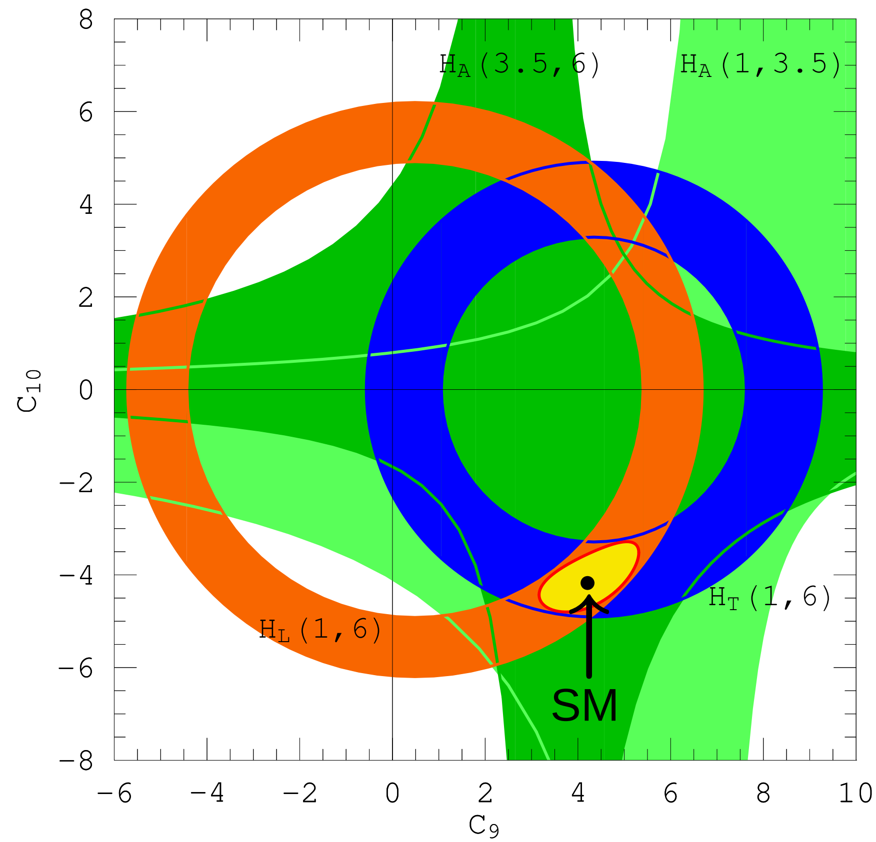

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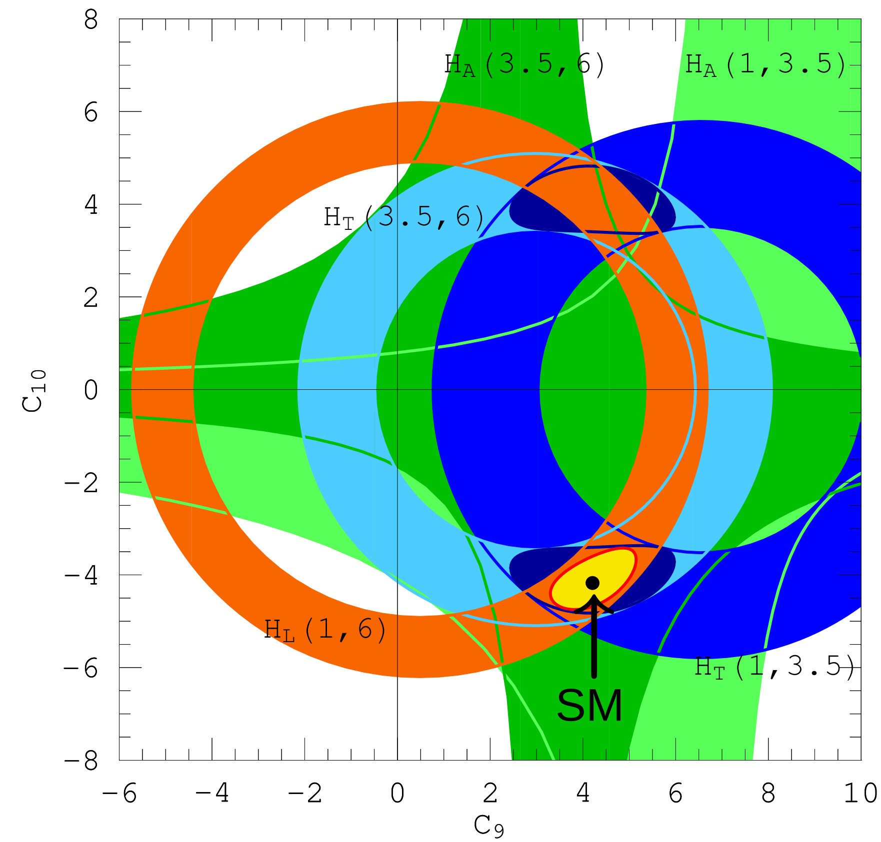


- Separating H_T and H_L is very powerful

- Can extract all information from a few integrated rates

Exclusive $B \to K^* \ell^+ \ell^-$

- Seven $B \to K^* \ell^+ \ell^-$ form factors $\Rightarrow$ fewer functions in the $E \gg \Lambda_{\rm QCD}$ limit

Angular decomposition: $\zeta(s)$ and $\zeta^J(s)$ $\sim$ non-factorizable and factorizable parts

$$H_T \sim s \left\{ \mathcal{C}_{10}^2 \, [\zeta_\perp(s)]^2 + \left| \mathcal{C}_9 \, \zeta_\perp(s) + \frac{2\mathcal{C}_7}{s} \frac{m_b}{m_B} \Big[\zeta_\perp(s) + (1-s) \zeta_\perp^J(s) \Big] \right|^2 \right\}$$

$$H_A \sim -2s \, \mathcal{C}_{10} \, \zeta_\perp(s) \left\{ \mathcal{C}_9 \, \zeta_\perp(s) + \frac{2\mathcal{C}_7}{s} \frac{m_b}{m_B} \Big[\zeta_\perp(s) + (1-s) \zeta_\perp^J(s) \Big] \right\}$$

$$H_L \sim \frac{1}{2} \left(\mathcal{C}_{10}^2 + \left| \mathcal{C}_9 + 2\mathcal{C}_7 \frac{m_b}{m_B} \right|^2 \right) \Big[\zeta_\parallel(s) - \zeta_\parallel^J(s) \Big]^2$$

Without nonperturbative input [or $SU(3)$], cannot use $H_L^{(B\to K^*\ell^+\ell^-)}$ & $B \to K \ell^+ \ell^-$

- Form factors $\Rightarrow$ few numbers: $\zeta_\perp^{(J)}(s) = \dfrac{\zeta_\perp^{(J)}(0)}{(1-s)^2} \Big[1 + \mathcal{O}\Big(\alpha_s, \dfrac{\Lambda}{E}\Big) \Big]$ $\quad (1.9 < E < 2.7 \text{GeV})$

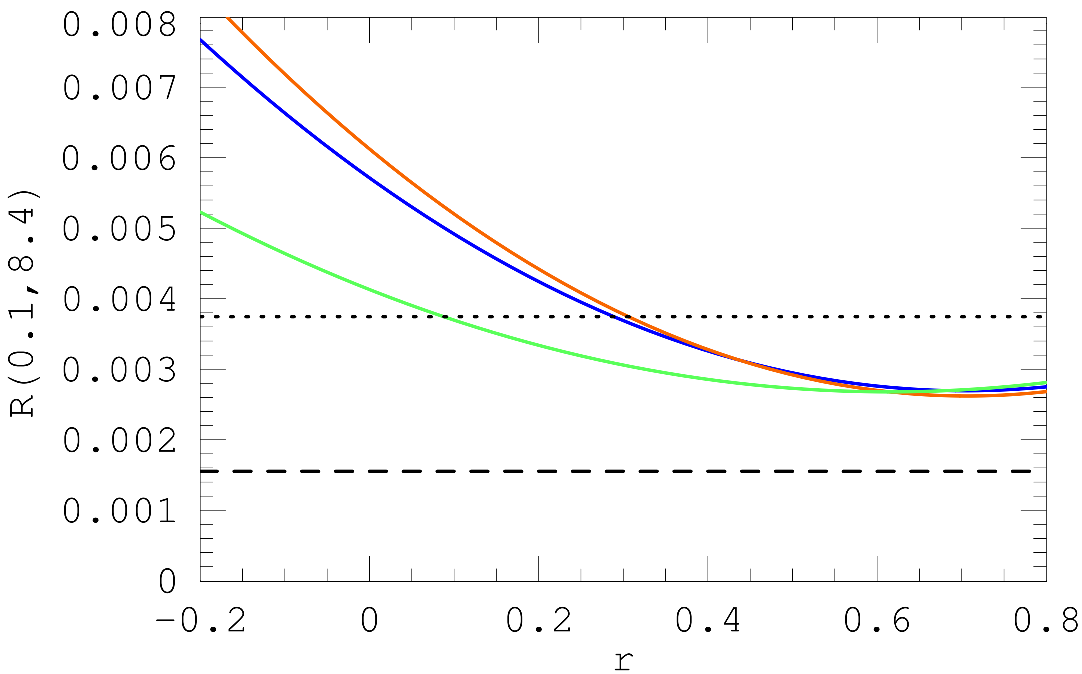
- Can extract from ratios of: $\Gamma(B \to K^* \gamma),\ H_T(0,8),\ H_A(0,4),\ H_A(4,8)$

$\Rightarrow$ $\mathcal{C}_{10}/\mathcal{C}_7,\ \mathcal{C}_9/\mathcal{C}_7$, and hadronic parameter $\zeta_\perp^J(0)/[\zeta_\perp(0) + \zeta_\perp^J(0)]$

Constraining hadronic physics

$$R(q_1^2, q_2^2) \equiv \frac{H_T(q_1^2, q_2^2)}{\Gamma(B \to K^*\gamma)} = \frac{\alpha_{\rm em}}{12\pi} \frac{m_B^2}{m_b^2} \int_{q_1^2/m_B^2}^{q_2^2/m_B^2} {\rm d}s\, \frac{\lambda^3\, s}{(1-\rho)^3\,(1-s)^4}$$

$$\times \left\{ \frac{\mathcal{C}_{10}^2}{\mathcal{C}_7^2}\,(1-r)^2 + \left[\frac{\mathcal{C}_9}{\mathcal{C}_7}\,(1-r) + \frac{2}{s}\frac{m_b}{m_B}\,(1-sr) \right]^2 \right\}$$



Plot $R(0.1, 8.41)$ and BaBar data

$$r \equiv \frac{\zeta_\perp^J(0)}{\zeta_\perp(0) + \zeta_\perp^J(0)}$$

1σ upper bound

central value

- Will be very interesting with more data (expect central value of H_T to increase)

$B \to X_s \ell^+ \ell^-$ with q^2 and m_X cuts

Recall NNLL for $B \to X_s \ell^+ \ell^-$

- Rate depends (mostly) on

$$O_7 = \overline{m}_b\, \bar{s}\sigma_{\mu\nu} e F^{\mu\nu} P_R b,$$

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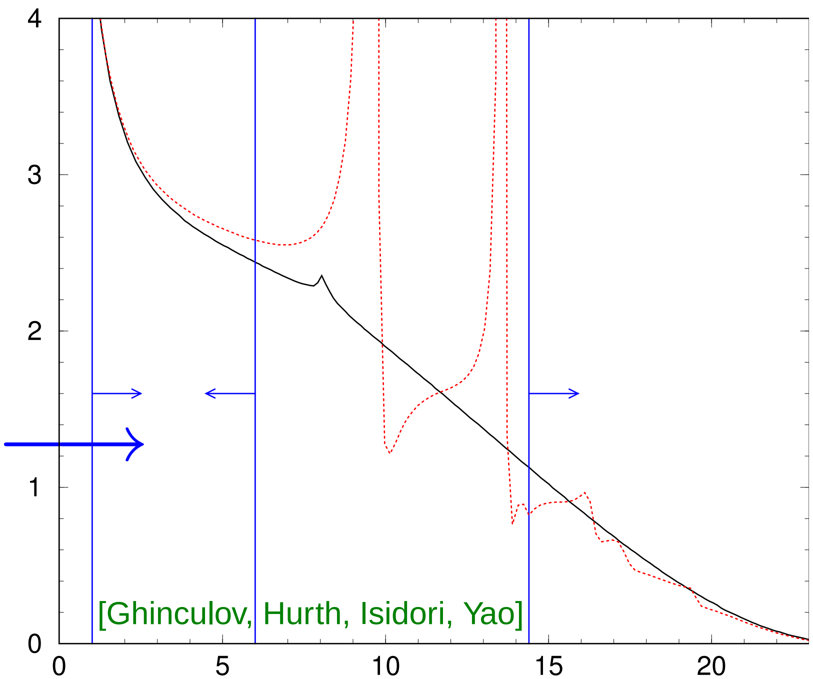
Theory most precise for $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$ ⟶

– NNLL $b \to s\ell^+\ell^-$ perturbative calculations

– Nonperturbative corrections to q^2 spectrum



- In small q^2 region experiments need additional cut $m_{X_s} \lesssim 2\,\mathrm{GeV}$
  ... to suppress $b \to c(\to s\ell^+\nu)\ell^-\bar{\nu}$ backgrounds

$\Rightarrow$ Similar issues as phase space cuts in inclusive $B \to X_u \ell \bar{\nu}$ measurements

$B \to X_s \ell^+ \ell^-$ kinematics, below the ψ

- Only two kinematic variables are symmetric in p_{ℓ^+} and p_{ℓ^-}

$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

q^2 not large and $m_X^2 \ll m_B^2 \Rightarrow E_X = \mathcal{O}(m_B) \Rightarrow E_X^2 \gg m_X^2$, so p_X near light-cone

$$p_X^+ = n \cdot p_X = \mathcal{O}(\Lambda_{\rm QCD}) \qquad p_X^- = \bar{n} \cdot p_X = \mathcal{O}(m_B) \qquad n, \bar{n} = (1, \pm\vec{p}_X/|\vec{p}_X|)$$

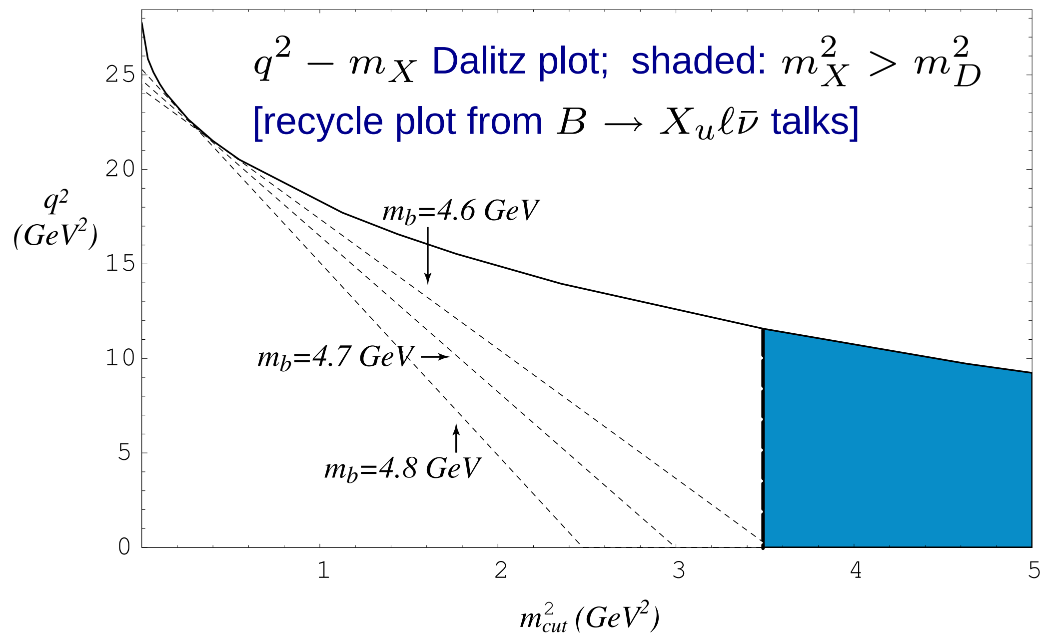
- $p_X^+ \ll p_X^-$: jet-like hadronic final state

- Parton level: $\Gamma \propto f(q^2)\, \delta[(m_b v - q)^2]$

$$m_X^2 \geq \bar{\Lambda}(m_B - q^2/m_b)$$

rate vanishes left of the dashed lines

$\Rightarrow$ nonperturbative physics important



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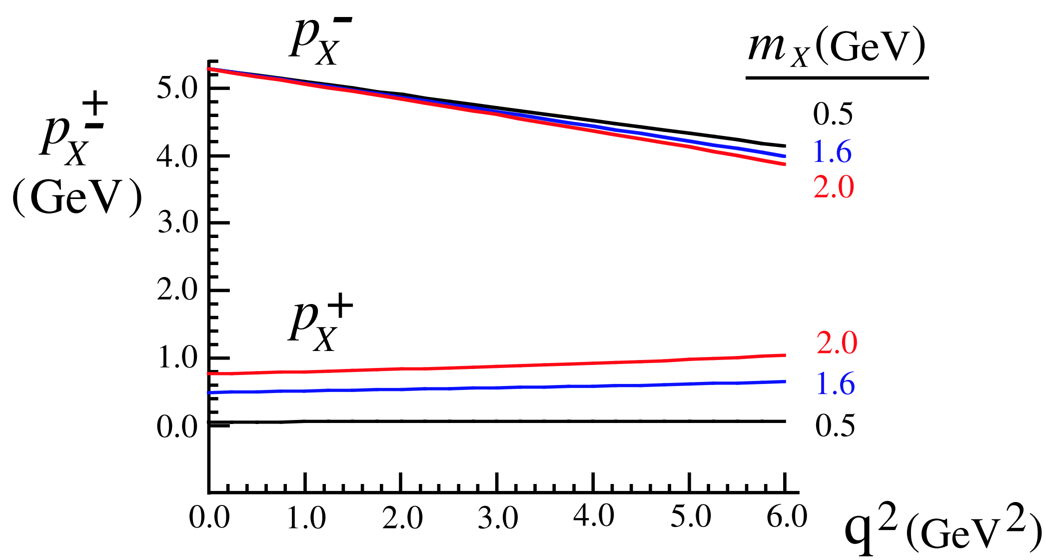


Effects of m_X cut at lowest order

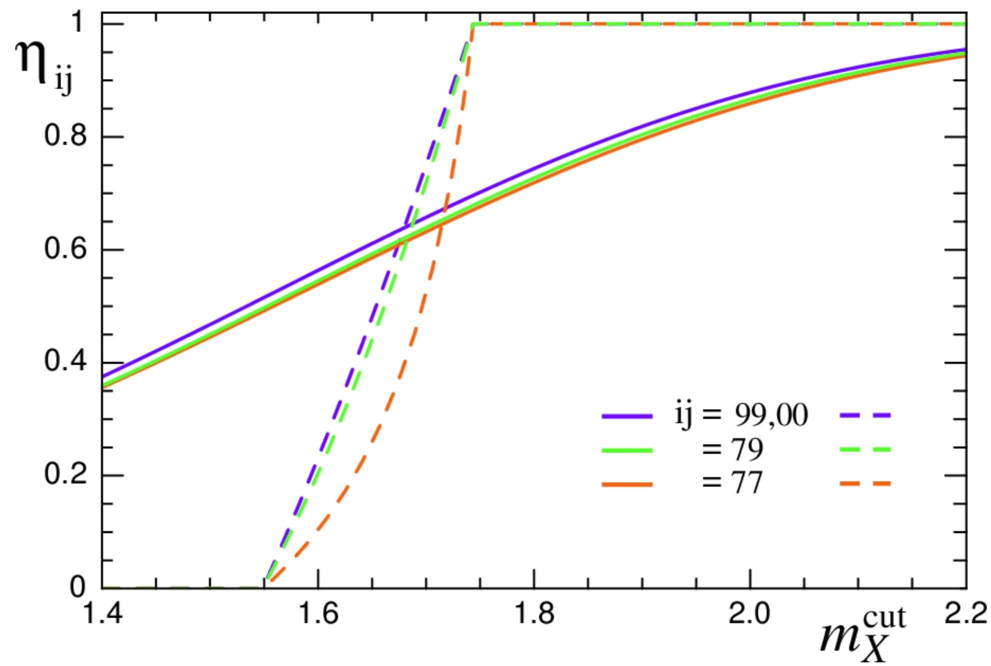
- Define:

$$\eta_{ij} = \frac{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2 \int_0^{m_X^{\mathrm{cut}}} \mathrm{d}m_X^2\, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2\, \mathrm{d}m_X^2}}{\displaystyle\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \mathrm{d}q^2\, \frac{\mathrm{d}\Gamma_{ij}}{\mathrm{d}q^2}}$$

ij: C_9^2 and C_{10}^2, C_7C_9, C_7^2 — different functionally for each contribution

Dashed: tree level in local OPE [wrong]

Solid: with a fixed shape function model



- η_{ij} determine fraction of rate that is measured in presence of m_X cut

I.e., a 30% deviation at $m_X^{\mathrm{cut}} = 1.8\,\mathrm{GeV}$ may be hadronic physics, not new physics

(Experimental papers have used Fermi-motion model to include effects of m_X^{cut})

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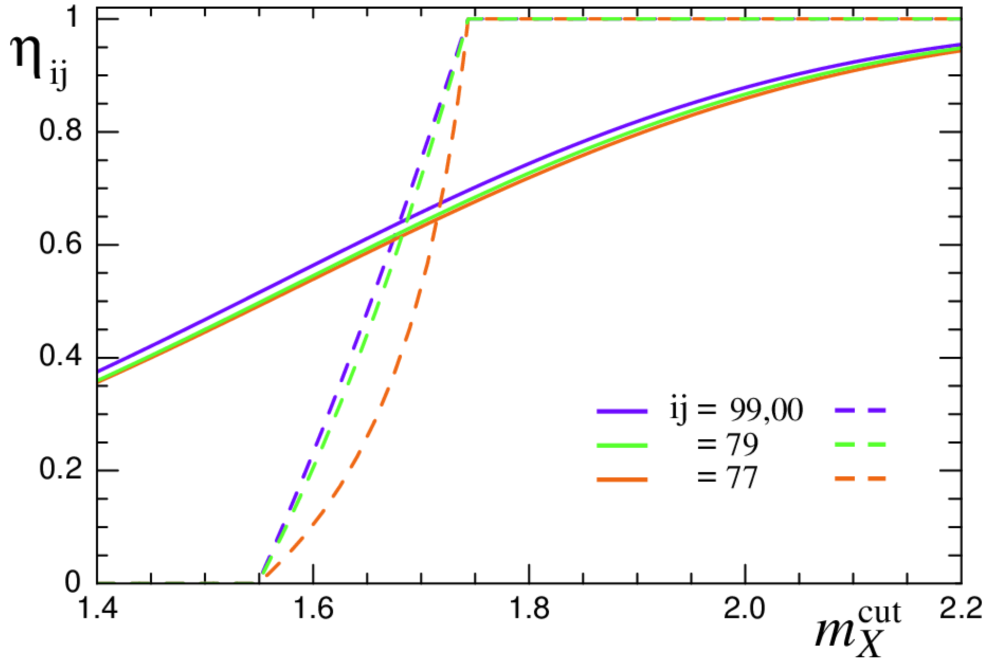

- Strong $m_X^{\rm cut}$ dependence: important to raise it, if possible

  Once $1-\eta$ is sizable, so will be its uncertainty

- Approximate universality of η_{ij}: because shape function varies on scale $p_X^+/\Lambda_{\rm QCD}$, while $\Gamma_{ij}^{\rm parton}$ varies on scale p_X^+/m_b $\Rightarrow$ $\eta \approx \eta_{ij}$

Perturbation theory for amplitude or rate?

- Usual power counting: expand $\langle s\ell^+\ell^-|\mathcal{H}|b\rangle$ in α_s, treating $\alpha_s \ln(m_W/m_b) = \mathcal{O}(1)$

  This is OK in local OPE region (e.g., rate or q^2 spectrum) where nonperturbative corrections ($\lambda_{1,2}$, etc.) are small and can be included at the end

- Shape function region: only the rate is calculable, $\Gamma \sim \text{Im}\,\langle B|\,T\{O_i^\dagger(x)\,O_j(0)\}|B\rangle$

  $C_9(m_b) \sim \ln(m_W/m_b) \sim 1/\alpha_s$ “enhancement”, but $|C_9(m_b)| \sim C_{10}$

  – Need to take it seriously to cancel scheme- and scale-dependence in running

  – Do not want power counting to imply that $\langle B|O_9^\dagger O_9|B\rangle$ at $\mathcal{O}(\alpha_s^2)$ is of same order as $\langle B|O_{10}^\dagger O_{10}|B\rangle$ at tree level

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- Matching onto SCET, can separate μ-dependence in matrix element that cancels that in running from $\mathcal{O}(m_W)$ to $\mathcal{O}(m_b)$, and dependence on scales $\sqrt{m_b\Lambda_{\text{QCD}}}$ and $\mu_{\text{hadr}} \sim 1\,\text{GeV}$ — can work to different orders

"Split matching" and running below m_b

- Match H_w at $\mu_0 \sim m_b$ onto scale invariant operators [Lee and Stewart]

$$C_9^{\rm mix}(\mu_0)\,(\bar s_L\gamma^\mu b_L)(\bar\ell\gamma_\mu\ell) + C_{10}(\bar s_L\gamma^\mu b_L)(\bar\ell\gamma_\mu\gamma_5\ell) - C_7^{\rm mix}(\mu_0)\,\frac{2\overline{m}_b q_\nu}{q^2}\,(\bar s_L\sigma^{\mu\nu}b_R)_{\mu=m_b}(\bar\ell\gamma_\mu\ell)$$

- Match onto SCET at $\mu_b \sim m_b$ [$\mu_b = \mu_0$ in practice; common anom. dim. below m_b]

- Run down to $\mu_i \sim \sqrt{m_b\Lambda_{\rm QCD}}$, calculate

$$\mathrm{d}^3\Gamma^{(0)} = H \int \mathrm{d}k\, J(k)\, f^{(0)}(k)$$

H and J perturbative, $f^{(0)}$ nonperturbative

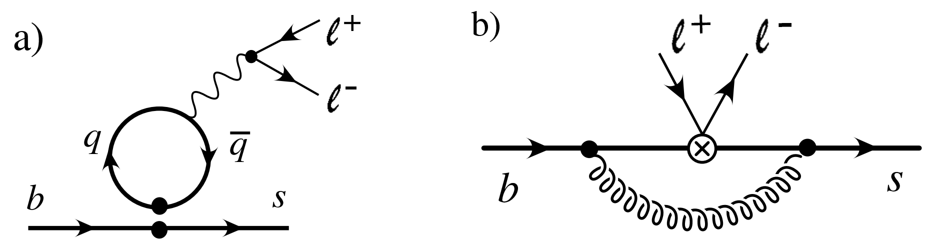
- Take $f^{(0)}(k)$ from $B \to X_s\gamma$, or run model from μ_0 to μ_i [Bosch, Lange, Neubert, Paz]
(recall: $\Lambda_{\rm QCD}/m_b$ suppressed shape functions are non-universal)

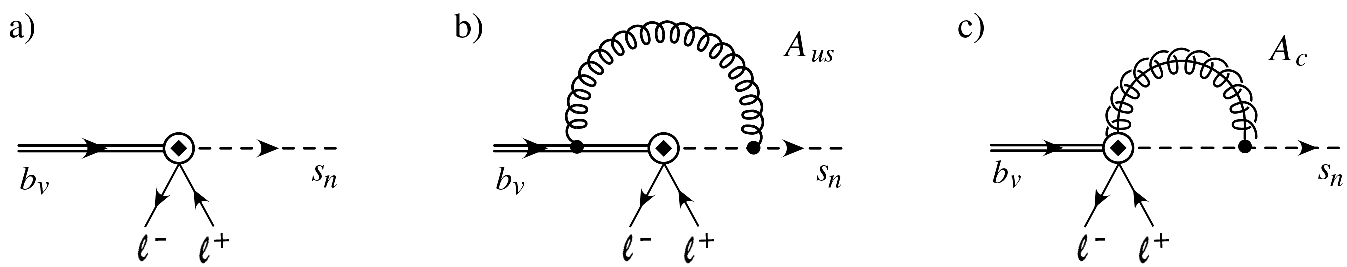
$$f^{(0)}(\hat\omega,\mu_i) = \frac{e^{V_S(\mu_i,\mu_0)}}{\Gamma(1+\eta)}\left(\frac{\hat\omega}{\mu_0}\right)^\eta \int_0^1 \mathrm{d}t\, f^{(0)}\Big[\hat\omega(1-t^{1/\eta}),\,\mu_0\Big] \qquad \eta = \frac{16}{27}\ln\frac{\alpha_s(\mu_0)}{\alpha_s(\mu_i)}$$

Matching onto SCET

- SCET operators: $J_{\ell\ell}^{(0)} = \sum_{i=a,b,c} C_{9i}(s)\Big(\bar\chi_{n,p}\,\Gamma_i^\mu\,\mathcal{H}_v\Big)\,(\bar\ell\gamma_\mu\ell) +$ similar $C_{10,7}$ terms

$$\mathcal{H}_v = Y^\dagger h_v\,, \qquad \chi_n = W^\dagger \xi_n\,, \qquad \Gamma^\mu_{a-c} = P_R\left\{\gamma^\mu, v^\mu, \frac{n^\mu}{n\cdot v}\right\}$$



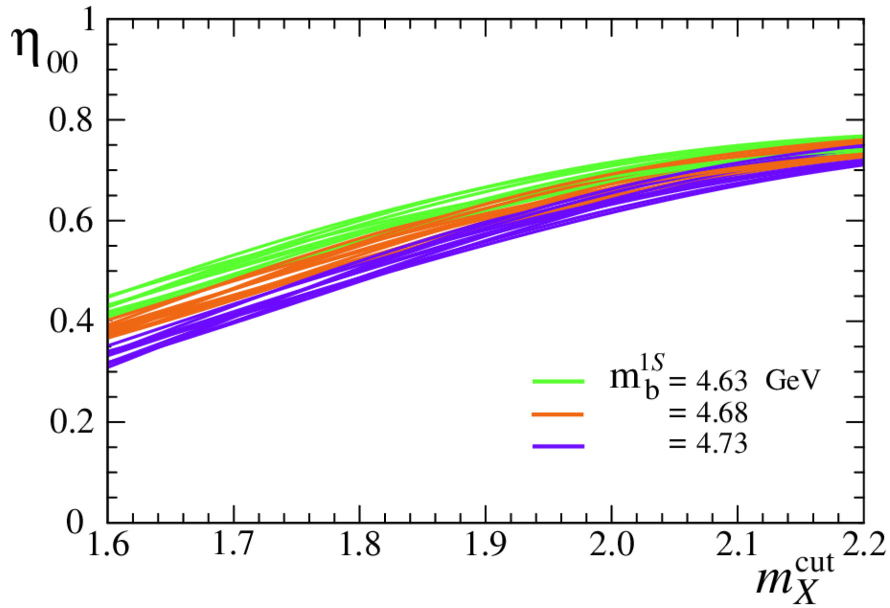


Wilson Coefficients: $C_{9a} = \widetilde{C}_9^{\rm eff}\,[1+\mathcal{O}(\alpha_s)] \qquad C_{9b,c} = \mathcal{O}(\alpha_s)$

- Some parts of the "usual" NLL $\mathcal{O}(\alpha_s)$ corrections included in $\widetilde{C}_9^{\rm eff}$ [Misiak; Buras, Munz] now contribute to the jet function, J, some others to the shape function, $f^{(0)}(k)$

Including NLL corrections

- Universality maintained; estimate shape function uncertainties using $B \to X_s\gamma$



NNLL reduces μ-dependence, effect on q^2 spectrum small $\Rightarrow$ $\eta^{(\mathrm{NLL})} \approx \eta^{(\mathrm{NNLL})}$

- If increasing m_X^{cut} above $2\,\mathrm{GeV}$ hard $\Rightarrow$ keep $m_X^{\mathrm{cut}} < m_D$, normalize to $B \to X_u \ell\bar\nu$ with same cuts:

$$R = \Gamma^{\mathrm{cut}}(B \to X_s \ell^+\ell^-)/\Gamma^{\mathrm{cut}}(B \to X_u \ell\bar\nu)$$

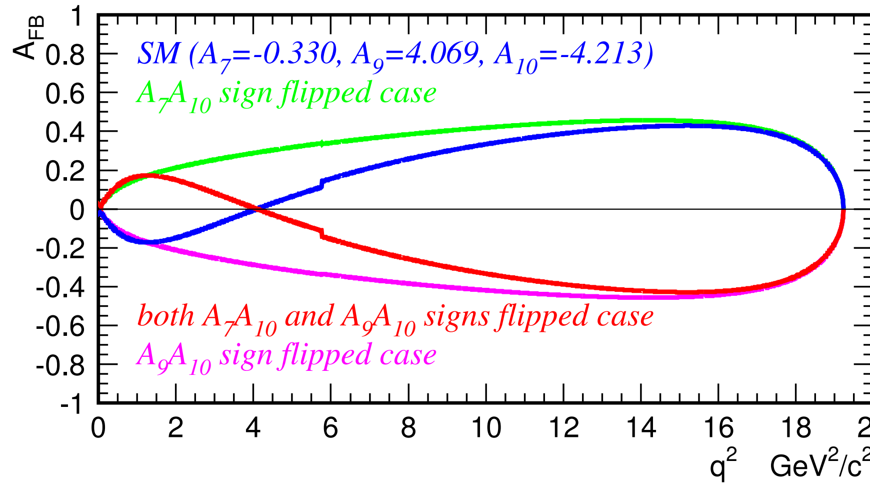
Both shape function (m_X^{cut}) and m_b dependence drastically reduced

Forward-backward asymmetry

- θ: angle between the ℓ^+ and the $\overline{B}^0, B^-$ momenta in the $\ell^+\ell^-$ rest frame

$$A_{\rm FB} = \int_{-1}^{1} {\rm d}\cos\theta \, \frac{{\rm d}^2\Gamma(B \to X_s \ell^+\ell^-)}{{\rm d}s\,{\rm d}\cos\theta}\, {\rm sgn}(\cos\theta)$$

$$A_{\rm FB}(s) \propto (1-s)^2\, C_{10} \left[2C_7^{\rm eff} + s\, {\rm Re}C_9^{\rm eff}(s) \right]$$



- No sensitivity in ${\rm d}\Gamma/{\rm d}q^2$ to these terms, odd under $\cos\theta \to -\cos\theta$:

$$\frac{{\rm d}\Gamma}{{\rm d}s} \propto (1-s)^2 \left[(|C_9^{\rm eff}|^2 + C_{10}^2)(1+2s) + \frac{4|C_7^{\rm eff}|^2}{s}(2+s) + 12{\rm Re}(C_7^{\rm eff} C_9^{\rm eff *}) \right]$$

- $q_0^2 \approx 2m_b[\overline{m}_b C_7^{\rm eff}]/{\rm Re}C_9^{\rm eff}$ where $A_{\rm FB}(q_0^2) = 0$ is sensitive to different C_i's (& signs)

- Approximate universality $\Rightarrow$ $A_{\rm FB}$ only slightly affacted (not at all trivial a priori)

  We find $q_0^2 \approx 3\,{\rm GeV}^2$, somewhat lower than earlier results

Summary and conclusions

- To achieve maximal sensitivity to NP, separate rate: $1+\cos^2\theta$, $1-\cos^2\theta$, $\cos\theta$ (both inclusive & exclusive)

- Experimental m_X cut $\Rightarrow$ rate sensitive to the shape function for small q^2 region
  SF region: expansion for rate, not the amplitude, reorganize perturbation theory

- Must take hadronic effects into account using $B \to X_s\gamma$ and / or $B \to X_u\ell\bar\nu$ data
  Sensitivity to NP not reduced

- First model independent direct calculation:
  For $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ obtain: $\Gamma^{\text{cut}}\,\tau_B = \begin{cases} (1.20 \pm 0.15)\times 10^{-6}\,, & m_X^{\text{cut}} = 1.8\text{GeV} \\ (1.48 \pm 0.14)\times 10^{-6}\,, & m_X^{\text{cut}} = 2.0\text{GeV} \end{cases}$

- Fits to 2-dim distributions and zero of A_{FB} not essential, just a few integrated rates